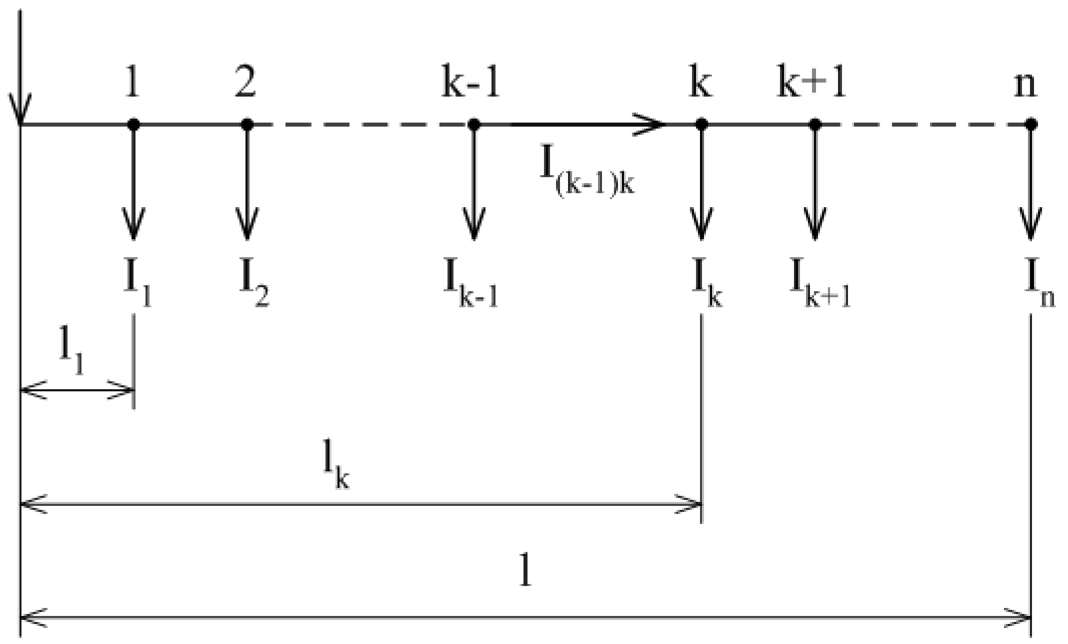
VOLTAGE DROP IN POWER SYSTEM

Simple DC line (LV, HV)

Double-wire circuit. Pre-condition: constant cross-section and resistance. El. traction, electrochemistry, light sources, long-distance transmission, power electronics.

Single load is fed from one side



a) addition method

Count voltage drops in circuit section by section.

(Voltage drop is counted for both conductors in section)

k-th section

$$U_{(k-1)} - U_k = \Delta U_{(k-1)k} = 2\frac{\rho}{S}\left(l_k - l_{(k-1)}\right)\cdot I_{(k-1)k} \quad (V;\, \Omega m,\, m^2,\, m,\, A)$$

Current in k-th section

$$I_{(k-1)k} = \sum_{y=k}^{n} I_y$$

Maximum voltage drop

$$\Delta U_n = \sum_{k=1}^{n} \Delta U_{(k-1)k} = 2\frac{\rho}{S}\sum_{k=1}^{n}\left(l_k - l_{(k-1)}\right)\cdot \sum_{y=k}^{n} I_y$$

b) superposition method

Voltage drops are counted in places of discrete loads:

$$\Delta U_n = 2\frac{\rho}{S}\sum_{k=1}^{n} l_k I_k$$

$l_k I_k$ …current moment to feed

Rated voltage drop:

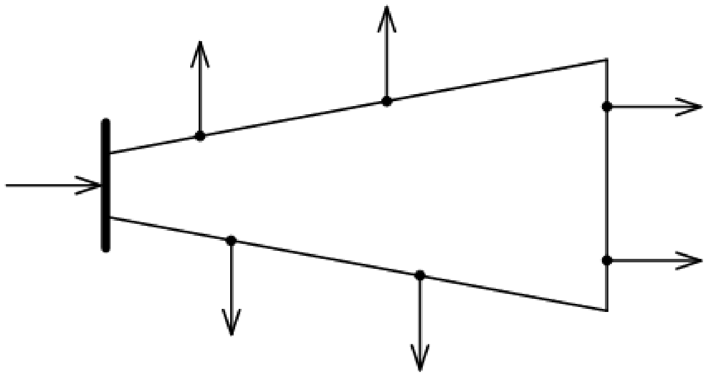
$$\varepsilon = \frac{\Delta U}{U_n} \quad (-;\, V,\, V)$$

