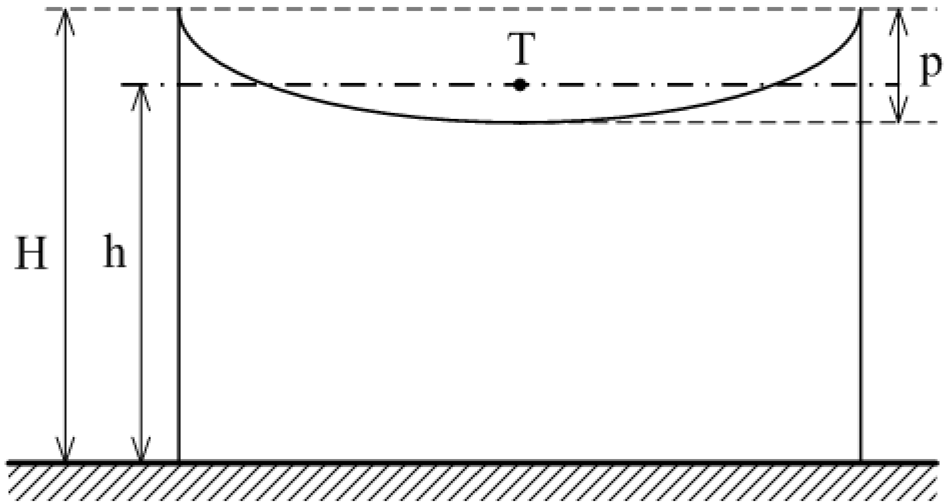
Capacity of overhead lines

Wires of the same straight line, parallel to each other and with the earth surface.



Catenary (cosh x) replaced by a straight line through the centre of gravity:

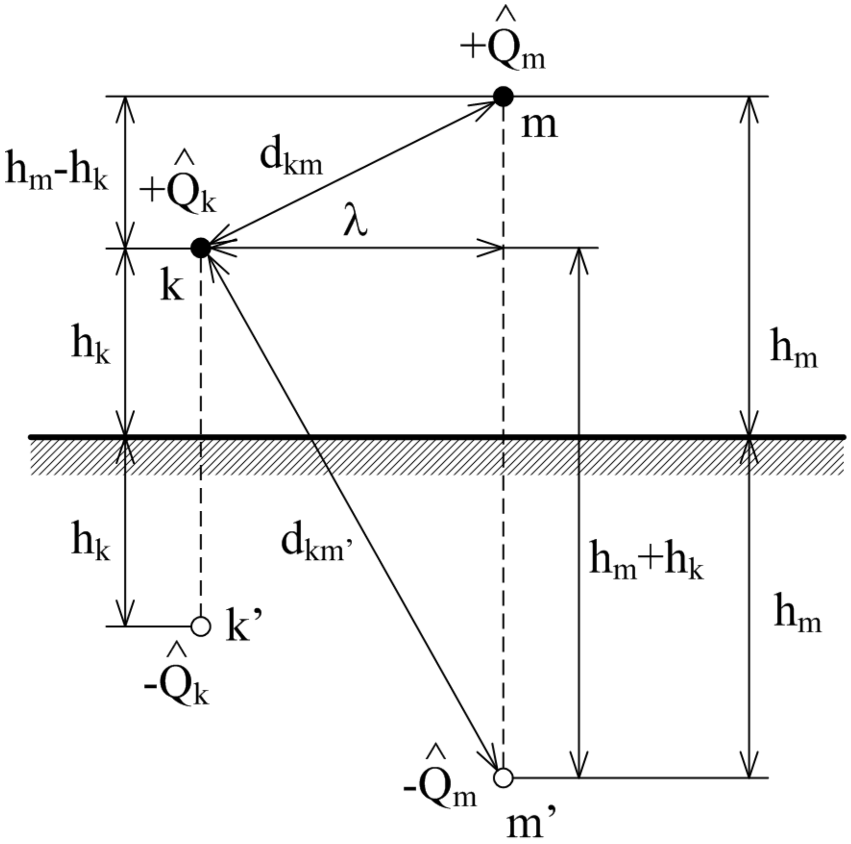
$$h = H - 0{,}7p \quad (m)$$

H…suspension height

p…sag

h…calculation height

El. potential at point P in the system of *n* parallel conductors ($d_{kk'} << l$) and ground with zero potential - mirror method



$$\hat{U}_P = \sum_{k=1}^{n} (\hat{U}_{Pk} + \hat{U}_{Pk'}) = \sum_{k=1}^{n} \frac{\hat{Q}_k}{2\pi\varepsilon} \ln \frac{d_{Pk'}}{d_{Pk}} \quad (V/m;\ C/m,\ m,\ m)$$

