Symmetrical system components

Decomposition of unsymmetrical (unbalanced) voltage:

$$\hat{U}_{A} = \hat{U}_{A1} + \hat{U}_{A2} + \hat{U}_{A0}$$

$$\hat{U}_{B} = \hat{U}_{B1} + \hat{U}_{B2} + \hat{U}_{B0}$$

$$\hat{U}_{C} = \hat{U}_{C1} + \hat{U}_{C2} + \hat{U}_{C0}$$

$$\hat{U}_{B} = \hat{U}_{B1} + \hat{U}_{B2} + \hat{U}_{B0}$$

$$\hat{U}_{C} = \hat{U}_{C1} + \hat{U}_{C2} + \hat{U}_{C0}$$

$$\hat{U}_{C} = \hat{U}_{C1} + \hat{U}_{C2} + \hat{U}_{C0}$$

Positive sequence (1), negative (2) and zero (0) sequence.

Hence (reference phase A)

$$\hat{U}_{A} = \hat{U}_{1} + \hat{U}_{2} + \hat{U}_{0} \qquad \qquad \hat{I}_{A} = \hat{I}_{1} + \hat{I}_{2} + \hat{I}_{0}$$

$$\hat{U}_{B} = \hat{a}^{2} \hat{U}_{1} + \hat{a} \hat{U}_{2} + \hat{U}_{0} \qquad \qquad \hat{I}_{B} = \hat{a}^{2} \hat{I}_{1} + \hat{a} \hat{I}_{2} + \hat{I}_{0}$$

$$\hat{U}_{C} = \hat{a} \hat{U}_{1} + \hat{a}^{2} \hat{U}_{2} + \hat{U}_{0} \qquad \qquad \hat{I}_{C} = \hat{a} \hat{I}_{1} + \hat{a}^{2} \hat{I}_{2} + \hat{I}_{0}$$

$$\hat{I}_{C} = \hat{a} \hat{I}_{1} + \hat{a}^{2} \hat{I}_{2} + \hat{I}_{0}$$

$$\hat{I}_{C} = \hat{a} \hat{I}_{1} + \hat{a}^{2} \hat{I}_{2} + \hat{I}_{0}$$

$$\hat{I}_{C} = \hat{a} \hat{I}_{1} + \hat{a}^{2} \hat{I}_{2} + \hat{I}_{0}$$

$$\hat{I}_{C} = \hat{a} \hat{I}_{1} + \hat{a}^{2} \hat{I}_{2} + \hat{I}_{0}$$

$$\hat{I}_{C} = \hat{a} \hat{I}_{1} + \hat{a}^{2} \hat{I}_{2} + \hat{I}_{0}$$

$$\hat{I}_{C} = \hat{a} \hat{I}_{1} + \hat{a}^{2} \hat{I}_{2} + \hat{I}_{0}$$

$$\hat{I}_{C} = \hat{a} \hat{I}_{1} + \hat{a}^{2} \hat{I}_{2} + \hat{I}_{0}$$

$$\hat{I}_{C} = \hat{a} \hat{I}_{1} + \hat{a}^{2} \hat{I}_{2} + \hat{I}_{0}$$

$$\hat{I}_{C} = \hat{a} \hat{I}_{1} + \hat{a}^{2} \hat{I}_{2} + \hat{I}_{0}$$

$$\hat{I}_{C} = \hat{a} \hat{I}_{1} + \hat{a}^{2} \hat{I}_{2} + \hat{I}_{0}$$

$$\hat{I}_{C} = \hat{a} \hat{I}_{1} + \hat{a}^{2} \hat{I}_{2} + \hat{I}_{0}$$

$$\hat{I}_{C} = \hat{a} \hat{I}_{1} + \hat{a}^{2} \hat{I}_{2} + \hat{I}_{0}$$

$$\hat{I}_{C} = \hat{a} \hat{I}_{1} + \hat{a}^{2} \hat{I}_{2} + \hat{I}_{0}$$

Matrix

$$(U_{ABC}) = \begin{pmatrix} \hat{U}_{A} \\ \hat{U}_{B} \\ \hat{U}_{C} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ \hat{a}^{2} & \hat{a} & 1 \\ \hat{a} & \hat{a}^{2} & 1 \end{pmatrix} \begin{pmatrix} \hat{U}_{1} \\ \hat{U}_{2} \\ \hat{U}_{0} \end{pmatrix} = (T)(U_{120})$$

Inversely

$$(U_{120}) = \begin{pmatrix} \hat{U}_1 \\ \hat{U}_2 \\ \hat{U}_0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & \hat{a} & \hat{a}^2 \\ 1 & \hat{a}^2 & \hat{a} \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \hat{U}_A \\ \hat{U}_B \\ \hat{U}_C \end{pmatrix} = (T^{-1})(U_{ABC})$$

3ph power

$$\hat{\mathbf{S}} = \hat{\mathbf{U}}_{\mathbf{A}} \hat{\mathbf{I}}_{\mathbf{A}}^* + \hat{\mathbf{U}}_{\mathbf{B}} \hat{\mathbf{I}}_{\mathbf{B}}^* + \hat{\mathbf{U}}_{\mathbf{C}} \hat{\mathbf{I}}_{\mathbf{C}}^* = (\hat{\mathbf{U}}_{\mathbf{A}} \quad \hat{\mathbf{U}}_{\mathbf{B}} \quad \hat{\mathbf{U}}_{\mathbf{C}})^{\mathsf{T}} \hat{\mathbf{I}}_{\mathbf{B}} = (\mathbf{U}_{\mathbf{A}\mathbf{B}\mathbf{C}})^{\mathsf{T}} (\mathbf{I}_{\mathbf{A}\mathbf{B}\mathbf{C}})^*$$

$$\hat{\mathbf{S}} = [(\mathbf{T})(\mathbf{U}_{120})]^{T} [(\mathbf{T})(\mathbf{I}_{120})]^{*} = (\mathbf{U}_{120})^{T} (\mathbf{T})^{T} (\mathbf{T})^{*} (\mathbf{I}_{120})^{*}$$