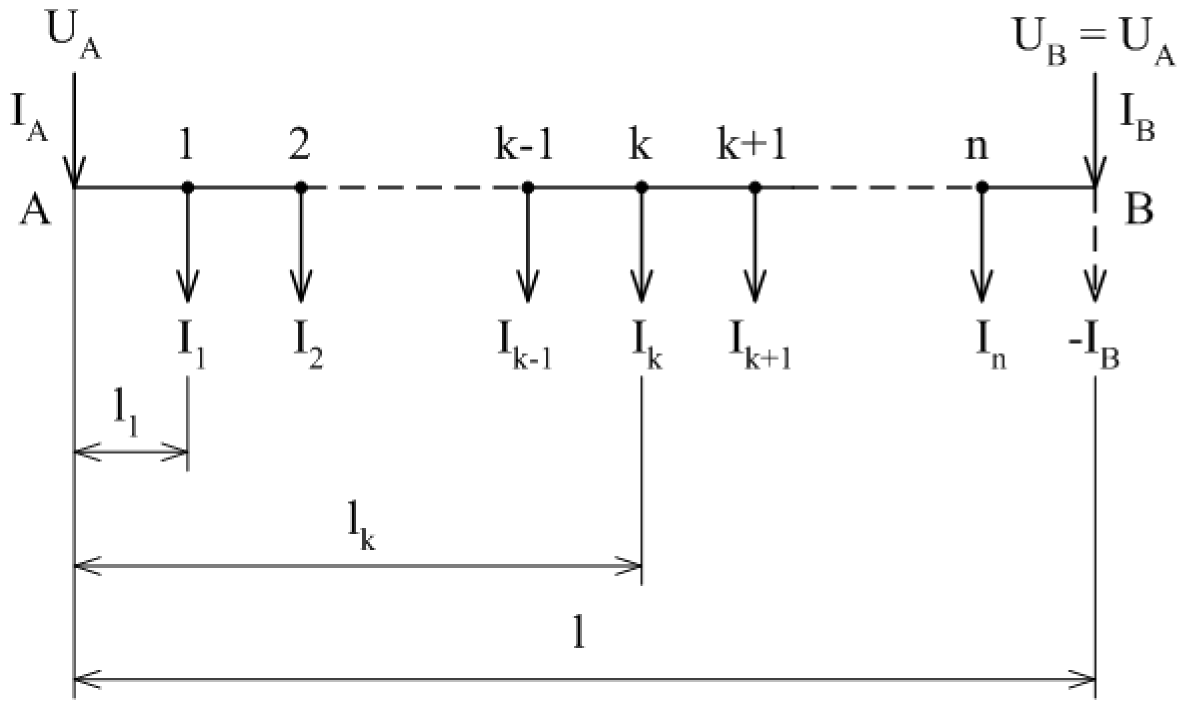
Note. Losses should be counted using additional!

$$\Delta P_{(k-1)k} = 2\frac{\rho}{S}\left(l_k - l_{(k-1)}\right)\cdot I^2_{(k-1)k} \quad (W;\, \Omega m,\, m^2,\, m,\, A)$$

$$\Delta P = \sum_{k=1}^{n} \Delta P_{(k-1)k}$$

Single load is fed from both sides – voltage level is the same





- Round grid, higher reliability of supply.
- Two one-side fed lines after fault.
- Evaluation of current distribution and voltage drops.

Consider I_B as negative load (power supply-feeder):

$$\Delta U_{AB} = U_A - U_B = 0 = 2\frac{\rho}{S}\sum_{k=1}^{n} l_k I_k - 2\frac{\rho}{S} l I_B$$

Hence (moment's theorem)

$$I_B = \frac{\sum_{k=1}^{n} l_k I_k}{l}$$

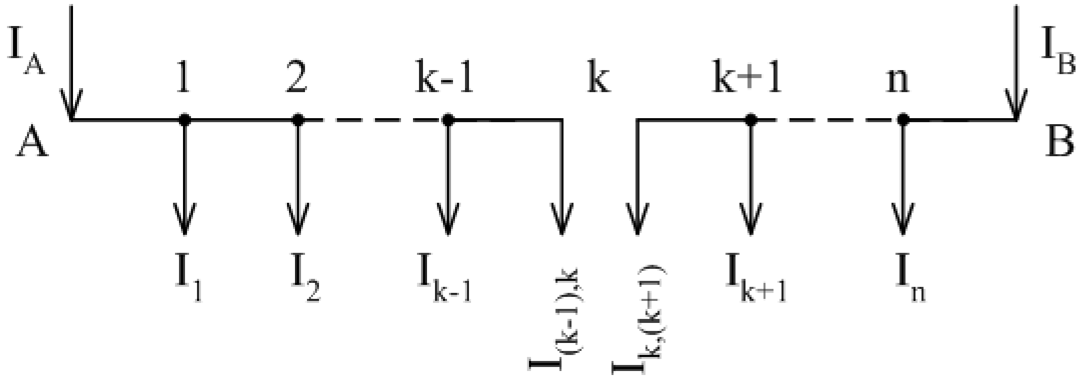
Analogically (current moments to other feeder)

$$I_A = \frac{\sum_{k=1}^{n} (l - l_k) I_k}{l}$$

Therefore

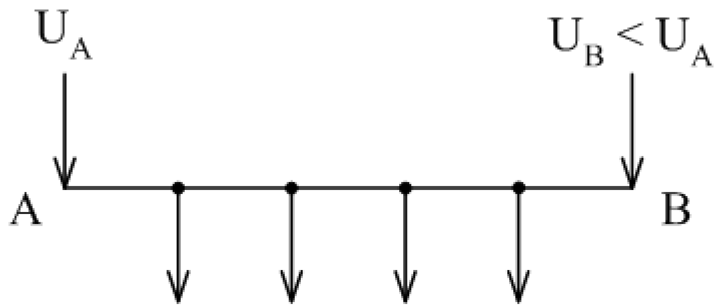
$$I_A + I_B = \sum_{y=1}^{n} I_y$$

Current distribution identify the place with the biggest voltage drop = the place with feeder division → split-up to two one-side fed lines.



<u>Single load is fed from both sides – voltage level is different</u>

Two different sources, mesh grid.