Point P on the surface of the real wire *k* ($r_k << d_{km}$):

$$\hat{U}_k = \sum_{m=1}^{n} \frac{\hat{Q}_m}{2\pi\varepsilon} \ln \frac{d_{km'}}{d_{km}} = \sum_{m=1}^{n} \delta_{mk} \hat{Q}_m$$

$$\left(d_{kk} = r_k ; d_{kk'} = 2h_k\right)$$

Point on the ground

$$d_{Zk} = d_{Zk'}$$

$$\hat{U}_Z = \sum_{m=1}^{n} \frac{\hat{Q}_m}{2\pi\varepsilon} \ln \frac{d_{Zm'}}{d_{Zm}} = \sum_{m=1}^{n} \frac{\hat{Q}_m}{2\pi\varepsilon} \ln 1 = 0$$

Self potential coefficient of the wire *k* (m=k)

$$\delta_{kk} = \frac{1}{2\pi\varepsilon} \ln \frac{2h_k}{r_k} \quad (\text{m} / \text{F}; \text{F} / \text{m}, \text{m}, \text{m})$$

The mutual potential coefficient ($m \neq k$)

$$\delta_{km} = \delta_{mk} = \frac{1}{2\pi\varepsilon} \ln \frac{d_{km'}}{d_{km}} \quad (m/F;\ F/m,\ m,\ m)$$

$$d_{km'} = \sqrt{(h_k + h_m)^2 + d_{km}^2 - (h_m - h_k)^2}$$

$$\delta_{km} = \delta_{mk} = \frac{1}{2\pi\varepsilon} \ln \frac{\sqrt{4h_k h_m + d_{km}^2}}{d_{km}}$$

Modified

$$\varepsilon_0 = 8{,}854 \cdot 10^{-12} \approx \frac{10^{-9}}{36\pi} \ F/m\ ; \ \ \varepsilon_r = 1\ ; \ \ \ln x = 2{,}3 \log x$$

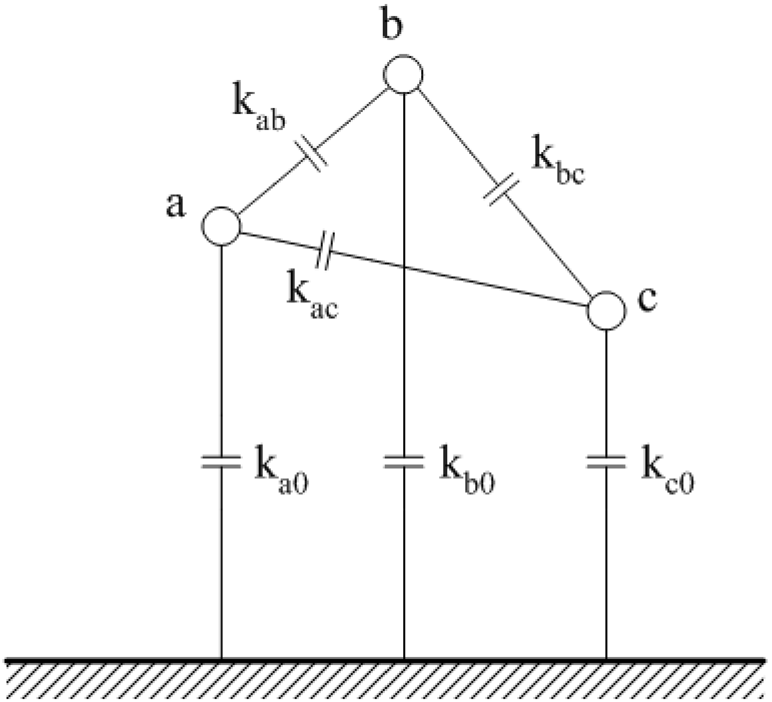
$$\Rightarrow \quad \delta_{kk} = \frac{1}{0{,}0242} \log \frac{2h_k}{r_k} \quad (km/\mu F)$$

$$\delta_{km} = \frac{1}{0{,}0242} \log \frac{\sqrt{4h_k h_m + d_{km}^2}}{d_{km}} \quad (km/\mu F)$$

Matrix

$$\left(\hat{U}\right) = \left(\delta_{km}\right)\left(\hat{Q}\right)$$

A simple three-phase line without ground wires



Partial capacity to the ground: k_{x0}

Partial mutual capacity: k_{xy}

Symmetrical voltage

$$\hat{U}_a = \hat{U}_a \qquad \hat{U}_b = \hat{a}^2 \hat{U}_a \qquad \hat{U}_c = \hat{a} \hat{U}_a$$

Charges of the individual wires

$$\hat{Q}_a = k_{a0}\hat{U}_a + k_{ab}\left(\hat{U}_a - \hat{U}_b\right) + k_{ac}\left(\hat{U}_a - \hat{U}_c\right)$$

$$\hat{Q}_b = k_{b0}\hat{U}_b + k_{ab}\left(\hat{U}_b - \hat{U}_a\right) + k_{bc}\left(\hat{U}_b - \hat{U}_c\right)$$

$$\hat{Q}_c = k_{c0}\hat{U}_c + k_{ac}\left(\hat{U}_c - \hat{U}_a\right) + k_{bc}\left(\hat{U}_c - \hat{U}_b\right)$$