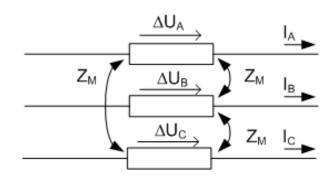
$$(T)^{T}(T)^{*} = \begin{pmatrix} 1 & \hat{a}^{2} & \hat{a} \\ 1 & \hat{a} & \hat{a}^{2} \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ \hat{a} & \hat{a}^{2} & 1 \\ \hat{a}^{2} & \hat{a} & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} = 3(E)$$

$$\hat{S} = 3(U_{120})^T (I_{120})^*$$

$$\hat{S} = 3(\hat{U}_1\hat{I}_1^* + \hat{U}_2\hat{I}_2^* + \hat{U}_0\hat{I}_0^*)$$

Series symmetrical segments in ES

$$\begin{pmatrix} \Delta \hat{\mathbf{U}}_{\mathrm{A}} \\ \Delta \hat{\mathbf{U}}_{\mathrm{B}} \\ \Delta \hat{\mathbf{U}}_{\mathrm{C}} \end{pmatrix} = \begin{pmatrix} \hat{\mathbf{Z}} & \hat{\mathbf{Z}}' & \hat{\mathbf{Z}}' \\ \hat{\mathbf{Z}}' & \hat{\mathbf{Z}} & \hat{\mathbf{Z}}' \\ \hat{\mathbf{Z}}' & \hat{\mathbf{Z}}' & \hat{\mathbf{Z}} \end{pmatrix} \begin{pmatrix} \hat{\mathbf{I}}_{\mathrm{A}} \\ \hat{\mathbf{I}}_{\mathrm{B}} \\ \hat{\mathbf{I}}_{\mathrm{C}} \end{pmatrix}$$



$$(\Delta U_{ABC}) = (Z_{ABC})(I_{ABC})$$

$$(T)(\Delta U_{120}) = (Z_{ABC})(T)(I_{120})$$

$$(\Delta U_{120}) = (T)^{-1} (Z_{ABC})(T)(I_{120}) = (Z_{120})(I_{120})$$

$$(Z_{120}) = (T)^{-1} (Z_{ABC})(T)$$

$$(Z_{120}) = \begin{pmatrix} \hat{Z}_1 & 0 & 0 \\ 0 & \hat{Z}_2 & 0 \\ 0 & 0 & \hat{Z}_0 \end{pmatrix} = \begin{pmatrix} \hat{Z} - \hat{Z}' & 0 & 0 \\ 0 & \hat{Z} - \hat{Z}' & 0 \\ 0 & 0 & \hat{Z} + 2\hat{Z}' \end{pmatrix}$$

Multiple lines

$$\begin{pmatrix} \left(\Delta U_{V1}\right) \\ \left(\Delta U_{V2}\right) \\ \dots \end{pmatrix}_{ABC} = \begin{pmatrix} \left(I_{V1}\right) \\ \left(I_{V2}\right) \\ \dots \end{pmatrix}_{ABC}$$

$$\begin{pmatrix} \left(T\right) & 0 & 0 \\ 0 & \left(T\right) & 0 \\ 0 & 0 & \dots \end{pmatrix} \begin{pmatrix} \left(\Delta U_{V1}\right) \\ \left(\Delta U_{V2}\right) \\ \dots \end{pmatrix}_{120} = \begin{pmatrix} \left(Z_{ABC}\right) \begin{pmatrix} \left(T\right) & 0 & 0 \\ 0 & \left(T\right) & 0 \\ 0 & 0 & \dots \end{pmatrix} \begin{pmatrix} \left(I_{V1}\right) \\ \left(I_{V2}\right) \\ \dots \end{pmatrix}_{120}$$

$$\begin{pmatrix} \left(\Delta U_{V1}\right) \\ \left(\Delta U_{V2}\right) \\ \dots \end{pmatrix}_{120} = \begin{pmatrix} \left(T\right) & 0 & 0 \\ 0 & \left(T\right) & 0 \\ 0 & 0 & \dots \end{pmatrix} \begin{pmatrix} \left(T\right) & 0 & 0 \\ \left(Z_{ABC}\right) \begin{pmatrix} \left(T\right) & 0 & 0 \\ 0 & \left(T\right) & 0 \\ 0 & 0 & \dots \end{pmatrix} \begin{pmatrix} \left(I_{V1}\right) \\ \left(I_{V2}\right) \\ \dots \end{pmatrix}_{120}$$

$$\begin{pmatrix} \left(\Delta U_{V1}\right) \\ \left(\Delta U_{V2}\right) \\ \dots \end{pmatrix}_{120} = \begin{pmatrix} \left(T\right)^{-1} & 0 & 0 \\ 0 & \left(T\right)^{-1} & 0 \\ 0 & 0 & \dots \end{pmatrix} \begin{pmatrix} \left(T\right) & 0 & 0 \\ \left(Z_{ABC}\right) \begin{pmatrix} \left(T\right) & 0 & 0 \\ 0 & \left(T\right) & 0 \\ 0 & 0 & \dots \end{pmatrix} \begin{pmatrix} \left(I_{V1}\right) \\ \left(I_{V2}\right) \\ \dots \end{pmatrix}_{120}$$