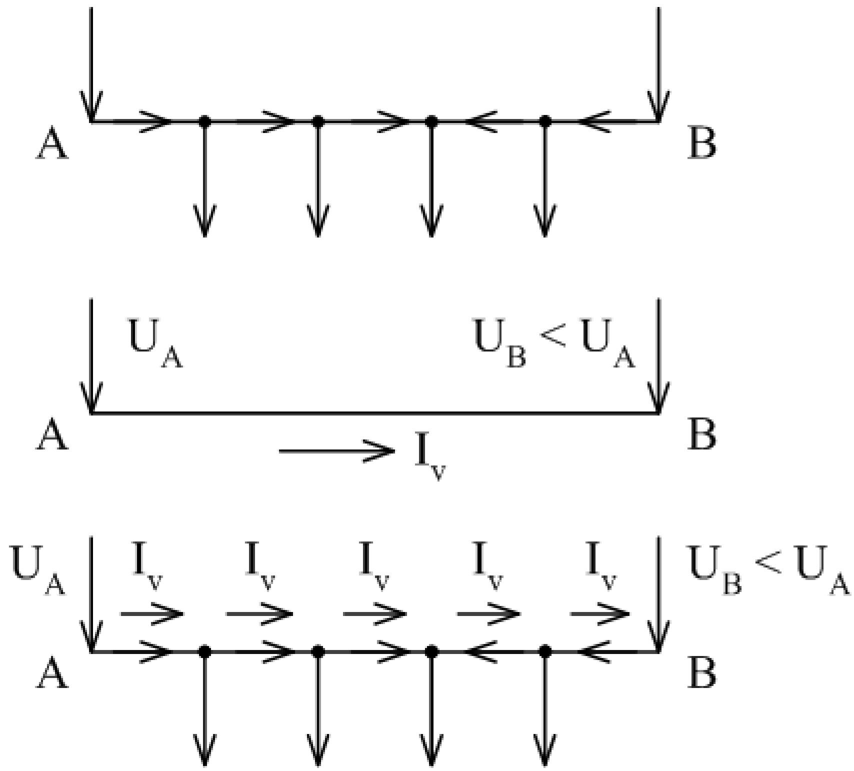


Superposition:

1) Current distribution with the same voltages.
2) Different voltage and zero loads → balancing current

$$I_v = \frac{U_A - U_B}{2\frac{\rho}{S} l}$$

3) Sum of the solutions 1+2



Further calculation is the same.

Or directly:

$$U_A - U_B = 2\frac{\rho}{S}\sum_{k=1}^{n} l_k I_k - 2\frac{\rho}{S} l I_B$$

$$I_B = \frac{2\frac{\rho}{S}\sum_{k=1}^{n} l_k I_k}{2\frac{\rho}{S} l} - \frac{U_A - U_B}{2\frac{\rho}{S} l}$$

Direct current transmission at UHV

Long distance energy transmission or local stations (AC-DC-AC).

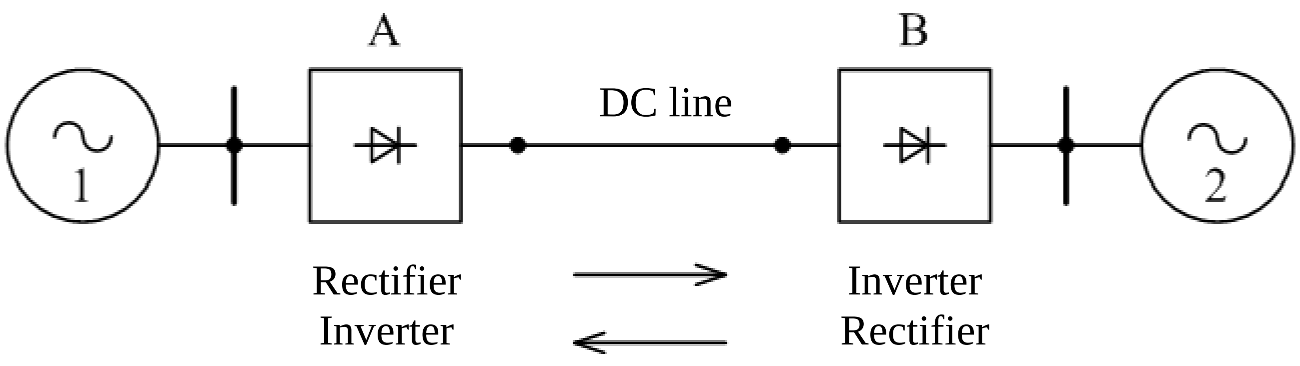
Reasons:

- transmission stability, power systems connection,
- short-circuit conditions,
- parameters compensation,
- economic aspects.