Modified

$$\hat{Q}_a = \left(k_{a0} + k_{ab} + k_{ac}\right)\hat{U}_a - k_{ab}\hat{U}_b - k_{ac}\hat{U}_c$$

$$\hat{Q}_b = -k_{ab}\hat{U}_a + \left(k_{b0} + k_{ab} + k_{bc}\right)\hat{U}_b - k_{bc}\hat{U}_c$$

$$\hat{Q}_c = -k_{ac}\hat{U}_a - k_{bc}\hat{U}_b + \left(k_{c0} + k_{ac} + k_{bc}\right)\hat{U}_c$$

The introduction of capacity coefficients

$$\hat{Q}_a = c_{aa}\hat{U}_a + c_{ab}\hat{U}_b + c_{ac}\hat{U}_c$$

$$\hat{Q}_b = c_{ab}\hat{U}_a + c_{bb}\hat{U}_b + c_{bc}\hat{U}_c$$

$$\hat{Q}_c = c_{ac}\hat{U}_a + c_{bc}\hat{U}_b + c_{cc}\hat{U}_c$$

Matrix

$$\left(\hat{Q}\right) = \left(c_{km}\right)\left(\hat{U}\right)$$

$$\left(\hat{Q}\right) = \left(\delta_{km}\right)^{-1}\left(\hat{U}\right) \quad \Rightarrow \quad \left(c_{km}\right) = \left(\delta_{km}\right)^{-1}$$

Calculation procedure:

geometry $\rightarrow \left(\delta_{km}\right) \rightarrow \left(c_{km}\right) \rightarrow$ capacity

$$m = k: \quad k_{k0} = \sum_{m=1}^{n} c_{km}$$

$$m \neq k: \quad k_{km} = -c_{km}$$

Operational capacity - the *k*th conductor alone has the same charge as in the system of *n* conductors

$$\hat{C}_k = \frac{\hat{Q}_k}{\hat{U}_k} = \frac{k_{k0}\hat{U}_k + \sum_{m=1,\, m\neq k}^{n} k_{km}\left(\hat{U}_k - \hat{U}_m\right)}{\hat{U}_k}$$

$$\hat{C}_a = \frac{(k_{a0} + k_{ab} + k_{ac})\hat{U}_a - k_{ab}\hat{U}_b - k_{ac}\hat{U}_c}{\hat{U}_a}$$

$$\hat{C}_b = \frac{-k_{ab}\hat{U}_a + (k_{b0} + k_{ab} + k_{bc})\hat{U}_b - k_{bc}\hat{U}_c}{\hat{U}_b}$$

$$\hat{C}_c = \frac{-k_{ac}\hat{U}_a - k_{bc}\hat{U}_b + (k_{c0} + k_{ac} + k_{bc})\hat{U}_c}{\hat{U}_c}$$

Generally

$$k_{a0} \neq k_{b0} \neq k_{c0}$$

$$k_{ab} \neq k_{bc} \neq k_{ac}$$

$$\hat{C}_a \neq \hat{C}_b \neq \hat{C}_c$$

$\rightarrow$ current unbalance $\left(\hat{I}_{kc} = j\omega\hat{Q}_k\right) \rightarrow$ transposition

Transposed lines

The potential coefficients matrix

$$(\delta_{km}) = \frac{1}{3}\left\{\begin{pmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{12} & \delta_{22} & \delta_{23} \\ \delta_{13} & \delta_{23} & \delta_{33} \end{pmatrix} + \begin{pmatrix} \delta_{33} & \delta_{13} & \delta_{23} \\ \delta_{13} & \delta_{11} & \delta_{12} \\ \delta_{23} & \delta_{12} & \delta_{22} \end{pmatrix} + \begin{pmatrix} \delta_{22} & \delta_{23} & \delta_{12} \\ \delta_{23} & \delta_{33} & \delta_{13} \\ \delta_{12} & \delta_{13} & \delta_{11} \end{pmatrix}\right\}$$

Let's introduce

$$\delta = \frac{1}{3}(\delta_{11} + \delta_{22} + \delta_{33})$$

$$\delta = \frac{1}{0{,}0242}\log\frac{2h}{r} \quad (\mathrm{km}/\mu\mathrm{F})$$