Shunt symmetrical segments in ES

$$\begin{aligned} &(U_{ABC}) = (Z_{ABC})(I_{ABC}) + (Z_{N})(I_{ABC}) \\ &(Z_{N}) = \begin{pmatrix} \hat{Z}_{N} & \hat{Z}_{N} & \hat{Z}_{N} \\ \hat{Z}_{N} & \hat{Z}_{N} & \hat{Z}_{N} \\ \hat{Z}_{N} & \hat{Z}_{N} & \hat{Z}_{N} \end{pmatrix} \\ &(U_{120}) = (T)^{-1}(Z_{ABC})(T)(I_{120}) + (T)^{-1}(Z_{N})(T)(I_{120}) \\ &(Z_{120}) = \begin{pmatrix} \hat{Z} - \hat{Z}' & 0 & 0 \\ 0 & \hat{Z} - \hat{Z}' & 0 \\ 0 & 0 & \hat{Z} + 2\hat{Z}' + 3Z_{N} \end{pmatrix} \end{aligned}$$

Symmetrical components voltages in the symmetrical segments depend only on the corresponding component current and component impedance.

Inductors and capacitors in ES

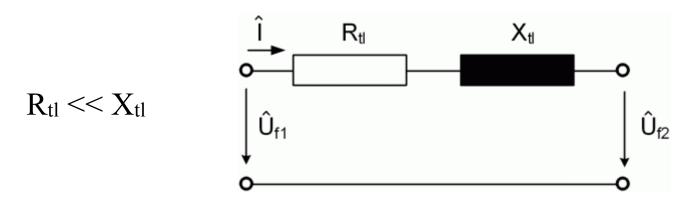
a) Series inductors

- reactors are used to limit short-circuit currents
- used in grids up to 35 kV, single-phase ($I_n > 200A$) or three-phase ($I_n < 200A$), usually air-cooled (small L)
- the same design in LC filters for harmonics suppression









Input: X_{tl}%, S_{tl}, U_n, I_n

Calculation:
$$S_{tl} = \sqrt{3}.U_n.I_n$$

$$\begin{split} X_{tl} &= \frac{X_{t\%} \cdot U_{n}}{100 \cdot \sqrt{3} \cdot I_{n}} = \frac{X_{t\%} \cdot U_{n}^{2}}{100 \cdot S_{tl}} \\ \Delta \hat{U}_{f} &= \hat{U}_{fl} - \hat{U}_{f2} = (R_{t} + jX_{t})\hat{I} = \hat{Z}_{t}\hat{I} \\ & \left[\hat{Z}_{tabc}\right] = \left[\hat{Z}_{t012}\right] = \hat{Z}_{t} \cdot [E] - 3ph \ inductor \end{split}$$

 \rightarrow self-impedance \hat{Z}_t , mutual impedances 0

In fault-free state the inductor can be bypassed by a fuse to reduce a voltage drop.

b) Shunt (parallel) inductors

- in the systems $U_N > 220 \text{ kV}$, oil cooling, Fe core
- used to compensate capacitive (charging) currents of overhead lines for no-load or small loads → U control:

$$X_{tl} = \frac{U_{tln}}{\sqrt{3} \cdot I_{tln}} = \frac{U_{tln}^{2}}{Q_{tln}}$$

$$\hat{Z}_{tl} = \hat{Z}_{tl1} = \hat{Z}_{t2}, Z_{tl0} \rightarrow \infty$$

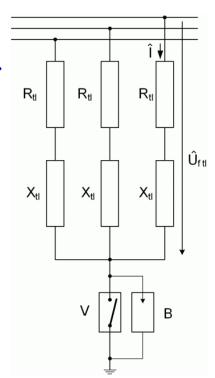
Connection in the system:

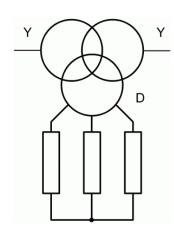
a) galvanic connection to the line

- Y winding, neutral point connected to the ground through V only during auto-reclosing (disturbances)

b) inductor connection to transformer tertiary winding

- lower voltage $U_n \approx 10 \div 35 \text{ kV}$
- problem with switch-off (purely inductive load)



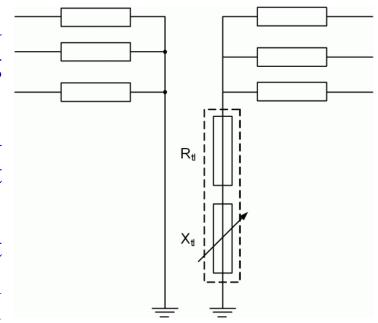


Kočín 400 kV



c) Neutral point inductors

- used in networks with indirectly earthed neutral point to compensate currents during ground fault
- value of the fault current does not depend on the ground fault position and the current is capacitive
- inductor reactance X_{tl} should assure that the value of the inductive current is equal to the value of capacitive current \rightarrow arc extinction



- for voltage 6 to 35 kV (rated at U_{fn}), reactor is single-phased!, oil cooling
- capacitive current change (network reconfiguration) → change in inductance (air gap correction in the magnetic circuit)
 - = arc-suppression coil (Peterson coil)
- it doesn't occur in positive and negative sequence component, $X_0 = 3X_{tl}$