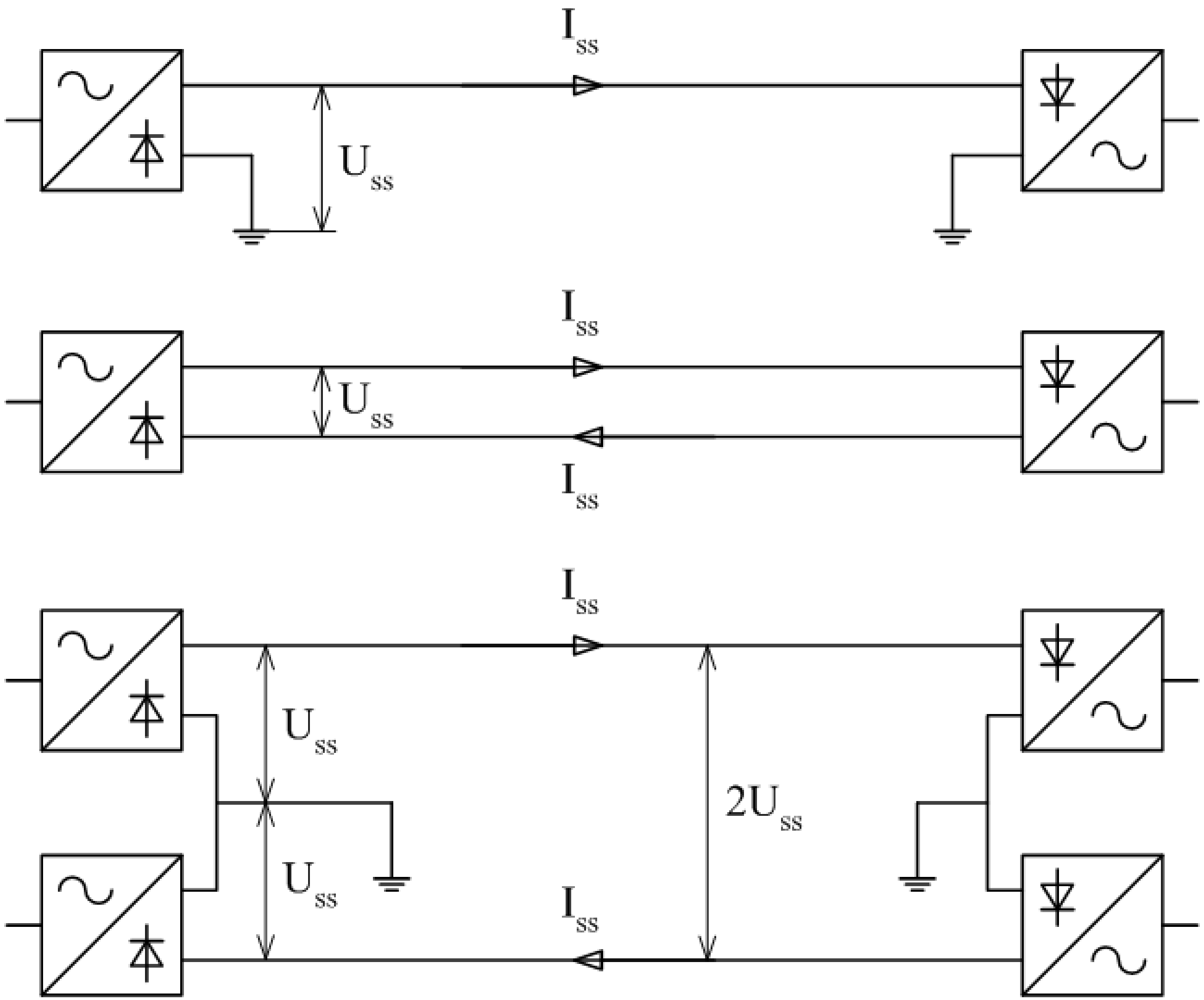
Lower insulation distances are available and higher transmission capacity than for AC systems.

DC voltage regulation.

Principle

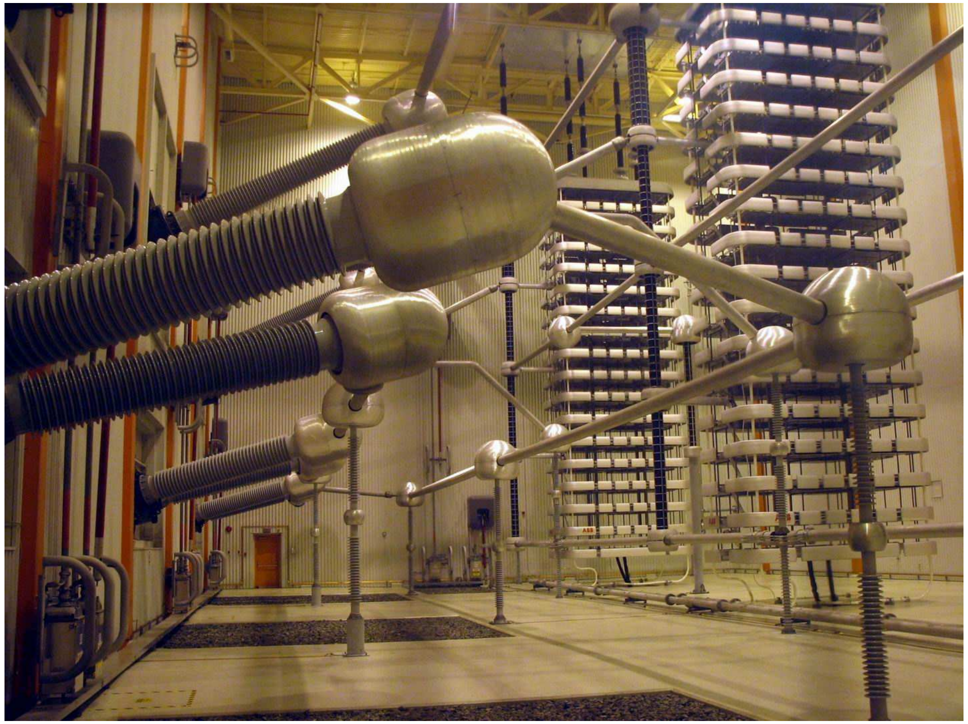


Applications:



a) Earth as back conductor. Ground resistance does not depend on line length but on ground connection. Suitability for unoccupied places (corroding process, EMC).
b) Two wires line. Compared to 3f less material, lighter towers.
c) Two convertors in series. Low balancing ground current caused by asymmetry. During fault transforms to a), half power.

450 kV DC Canada



Complex power in AC grids

3phase

$$P = 3U_f I \cos\varphi = \sqrt{3} UI \cos\varphi \quad (W)$$

$$Q = 3U_f I \sin\varphi = \sqrt{3} UI \sin\varphi \quad (VAr)$$

$$S = 3U_f I = \sqrt{3} UI = \sqrt{P^2 + Q^2} \quad (VA)$$

Complex (1phase)

$$\hat{S}_f = P_f \pm jQ_f = U_f I(\cos\varphi \pm j\sin\varphi) = S_f e^{\pm j\varphi}$$

Sign according convention.