mean geometrical height

$$h = \sqrt[3]{h_1 h_2 h_3}$$

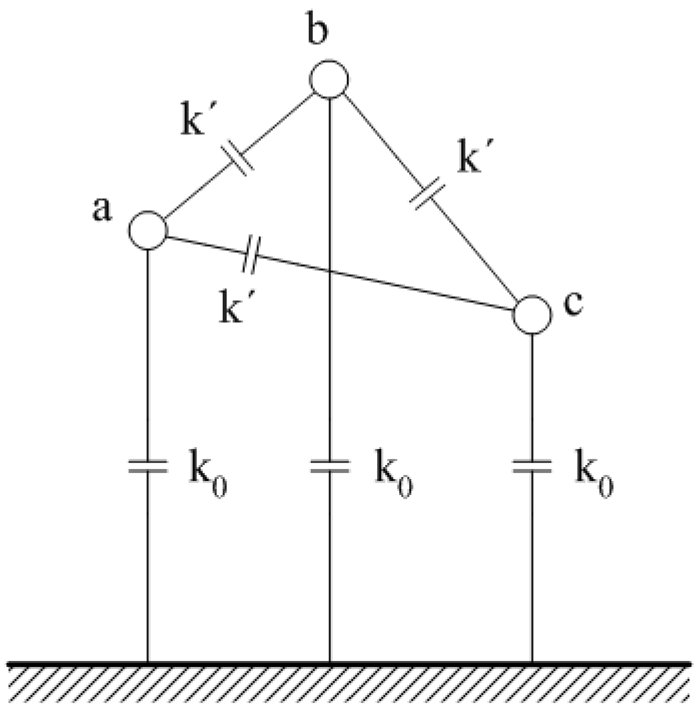
$$\delta' = \frac{1}{3}(\delta_{12} + \delta_{13} + \delta_{23})$$

$$\delta' = \frac{1}{0{,}0242}\log\frac{\sqrt{4h^2 + d^2}}{d} \quad (km/\mu F)$$

$$d = \sqrt[3]{d_{12}d_{13}d_{23}}$$

Then

$$\begin{pmatrix} \hat{U}_a \\ \hat{U}_b \\ \hat{U}_c \end{pmatrix} = \begin{pmatrix} \delta & \delta' & \delta' \\ \delta' & \delta & \delta' \\ \delta' & \delta' & \delta \end{pmatrix} \begin{pmatrix} \hat{Q}_a \\ \hat{Q}_b \\ \hat{Q}_c \end{pmatrix}$$



The diagrams include only 2 capacities

$$k_0 = k_{a0} = k_{b0} = k_{c0}$$

$$k' = k_{ab} = k_{bc} = k_{ac}$$

For the charges

$$\begin{pmatrix} \hat{Q}_a \\ \hat{Q}_b \\ \hat{Q}_c \end{pmatrix} = \begin{pmatrix} k_0 + 2k' & -k' & -k' \\ -k' & k_0 + 2k' & -k' \\ -k' & -k' & k_0 + 2k' \end{pmatrix} \begin{pmatrix} \hat{U}_a \\ \hat{U}_b \\ \hat{U}_c \end{pmatrix}$$

Solution:

Capacity to the ground

$$k_0 = \frac{1}{\delta + 2\delta'}$$

Capacity between conductors

$$k' = \frac{\delta'}{(\delta + 2\delta') \cdot (\delta - \delta')}$$

Operational capacity (real number)

$$C = C_a = C_b = C_c = k_0 + 3k'$$

$$C = \frac{1}{\delta - \delta'}$$

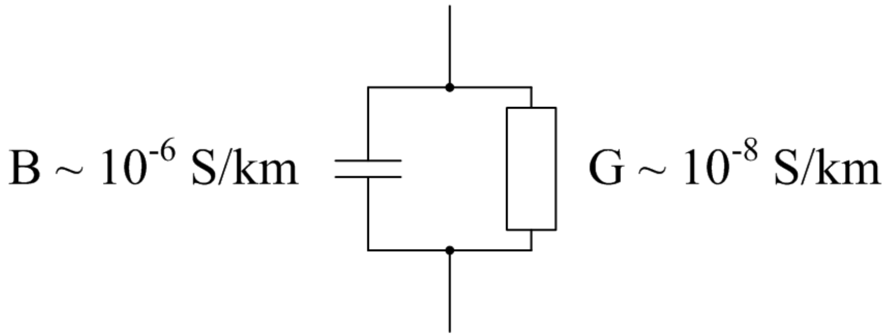
$$C = \frac{0,0242}{\log \frac{2hd}{r\sqrt{4h^2 + d^2}}} \quad (\mu F / km)$$

Influence of ground wires: k_0 increase, k' decrease, C no change

Influence of metal towers: k_0 increase (by 3-6%)

Values:

- 400 kV - $B_1 \approx (3,5 \div 4,5)\,\mu S \cdot km^{-1}$
- 110, 220 kV- $B_1 \approx (2,5 \div 3)\,\mu S \cdot km^{-1}$
- 22 kV- $B_1 \approx 1,4\,\mu S \cdot km^{-1}$



Conductance is negligible in relation to capacities.