6 MVAr, 13 kV, Sokolnice

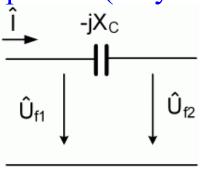




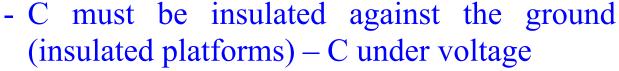
d) Series capacitors

- capacitors in ES = capacitor banks = series and parallel connection
- to improve voltage conditions (MV lines) or adjusting parameters (long HV lines)
- voltage and power of the capacitor varies with the load
- during short-circuits and overcurrents there appears overvoltage on the

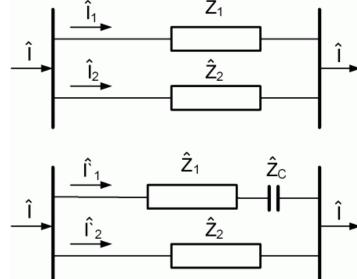
capacitor (very fast protections are used)



$$\hat{\mathbf{U}}_{\mathbf{C}} = -\mathbf{j} \frac{1}{\omega \mathbf{C}} \hat{\mathbf{I}}$$



- drawback allows harmonic currents flow
- current distribution among parallel transmission lines could be achieved



Canada 750 kV





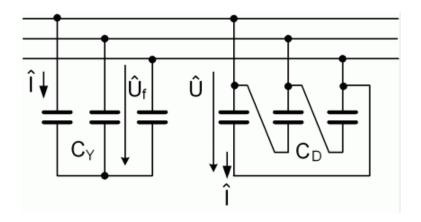
e) Shunt capacitors

- used in industrial networks up to 1 kV

- connection:

a) wye - Y

b) delta - Δ (D)



$$\begin{aligned} Q_f &= U_f \cdot I_C = U_f^2 \omega C_Y & Q_f &= U \cdot I_C = U^2 \omega C_\Delta \\ Q &= 3U_f^2 \omega C_Y = U^2 \omega C_Y & Q &= 3U^2 \omega C_\Delta \end{aligned}$$

- with the same reactive power

$$U^2 \omega C_Y = 3U^2 \omega C_\Delta \rightarrow C_Y = 3C_\Delta \rightarrow rather delta$$

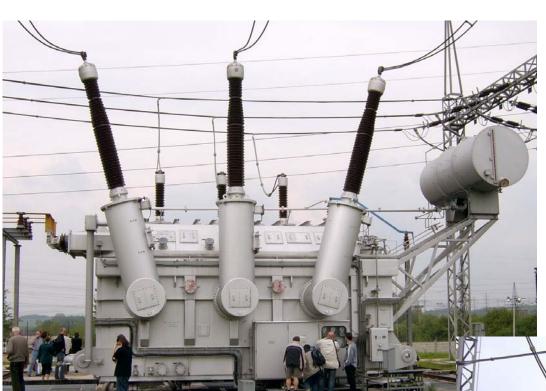


- reactive power compensation
- a) $Q_C \le Q$ under-compensated
- b) $Q_C = Q$ exact compensation
- c) $Q_C > Q$ over-compensated
- → power factor improvement, lower power losses, voltage drops
- individual or group compensation could be used

Transformer parameters







350 MVA, 400/110 kV YNauto - d1, Sokolnice



a) Two-winding transformers

- winding connection Y, Yn, D, Z, Zn, V

Yzn – distribution TRF MV/LV up to 250 kVA, for unbalanced load

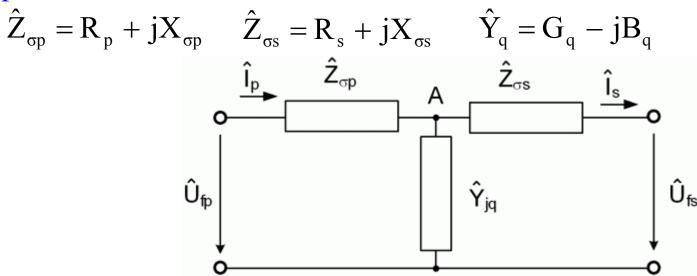
Dyn – distribution TRF MV/LV from 400 kVA

Yd – block TRF in power plants, the 3rd harmonic suppression

Yna-d, YNynd – power grid transformer (400, 220, 110 kV)

YNyd – power grid transformer (e.g. 110/23/6,3 kV)

- equivalent circuit: T – network



- each phase can be considered separately (unbalance is neglected), i.e. operational impedance (positive sequence) is used
- values of the parameters are calculated, then verified by no-load and short-circuit tests
 - o *no-load test* secondary winding open, primary w. supplied by rated voltage, no-load current flows (lower than rated one)
 - o short-circuit test secondary winding short-circuit, primary w. supplied by short-circuit voltage (lower than rated one) so that rated current flows

```
\Delta P_0 (W), i_0 (%), \Delta P_k (W), z_k = u_k (%), S_n (VA), U_n (V) u_k \approx 4 \div 14 % (increases with TRF power) p_k \approx 0.1 \div 1 % (decreases with TRF power) p_0 \approx 0.01 \div 0.1 % (decreases with TRF power)
```