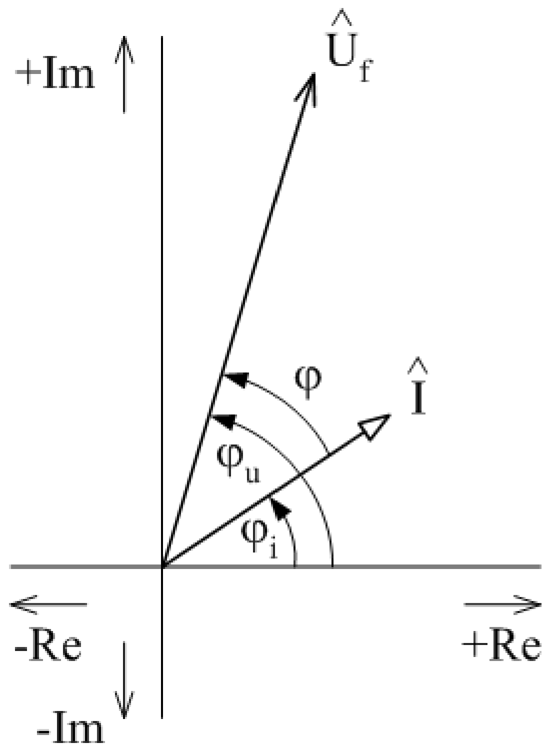
Inductive load:

$$\hat{U}_f = U_f e^{j\varphi_u} \;, \hat{I}_f = I e^{j\varphi_i}$$

Complex current sum:

$$\hat{S}_f = \hat{U}_f \hat{I}^* = U_f I e^{j(\varphi_u - \varphi_i)} = U_f I e^{j\varphi}$$

$$\hat{S}_f = \hat{U}_f \hat{I}^* = P_f \pm jQ_f \quad \begin{matrix} IND \\ CAP \end{matrix}$$

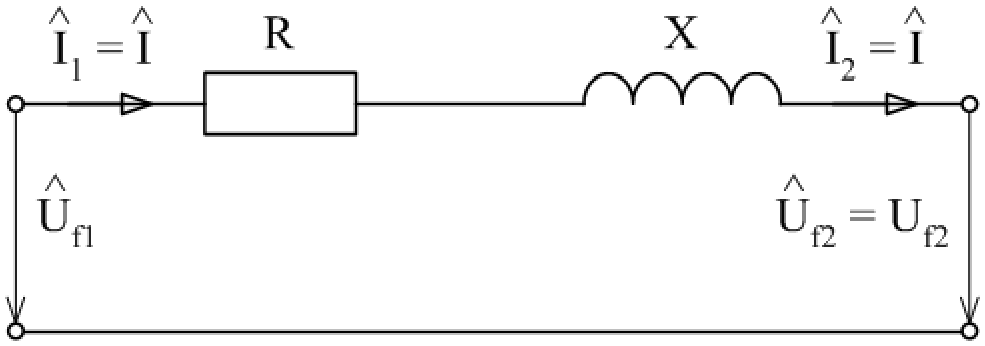


3 phase lines of LV, HV

Series parameters are applied, for LV X→0.

3 phase line HV, 1 load at the end

Symmetrical load → 1phase scheme, operating parameters.



Complex voltage drop:

$$\Delta\hat{U}_f = \hat{Z}_l\hat{I} = (R + jX)(I_č \mp jI_j) \begin{matrix} IND \\ CAP \end{matrix}$$

$$\Delta\hat{U}_f = RI_č \pm XI_j + j(XI_č \mp RI_j) \begin{matrix} IND \\ CAP \end{matrix}$$

value phase

Phasor diagram (input U_{f2}, I, φ_2)
(angle υ usually small, up to 3°)
After imag. part neglect and modification:

$$\Delta U_f = \frac{R3U_f I_č \pm X3U_f I_j}{3U_f} = \frac{RP \pm XQ}{3U_f}$$

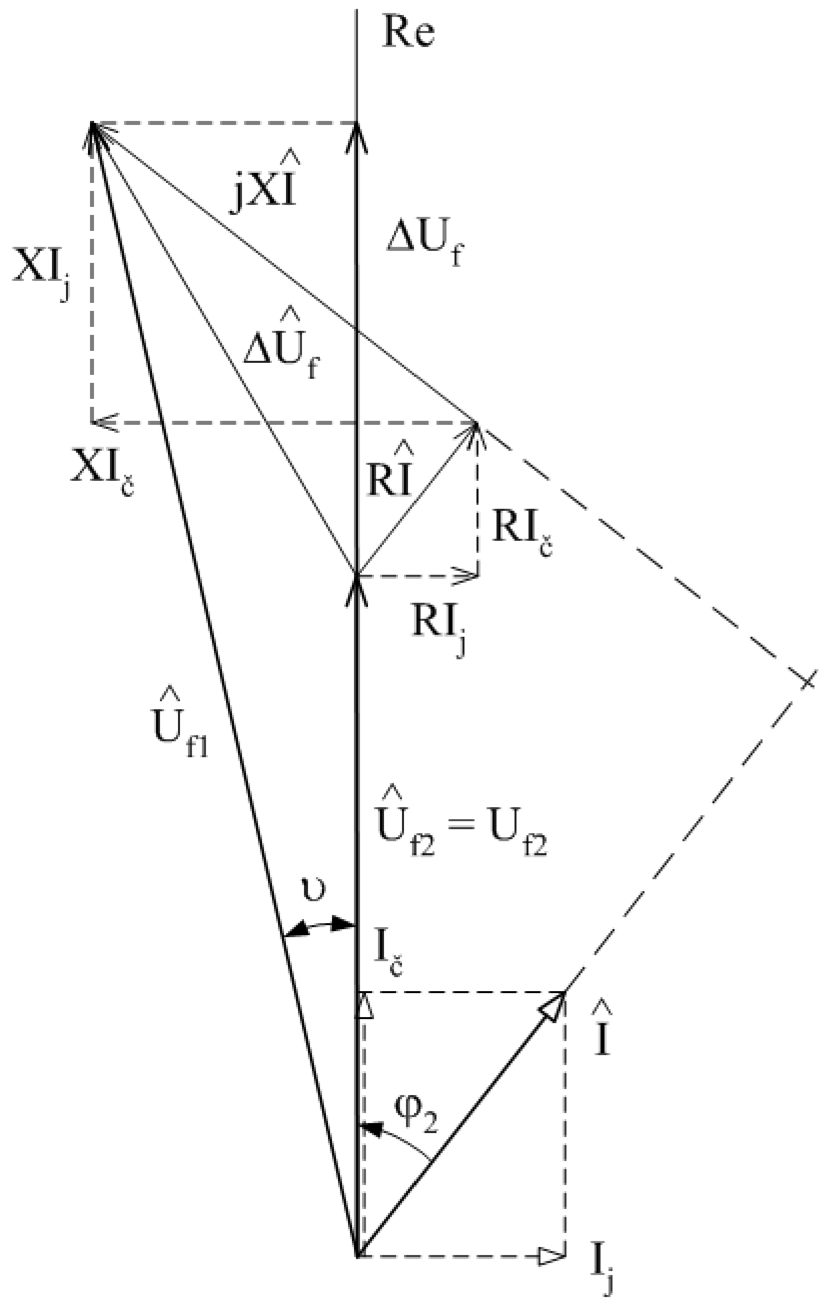
Percentage voltage drop:

$$\varepsilon = \frac{\Delta U_f}{U_f} = \frac{RP \pm XQ}{3U_f^2} = \frac{RP \pm XQ}{U^2}$$

3 phase active power losses:

$$\Delta\hat{S} = 3\Delta\hat{U}_f \hat{I}^* = 3\hat{Z}_1 \hat{I} \cdot \hat{I}^* = 3\hat{Z}_1 I^2 =$$
$$= 3(R + jX)I^2 = 3RI^2 + j3XI^2$$
$$\Delta P = 3RI^2 = 3R\left(I_č^2 + I_j^2\right) \quad (W; \Omega, A)$$

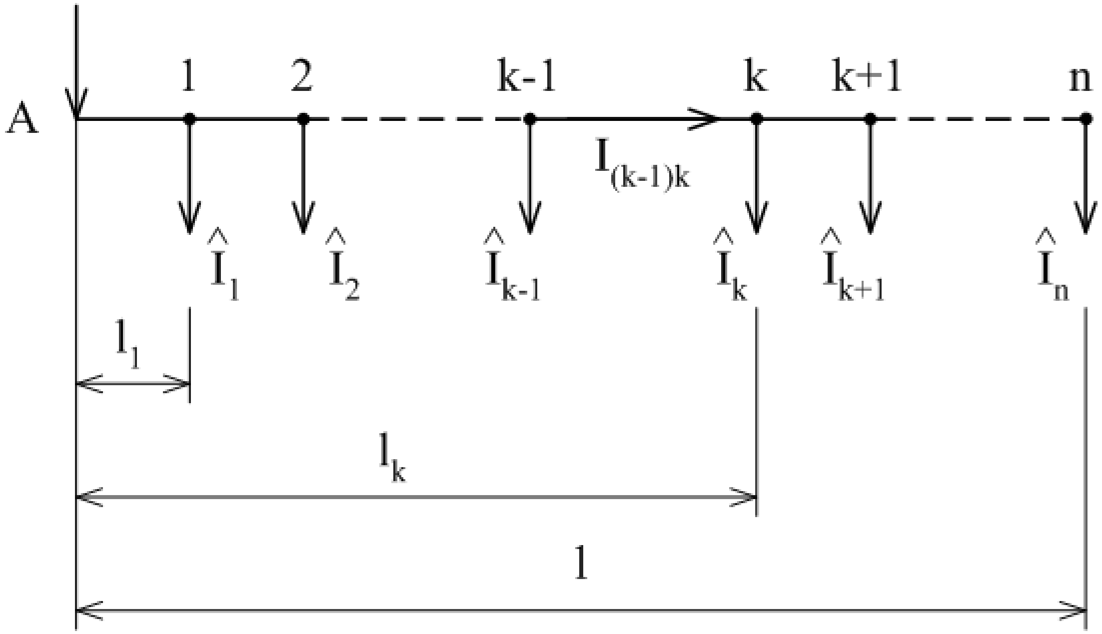
! Even reactive current causes active power losses! → Reactive power compensation



3 phase HV line one-side fed

Constant series impedance

$$\hat{Z}_{l_1} = R_1 + jX_1 \quad (\Omega / km)$$



Voltage drop at the end (does not have to be the highest, it depends on load character)

$$\Delta\hat{U}_{fAn} = \hat{Z}_{l_1} \sum_{k=1}^{n} l_k \hat{I}_k$$

After imag. part neglect:

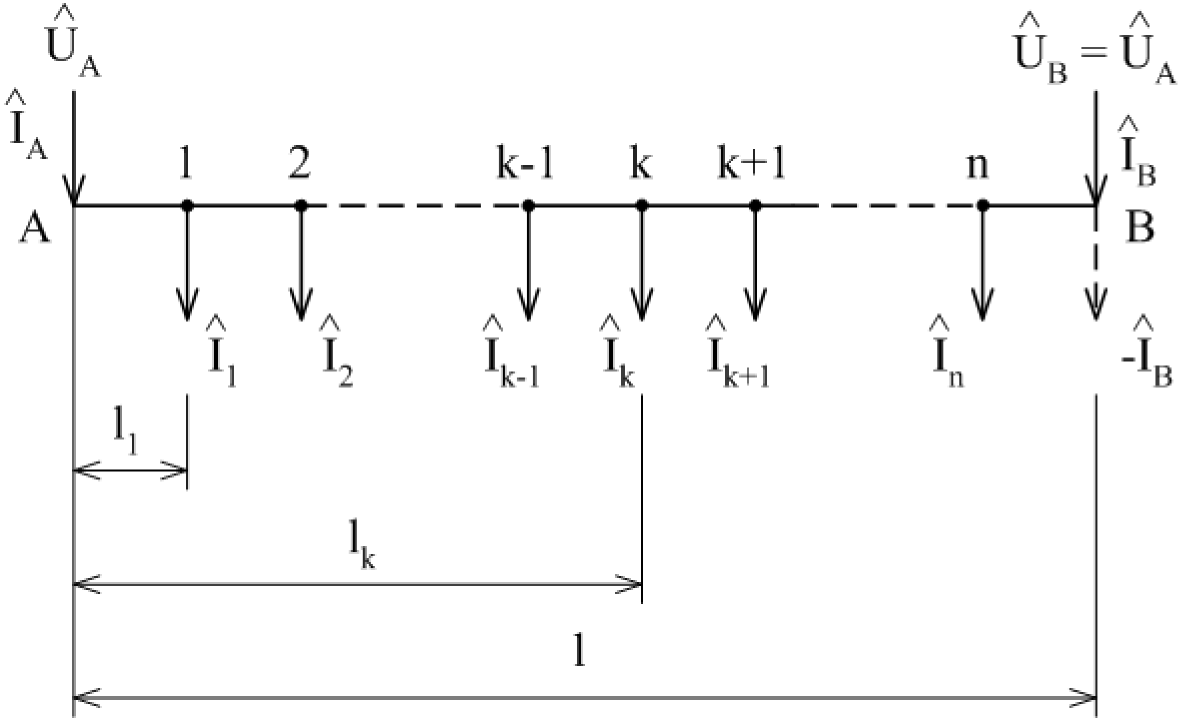
$$\Delta U_{fAn} \doteq R_1 \sum_{k=1}^{n} l_k I_{\check{c}k} \pm X_1 \sum_{k=1}^{n} l_k I_{jk} \quad \begin{matrix} IND \\ CAP \end{matrix}$$

$$\Delta U_{fAn} \doteq \frac{R_1 \sum_{k=1}^{n} l_k P_k \pm X_1 \sum_{k=1}^{n} l_k Q_k}{3U_f} \quad \begin{matrix} IND \\ CAP \end{matrix}$$

Voltage drop up to point X (superposition)

$$\Delta \hat{\mathrm{U}}_{\mathrm{fAX}} = \hat{\mathrm{Z}}_{\mathrm{l}_1} \sum_{\mathrm{k=1}}^{\mathrm{X}} \mathrm{l}_\mathrm{k} \hat{\mathrm{I}}_\mathrm{k} + \hat{\mathrm{Z}}_{\mathrm{l}_1} \mathrm{l}_{\mathrm{AX}} \sum_{\mathrm{k=X+1}}^{\mathrm{n}} \hat{\mathrm{I}}_\mathrm{k}$$

3 phase HV line doth-side fed



Calculation as for DC line (feeder is negative load, zero voltage drop)

$$\Delta \hat{U}_{AB} = 0 = \hat{Z}_{l_1} \sum_{k=1}^{n} l_k \hat{I}_k - \hat{Z}_{l_1} l \cdot \hat{I}_B$$

Moment's theorems:

$$\hat{I}_B = \frac{\sum_{k=1}^{n} l_k \hat{I}_k}{l} \qquad \hat{I}_A = \frac{\sum_{k=1}^{n} (l - l_k)\hat{I}_k}{l} \qquad \hat{I}_A + \hat{I}_B = \sum_{y=1}^{n} \hat{I}_y$$

(in principle is talking about current divider for each load)
Active and reactive power sign change could be in different nodes → maximum voltage drop should be controlled at both feeders.

Nodal grid HV

NO!