Double power lines with two ground wires

$$\begin{pmatrix} \hat{U}_a \\ \hat{U}_b \\ \hat{U}_c \\ \hat{U}_A \\ \hat{U}_B \\ \hat{U}_C \\ \hat{U}_{z1} \\ \hat{U}_{z2} \end{pmatrix} = \begin{pmatrix} \delta_{aa} & \delta_{ab} & \delta_{ac} & \delta_{aA} & \delta_{aB} & \delta_{aC} & \delta_{az1} & \delta_{az2} \\ \delta_{ba} & \delta_{bb} & \delta_{bc} & \delta_{bA} & \delta_{bB} & \delta_{bC} & \delta_{bz1} & \delta_{bz2} \\ \delta_{ca} & \delta_{cb} & \delta_{cc} & \delta_{cA} & \delta_{cB} & \delta_{cC} & \delta_{cz1} & \delta_{cz2} \\ \delta_{Aa} & \delta_{Ab} & \delta_{Ac} & \delta_{AA} & \delta_{AB} & \delta_{AC} & \delta_{Az1} & \delta_{Az2} \\ \delta_{Ba} & \delta_{Bb} & \delta_{Bc} & \delta_{BA} & \delta_{BB} & \delta_{BC} & \delta_{Bz1} & \delta_{Bz2} \\ \delta_{Ca} & \delta_{Cb} & \delta_{Cc} & \delta_{CA} & \delta_{CB} & \delta_{CC} & \delta_{Cz1} & \delta_{Cz2} \\ \delta_{z1a} & \delta_{z1b} & \delta_{z1c} & \delta_{z1A} & \delta_{z1B} & \delta_{z1C} & \delta_{z1z1} & \delta_{z1z2} \\ \delta_{z2a} & \delta_{z2b} & \delta_{z2c} & \delta_{z2A} & \delta_{z2B} & \delta_{z2C} & \delta_{z2z1} & \delta_{z2z2} \end{pmatrix} \begin{pmatrix} \hat{Q}_a \\ \hat{Q}_b \\ \hat{Q}_c \\ \hat{Q}_A \\ \hat{Q}_B \\ \hat{Q}_C \\ \hat{Q}_{z1} \\ \hat{Q}_{z2} \end{pmatrix}$$

In blocks

$$\left(\hat{U}_v\right) = \left(\delta_{vv}\right)\left(\hat{Q}_v\right) + \left(\delta_{vV}\right)\left(\hat{Q}_V\right) + \left(\delta_{vz}\right)\left(\hat{Q}_z\right)$$

$$\left(\hat{U}_V\right) = \left(\delta_{Vv}\right)\left(\hat{Q}_v\right) + \left(\delta_{VV}\right)\left(\hat{Q}_V\right) + \left(\delta_{Vz}\right)\left(\hat{Q}_z\right)$$

$$0 = \left(\hat{U}_z\right) = \left(\delta_{zv}\right)\left(\hat{Q}_v\right) + \left(\delta_{zV}\right)\left(\hat{Q}_V\right) + \left(\delta_{zz}\right)\left(\hat{Q}_z\right)$$

We can calculate again with the modified power line without ground wires (for impedances transfer to symmetrical components system)

$$\left(\hat{Q}_z\right) = -\left(\delta_{zz}\right)^{-1}\left[\left(\delta_{zv}\right)\left(\hat{Q}_v\right) + \left(\delta_{zV}\right)\left(\hat{Q}_V\right)\right]$$

Operational states calculation

- power line A, B, C no-load state

$$\left(\hat{Q}_V\right) = 0$$

$$\left(\hat{U}_v\right) = \left(\delta_{vv}\right)\left(\hat{Q}_v\right) + \left(\delta_{vz}\right)\left(\hat{Q}_z\right)$$

$$\rightarrow \quad \begin{aligned} \left(\hat{U}_V\right) &= \left(\delta_{Vv}\right)\left(\hat{Q}_v\right) + \left(\delta_{Vz}\right)\left(\hat{Q}_z\right) \\ 0 &= \left(\delta_{zv}\right)\left(\hat{Q}_v\right) + \left(\delta_{zz}\right)\left(\hat{Q}_z\right) \end{aligned} \quad \rightarrow \quad \left(\hat{Q}_z\right) = -\left(\delta_{zz}\right)^{-1}\left(\delta_{zv}\right)\left(\hat{Q}_v\right)$$

Modified

$$\left(\hat{U}_v\right) = \left[\left(\delta_{vv}\right) - \left(\delta_{vz}\right)\left(\delta_{zz}\right)^{-1}\left(\delta_{zv}\right)\right]\left(\hat{Q}_v\right) = \left(\delta_{k1}\right)\left(\hat{Q}_v\right)$$

$$\left(\hat{U}_V\right) = \left[\left(\delta_{Vv}\right) - \left(\delta_{Vz}\right)\left(\delta_{zz}\right)^{-1}\left(\delta_{zv}\right)\right]\left(\hat{Q}_v\right) = \left(\delta_{k2}\right)\left(\hat{Q}_v\right)$$

Voltages induced on the no-load lines by capacitive couplings

$$\left(\hat{Q}_v\right) = \left(\delta_{k1}\right)^{-1}\left(\hat{U}_v\right)$$

$$\left(\hat{U}_V\right) = \left(\delta_{k2}\right)\left(\delta_{k1}\right)^{-1}\left(\hat{U}_v\right)$$

Currents flowing to the ground wires through capacitive couplings

$$\left(\hat{I}_{zc}\right) = j\omega\left(\hat{Q}_z\right) = -\left(\delta_{zz}\right)^{-1}\left(\delta_{zv}\right)\left(\delta_{k1}\right)^{-1}\left(\hat{U}_v\right)$$

Electrical parameters of cables

Resistance

The same as for overhead lines.
Single-core – R increased due to eddy currents and hysteresis losses in the metal case (screen).