- shunt branch:

$$g_{q} = \frac{\Delta P_{0}}{S_{n}}$$
 $y_{q} = \frac{i_{0\%}}{100}$ $b_{q} = \sqrt{y_{q}^{2} - g_{q}^{2}}$

$$\hat{y}_{q} = \frac{\Delta P_{0}}{S_{n}} - j \sqrt{\left(\frac{i_{0\%}}{100}\right)^{2} - \left(\frac{\Delta P_{0}}{S_{n}}\right)^{2}} = g_{q} - j \cdot b_{q}$$

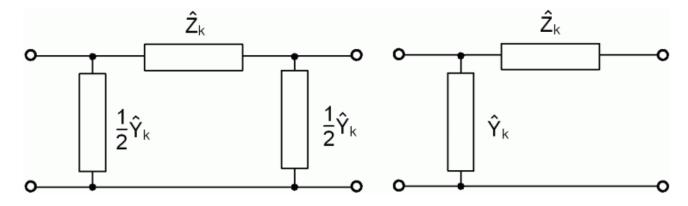
$$\hat{Y}_{q} = \hat{y}_{q} \frac{S_{n}}{U_{n}^{2}} = \frac{S_{n}}{U_{n}^{2}} \left[\frac{\Delta P_{0}}{S_{n}} - j \sqrt{\left(\frac{i_{0\%}}{100}\right)^{2} - \left(\frac{\Delta P_{0}}{S_{n}}\right)^{2}} \right] = G_{q} - j \cdot B_{q}$$

- series branch:

$$\begin{split} r_k &= \frac{\Delta P_k}{S_n} \qquad z_k = \frac{u_{k\%}}{100} \qquad x_k = \sqrt{z_k^2 - r_k^2} \\ \hat{z}_k &= \frac{\Delta P_k}{S_n} + j \sqrt{\left(\frac{u_{k\%}}{100}\right)^2 - \left(\frac{\Delta P_k}{S_n}\right)^2} = r_k + j \cdot x_k \\ \hat{Z}_k &= \hat{z}_k \frac{U_n^2}{S_n} = \frac{U_n^2}{S_n} \Bigg[\frac{\Delta P_k}{S_n} + j \sqrt{\left(\frac{u_{k\%}}{100}\right)^2 - \left(\frac{\Delta P_k}{S_n}\right)^2} \Bigg] = R_k + j \cdot X_k \\ \hat{Z}_{\sigma p s} &= \hat{Z}_k = \left(R_p + R_s\right) + j \left(X_{\sigma p} + X_{\sigma s}\right) \\ - \text{ we choose } \hat{Z}_{\sigma p} = 0.5 \hat{Z}_{\sigma p s} = \hat{Z}_{\sigma s} \end{split}$$

- this division is not physically correct (different leakage flows, different resistances)

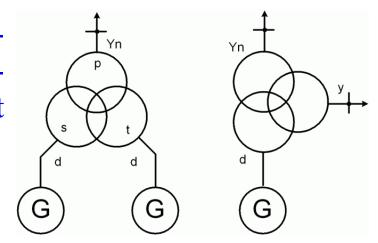
- usage of T-network to calculate meshed systems is not appropriate sometimes (it adds another node)
- therefore calculation using π -network, Γ -network



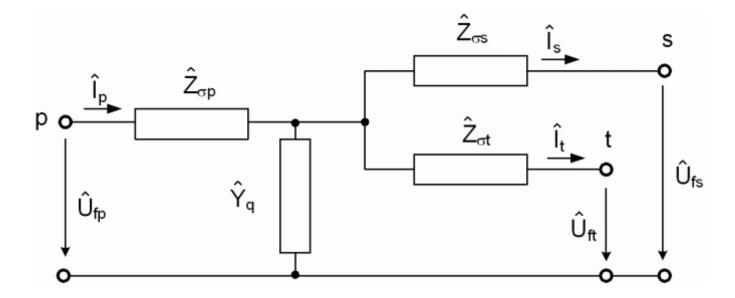
b) Three-winding transformers

- parameters are calculated, then verified by noload and short-circuit measurements (3 shortcircuit tests: 1 winding no-load, 1 short-circuit and 1 supplied):

$$\begin{split} &\Delta P_{0}\left(W\right),\,i_{0}\left(\%\right),\,\Delta P_{k}\left(W\right),\,z_{K}=u_{K}\left(\%\right),\\ &S_{n}\left(VA\right),\,U_{n}\left(V\right) \end{split}$$



- powers needn't be the same: e.g. $S_{Sn} = S_{Tn} = 0.5 \cdot S_{Pn}$
- equivalent circuit:



- <u>no-load measurement:</u> related to the primary rated power and rated voltage S_{Pn} a U_{PN} (supplied)

$$\hat{y}_{q} = g_{q} - j \cdot b_{q} = \frac{\Delta P_{0}}{S_{Pn}} - j \sqrt{\left(\frac{i_{0\%}}{100}\right)^{2} - \left(\frac{\Delta P_{0}}{S_{Pn}}\right)^{2}}$$

denominated value (S) – related to U_{PN}

$$\hat{Y}_{q} = \hat{y}_{q} \frac{S_{Pn}}{U_{Pn}^{2}} = G_{q} - j \cdot B_{q} = \frac{S_{Pn}}{U_{Pn}^{2}} \left[\frac{\Delta P_{0}}{S_{Pn}} - j \sqrt{\left(\frac{i_{0\%}}{100}\right)^{2} - \left(\frac{\Delta P_{0}}{S_{Pn}}\right)^{2}} \right]$$

- <u>short-circuit measurement:</u> (3x, supply – short-circuit – no-load) provided: $S_{Pn} \neq S_{Sn} \neq S_{Tn}$

measurement between	P - S	P - T	S - T
short-circuit losses (W)	ΔP_{kPS}	ΔP_{kPT}	ΔP_{kST}
short-circuit voltage (%)	u_{kPS}	U _{kPT}	U _k ST
measurement corresponds to power (VA)	S_{Sn}	S_{Tn}	S_{Tn}

short-circuit tests S - T:

parameter to be found:

$$\begin{split} \hat{Z}_{ST} &= \hat{Z}_{\sigma S} + \hat{Z}_{\sigma T} \ \left(\hat{Z}_{\sigma S} = R_S + j \cdot X_{\sigma S} \right) \text{- recalculated to } U_{PN} \\ \hat{Z}_{ST} &= \hat{Z}_{\sigma S} + \hat{Z}_{\sigma T} \text{- recalculated to } U_{PN}, S_{PN} \end{split}$$

$$\Delta P_k \text{ for } I_{Tn} \rightarrow \Delta P_{kST} = 3 \cdot R^+_{ST} \cdot I^2_{Tn}, \quad I_{Tn} = \frac{S_{Tn}}{\sqrt{3} \cdot U_{Tn}}$$

R⁺_{ST}....resistance of secondary and tertiary windings (related to U_{Tn})

$$R^{+}_{ST} = \frac{\Delta P_{kST}}{S_{Tn}^{2}} \cdot U_{Tn}^{2}$$

$$R_{ST} = R^{+}_{ST} \cdot \frac{U_{Pn}^{2}}{U_{Tn}^{2}} \longrightarrow R_{ST} = R_{S} + R_{T} = \frac{\Delta P_{kST}}{S_{Tn}^{2}} \cdot U_{Pn}^{2}$$

R_S (R_T)...resistance of sec. and ter. windings recalculated to primary

$$\mathbf{r}_{\mathrm{ST}} = \mathbf{R}_{\mathrm{ST}} \cdot \frac{\mathbf{S}_{\mathrm{Pn}}}{\mathbf{U}_{\mathrm{Pn}}^2} = \frac{\Delta \mathbf{P}_{\mathrm{kST}}}{\mathbf{S}_{\mathrm{Tn}}^2} \cdot \mathbf{S}_{\mathrm{Pn}}$$

- impedance:

$$z_{ST} = \frac{u_{kST\%}}{100} \cdot \frac{S_{Pn}}{S_{Tn}}, Z_{ST} = z_{ST} \cdot \frac{U_{Pn}^2}{S_{Pn}} = \frac{u_{kST\%}}{100} \cdot \frac{U_{Pn}^2}{S_{Tn}}$$

$$\hat{z}_{ST} = r_{ST} + j \cdot x_{ST}, x_{ST} = \sqrt{z_{ST}^2 - r_{ST}^2}, x_{ST} = x_{\sigma S} + x_{\sigma T}$$

- based on the derived relations we can write:

<u>P - S:</u>

$$\hat{z}_{_{PS}} = r_{_{PS}} + j \cdot x_{_{PS}} = \frac{\Delta P_{_{kPS}}}{S_{_{Sn}}^2} \cdot S_{_{Pn}} + j \cdot \sqrt{\left(\frac{u_{_{kPS\%}}}{100} \cdot \frac{S_{_{Pn}}}{S_{_{Sn}}}\right)^2 - \left(\frac{\Delta P_{_{kPS}}}{S_{_{Sn}}^2} \cdot S_{_{Pn}}\right)^2}$$

$$\hat{Z}_{_{_{PS}}} = R_{_{PS}} + j \cdot X_{_{PS}} = \frac{\Delta P_{_{kPS}}}{S_{_{Sn}}^2} \cdot U_{_{Pn}}^2 + j \cdot \sqrt{\left(\frac{u_{_{kPS\%}}}{100} \cdot \frac{U_{_{Pn}}^2}{S_{_{Sn}}}\right)^2 - \left(\frac{\Delta P_{_{kPS}}}{S_{_{Sn}}^2} \cdot U_{_{Pn}}^2\right)^2}$$

- analogous for P - T and S - T

- leakage reactances for P, S, T:

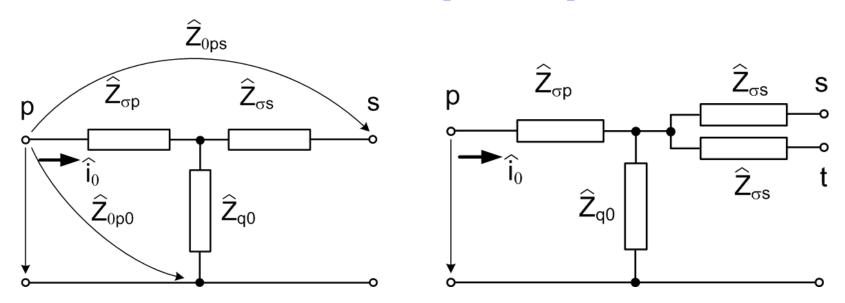
$$\hat{Z}_{\sigma P} = R_{P} + j \cdot X_{\sigma P} = 0,5 \cdot (\hat{Z}_{PS} + \hat{Z}_{PT} - \hat{Z}_{ST})$$

$$\hat{Z}_{\sigma S} = R_{S} + j \cdot X_{\sigma S} = 0,5 \cdot (\hat{Z}_{PS} + \hat{Z}_{ST} - \hat{Z}_{PT})$$

$$\hat{Z}_{\sigma T} = R_{T} + j \cdot X_{\sigma T} = 0,5 \cdot (\hat{Z}_{PT} + \hat{Z}_{ST} - \hat{Z}_{PS})$$

- knowledge of the series impedances and shunt admittances allows to study voltage and power conditions of 3-winding transformers
- mentioned impedances are valid for positive and negative sequences

Transformers zero sequence impedances



Series parameters are the same as for the positive sequence, the shunt always need to be determined.