Inductance

Three-core – the same as for transposed power lines.

$$L_1 = 0{,}46 \log \frac{d}{\xi r} \quad (mH/km)$$

Not valid d >> r → L values less precise but technically applicable.

6 kV	$X \sim 0{,}06\ \Omega/km$	$X_0 \approx X_1$ for three-core
22 kV	$X \sim 0{,}1\ \Omega/km$	$X_0 \approx 3X_1$ for single-core

Conductance

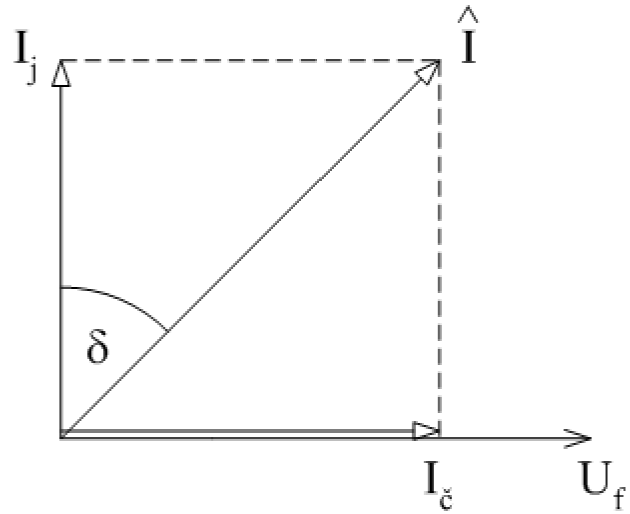
Determined by dielectric losses.

3 phase

$$P_d = 3U_f I_č = 3U_f I_j tg\delta \quad (W)$$

$$P_d = 3U_f \omega C U_f tg\delta = \omega C U^2 tg\delta = Q_c tg\delta$$

Q_c…charging power



Conductance per length unit

$$G_1 = \frac{P_{d1}}{U^2} \quad (S/km;\ W/km,\ V)$$

Capacities

3 cable types:

a) full plastic (without conducting screen)
b) single-core with a metal screen or multi-core with a screen for each conductor
c) three-core with a common metal screen

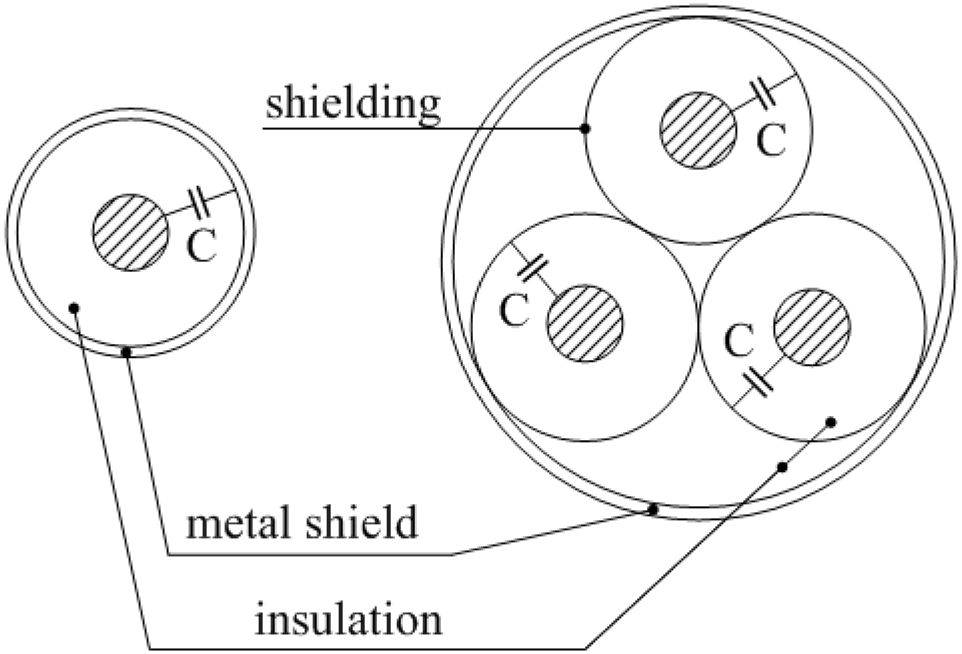
ad a)
C is changing with cable placement and environment. It is measured.