Assumptions:

- Zero sequence voltage supplies the primary winding.
- The relative values are related to U_{Pn} and S_{Pn} .
- We distinguish free and tied magnetic flows (core x shell TRF). Z₀ depends on the winding connection.

note: reluctance, inductance

magnetic resistance (reluctance)

$$R_m = \frac{1}{\mu} \frac{1}{S}$$
 analogy $R_e = \frac{1}{\gamma} \frac{1}{S} = \rho \frac{1}{S}$

magnetic flux (Hopkinson's law)

$$\Phi = \frac{N \cdot I}{R_{m}}$$
 analogy (Ohm's law)
$$I = \frac{U}{R_{e}}$$

$$L = \frac{\Phi_{c}}{I} = \frac{N \cdot \Phi}{I} = \frac{N^{2}}{R_{m}}$$

$$\mu_{Fe} >> \mu_{0} \implies R_{mFe} << R_{m0} \implies L_{Fe} >> L_{\sigma}$$

TRF magnetic circuit

$$\Phi = \Phi_{h} + \Phi_{\sigma} = \frac{N \cdot I}{R_{mFe}} + \frac{N \cdot I}{R_{m0}} = \frac{L_{Fe} \cdot I}{N} + \frac{L_{\sigma} \cdot I}{N} = \frac{I}{N} \left(L_{Fe} + L_{\sigma} \right)$$

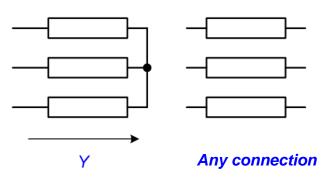
a) Y / any connection

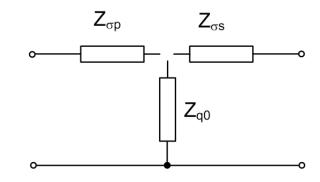
$$3i_0 = 0$$

$$z_0 = \frac{u_0}{i_0} \to \infty$$

$$z_{0p0} \to \infty$$

$$z_{0ps} \to \infty$$





b) D / any connection

Zero sequence voltage is attached to $D \rightarrow voltage$ at each phase

Secondary winding

$$u_0 - u_0 = 0 \longrightarrow i_a = i_b = i_c = 0 \longrightarrow i_0 = 0$$

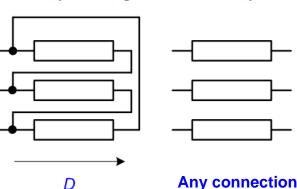
$$z_0 = \frac{u_0}{i_0} \to \infty$$

$$z^{0b0} \rightarrow \infty$$

$$z_{0ps} \rightarrow \infty$$



D



$$Z_{\sigma p}$$
 $Z_{\sigma s}$
 Z_{q0}

c) YN/D

Currents in the primary winding i₀ induce currents i₀' in the secondary winding to achieve magnetic balance.

Currents i₀' in the secondary winding are short-closed and do not flow further into the grid.

$$\hat{\mathbf{z}}_{0p0} = \hat{\mathbf{z}}_{\sigma p} + \hat{\mathbf{z}}_{q0}$$

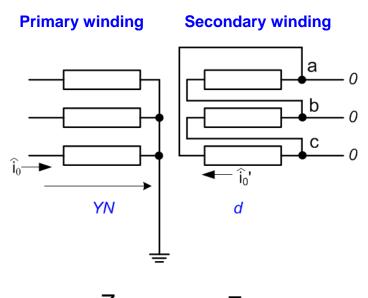
$$\hat{z}_{0} = \frac{\hat{u}_{0}}{\hat{i}_{0}} = \hat{z}_{\sigma p} + \frac{\hat{z}_{\sigma s} \cdot \hat{z}_{q0}}{\hat{z}_{\sigma s} + \hat{z}_{q0}}$$

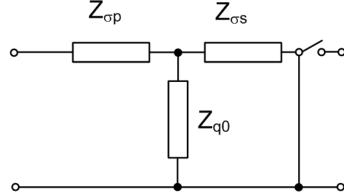
shell

$$\hat{z}_{q0} = \hat{y}_q^{-1} >> \hat{z}_{\sigma s} \longrightarrow \hat{z}_0 \approx \hat{z}_{\sigma p s} = \hat{z}_{1k}$$

3-core

$$\left|\hat{\mathbf{z}}_{q0}\right| < \left|\hat{\mathbf{y}}_{q}^{-1}\right| \longrightarrow \left|\hat{\mathbf{z}}_{0}\right| \approx (0.7 \div 0.9) \left|\hat{\mathbf{z}}_{\sigma ps}\right|$$

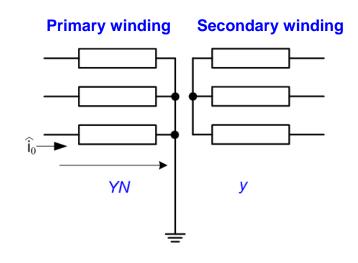


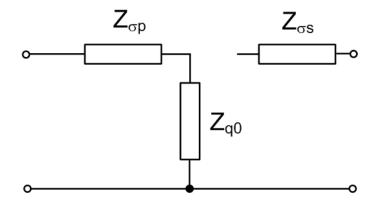


d) YN/Y

Zero sequence current can't flow through the secondary winding. Current i₀ corresponds to the magnetization current.

$$\begin{split} z_{0ps} &\to \infty \\ \hat{z}_{0} &= \hat{z}_{0p0} = \hat{z}_{\sigma p} + \hat{z}_{q0} \\ \text{shell} \\ \hat{z}_{q0} &= \hat{y}_{q}^{-1} \longrightarrow z_{0} \to \infty \\ \text{3-core} \\ \left| \hat{z}_{q0} \right| < \left| \hat{y}_{q}^{-1} \right| \longrightarrow \left| \hat{z}_{0} \right| \approx \left(0.3 \div 1 \right) \end{split}$$

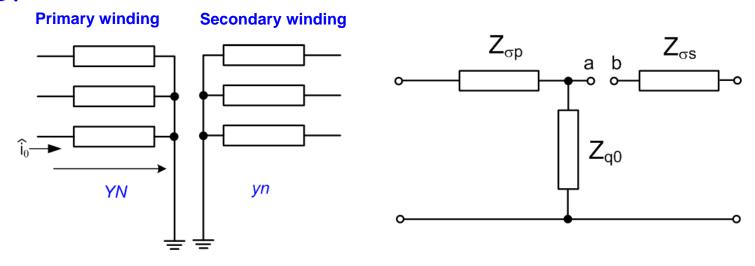




e) YN/YN

If element with YN or ZN behind TRF \rightarrow points a-b are connected \rightarrow as the positive sequence.

If element with Y, Z or D behind TRF \rightarrow a-b are disconnected \rightarrow as YN / Y.



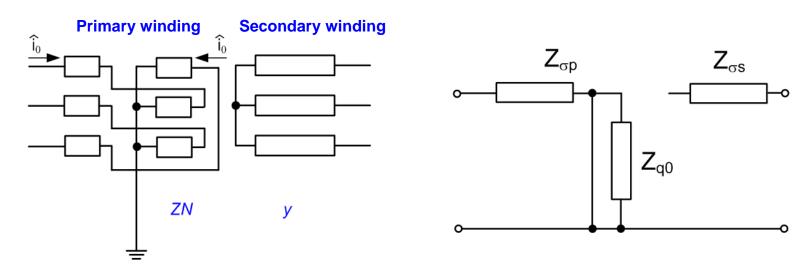
f) ZN / any connection

Currents i_0 induce mag. balance on the core themselves \rightarrow only leakages between the halves of the windings.

$$z_{0ps} \rightarrow \infty$$

$$\hat{z}_{0} = \hat{z}_{0p0} \approx (0,1 \div 0,3)\hat{z}_{\sigma ps}$$

$$r_{0} = r_{p}$$



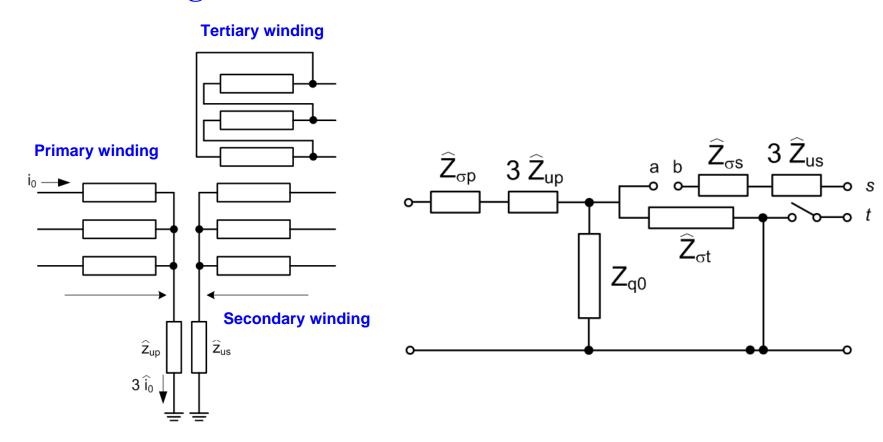
g) impedance in the neutral point

Current flowing through the neutral point is 3i₀.

Voltage drop:
$$\Delta \hat{\mathbf{u}}_{uz} = \hat{\mathbf{z}}_{u} \cdot 3\hat{\mathbf{i}}_{0} = 3\hat{\mathbf{z}}_{u} \cdot \hat{\mathbf{i}}_{0}$$

 \rightarrow in the model $3\hat{z}_u$ in series with the leakage reactance

h) three-winding TRF



System equivalent

Impedance (positive sequence) is given by the nominal voltage and shortcircuit current (power).

Three-phase (symmetrical) short-circuit: S''_k (MVA), I''_k (kA)

Three-phase (symmetrical) short-circuit:
$$S_k$$
 (WVA), I_k (RA)
$$S_k'' = \sqrt{3}U_n I_k''$$

$$Z_s = \frac{U_n^2}{S_k''} = \frac{U_n}{\sqrt{3} \cdot I_k''}$$
 CR: 400 kV $S_k'' \approx (6000 \div 30000) \text{ MVA}$ $I_k'' \approx (9 \div 45) \text{ kA}$ 220 kV $S_k'' \approx (2000 \div 12000) \text{ MVA}$ $I_k'' \approx (2 \div 30) \text{ kA}$ 110 kV $S_k'' \approx (100x \div 3000) \text{ MVA}$ $I_k'' \approx (x \div 15) \text{ kA}$