ad b)
Only capacity of the conductor to the screen = operational.
For coaxial cylinder

$$C = \frac{0{,}0242\varepsilon_r}{\log \frac{r_2}{r_1}} \quad (\mu F / km)$$

ε_r…insulation relative permittivity

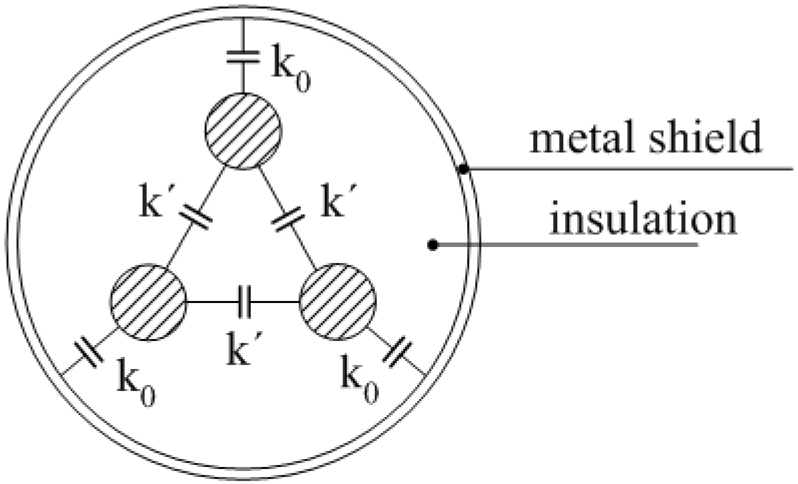
r_1…conductor radius
r_2…screen mean radius

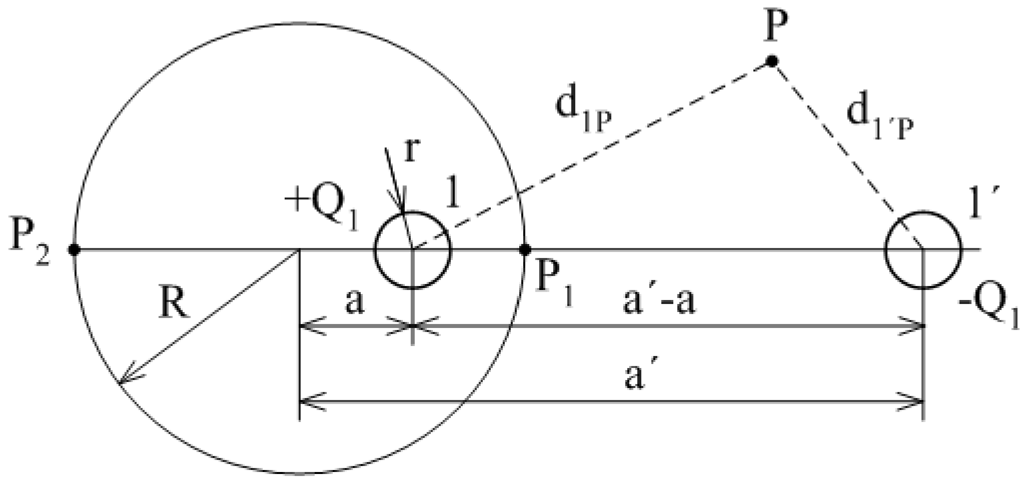


ad c)

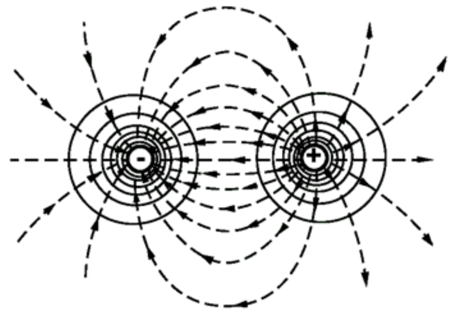
As three phase symmetrical power lines.

Mirror method x along the metal screen.





Potential in the point P (from 1, 1´)

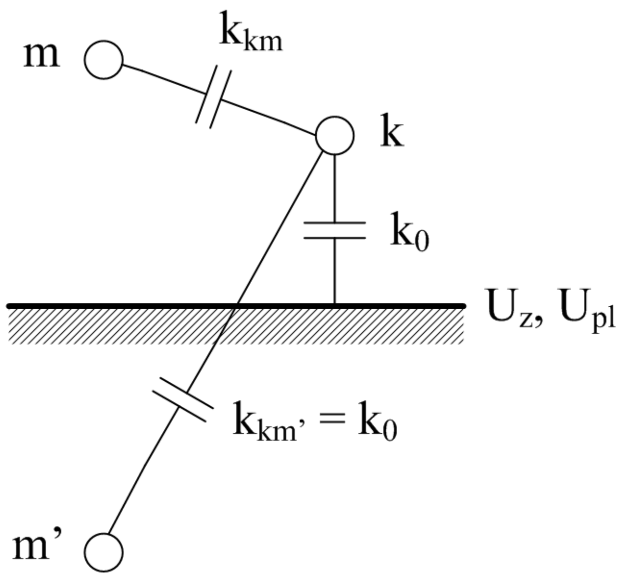


$$\hat{U}_P = \frac{\hat{Q}_1}{2\pi\varepsilon} \ln \frac{d_{1'P}}{d_{1P}}$$

Potential on the screen surface is the same everywhere (from 1, 1´)

$$\hat{U}_{pl} = \hat{U}_{P1} = \hat{U}_{P2}$$

$$\frac{\hat{Q}_1}{2\pi\varepsilon}\ln\frac{a'-R}{R-a}=\frac{\hat{Q}_1}{2\pi\varepsilon}\ln\frac{a'+R}{R+a}$$



Hence

$$a'=\frac{R^2}{a}$$

$$\hat{U}_{pl}=\frac{\hat{Q}_1}{2\pi\varepsilon}\ln\frac{R}{a}$$

Capacities k_0 to the screen

→ contribution to the conductor *k* potential from the conductors *m* and *m′*

$$\hat{U}'_{km}\approx(\hat{U}_k-\hat{U}_m)+(\hat{U}_k-\hat{U}_{m'})\approx(\hat{U}_k-\hat{U}_m)+(\hat{U}_k-\hat{U}_z)$$

$$\rightarrow\quad \hat{U}'_{km}=\hat{U}_{km}-\hat{U}_{pl}=\frac{\hat{Q}_1}{2\pi\varepsilon}\ln\frac{d_{km'}}{d_{km}}-\frac{\hat{Q}_1}{2\pi\varepsilon}\ln\frac{R}{a}=\frac{\hat{Q}_1}{2\pi\varepsilon}\ln\frac{d_{km'}\cdot a}{d_{km}\cdot R}$$

$$\delta^{x}_{km} = \delta^{x}_{mk} = \frac{1}{2\pi\varepsilon} \ln \frac{d_{km'} \cdot a}{d_{km} \cdot R}$$

Dimensions

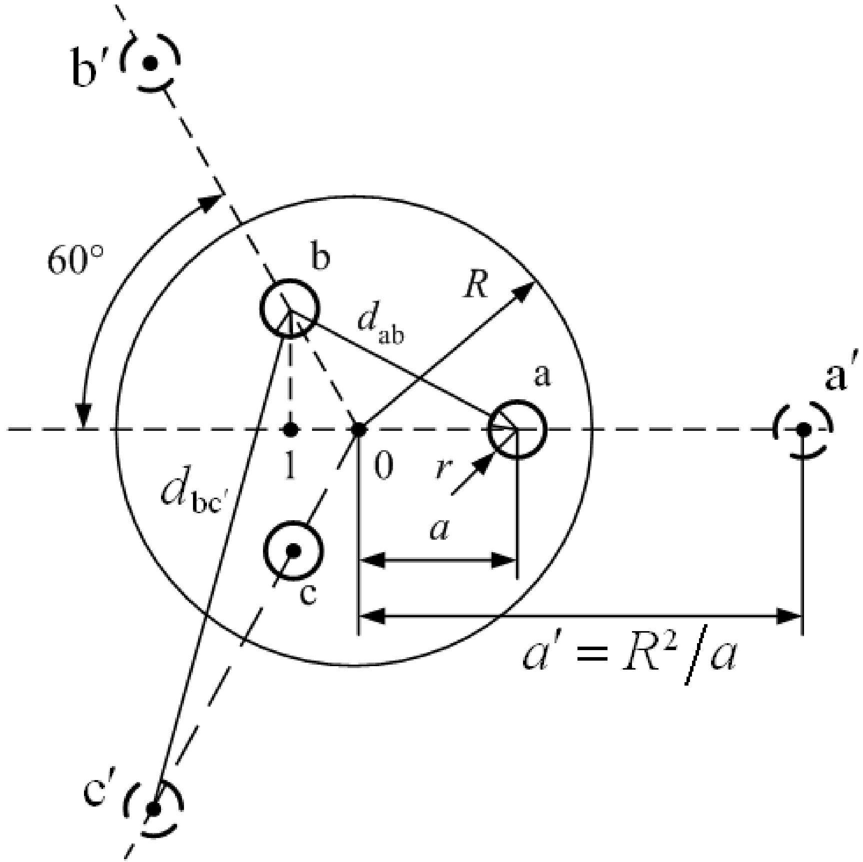
$$d_{kk} = r$$

$$d_{km} = a\sqrt{3}$$

$$d_{kk'} = a' - a = \frac{R^2 - a^2}{a}$$

$$d_{km'} = \sqrt{(a' + a\cos 60°)^2 + (a\sin 60°)^2}$$

$$= R\sqrt{1 + \frac{R^2}{a^2} + \frac{a^2}{R^2}}$$



Potential coefficients

$$\delta = \frac{1}{0{,}0242\varepsilon_r} \log \frac{R^2 - a^2}{R \cdot r} \quad (\mathrm{km}/\mu\mathrm{F})$$

$$\delta' = \frac{1}{0{,}0242\varepsilon_r} \log \sqrt{\frac{1 + \left(\frac{R}{a}\right)^2 + \left(\frac{a}{R}\right)^2}{3}} \quad (\mathrm{km}/\mu\mathrm{F})$$

Capacity to the screen

$$k_0 = \frac{1}{\delta + 2\delta'}$$

Mutual capacity

$$k' = \frac{\delta'}{(\delta + 2\delta') \cdot (\delta - \delta')}$$

Operational capacity

$$C = \frac{1}{\delta - \delta'}$$

Cable capacities are much higher than for overhead lines (c. 30÷50 times) → limited lengths of cable networks because of charging currents (10x km).

- 22kV - $B_1 \approx (70 \div 90)\,\mu S \cdot km^{-1}$