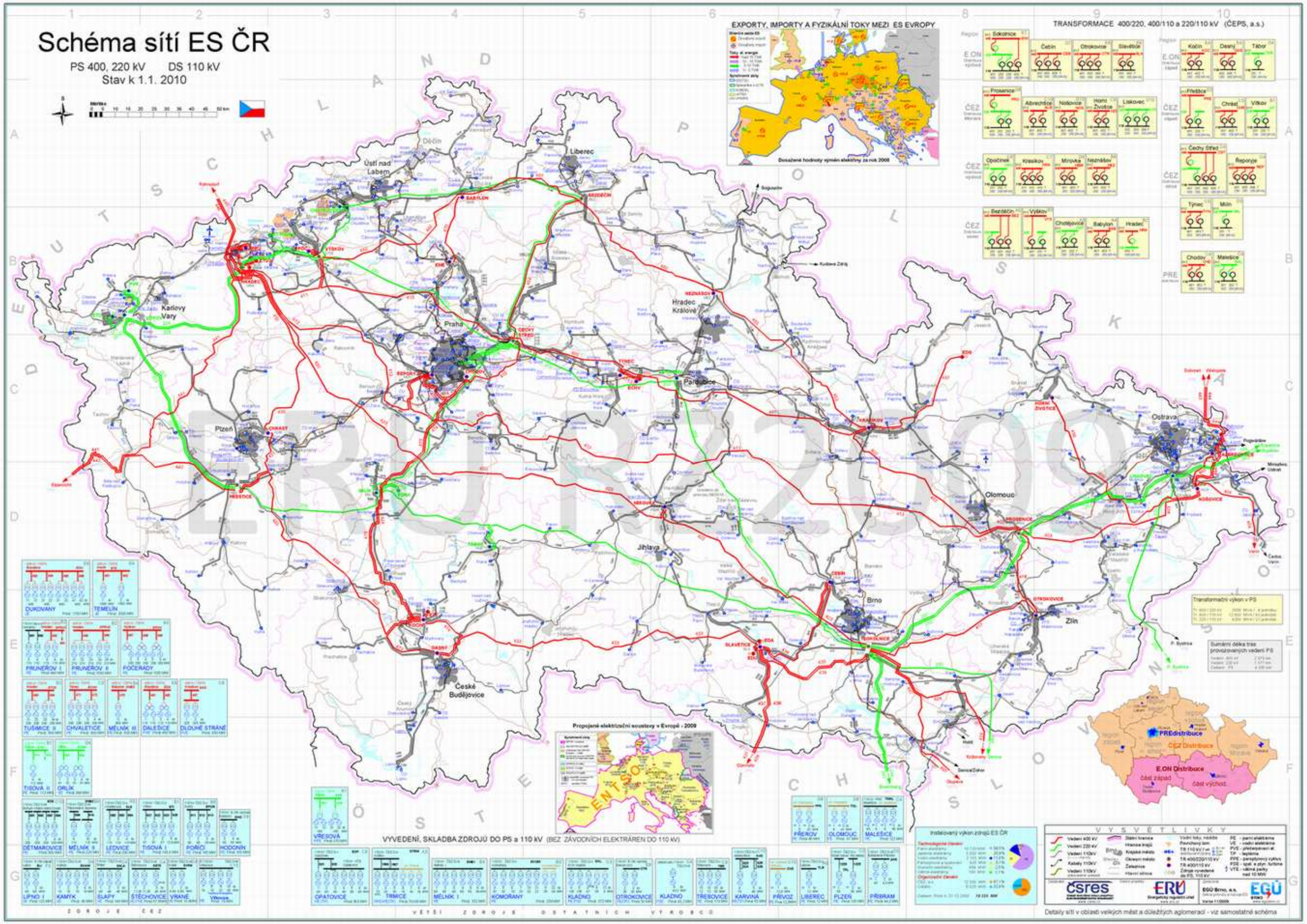
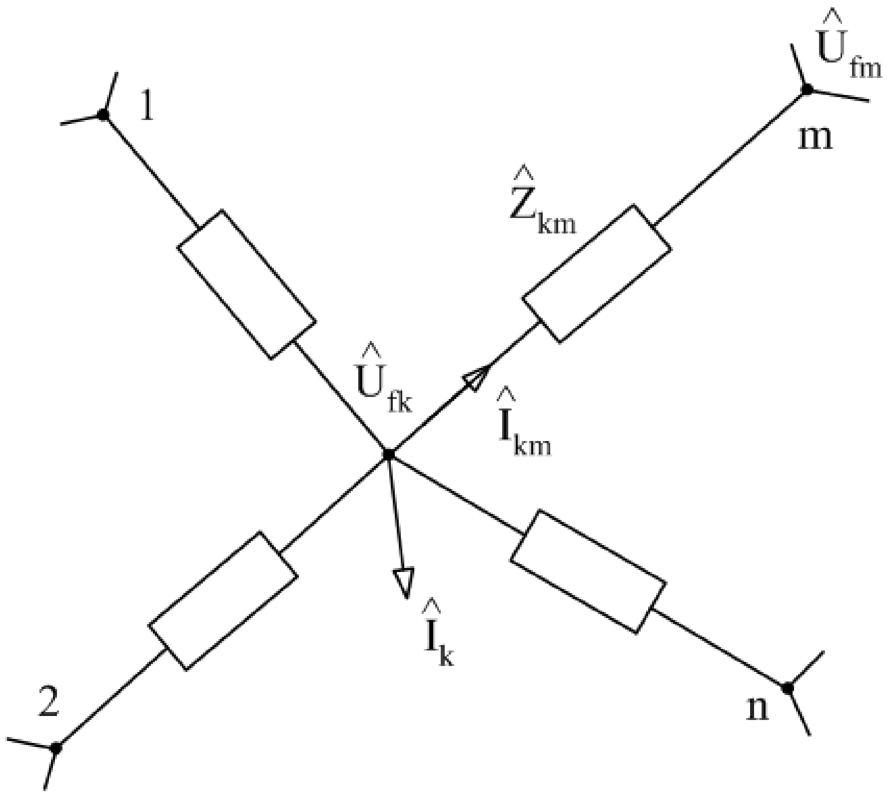


Schéma sítí ES ČR
PS 400, 220 kV DS 110 kV
Stav k 1.1. 2010
EXPORTY, IMPORTY A FYZIKÁLNÍ TOKY MEZI ES EVROPY
TRANSFORMACE 400/220, 400/110 a 220/110 kV (ČEPS, a.s.)
Liberec
Ústí nad Labem
Karlovy Vary
Praha
Hradec Králové
Pardubice
Plzeň
Ostrava
Olomouc
Jihlava
Brno
Zlín
České Budějovice
Propojené elektrizační soustavy v Evropě - 2009
VYVEDENÍ, SKLADBA ZDROJŮ DO PS a 110 kV (BEZ ZÁVODNÍCH ELEKTRÁREN DO 110 kV)
VYSVĚTLIVKY
PREdistribuce
ČEZ Distribuce
E.ON Distribuce
Detaily sítí v oblasti velkých měst a důležitých aglomerací - viz samostatná schémata

YES!

Nodal voltage method

Grid with *n* nodes. Set series branch parameters $\hat{Z}_{km}$, load current (nodal current) $\hat{I}_k$, min. 1 nodal voltage $\hat{U}_{fk}$.



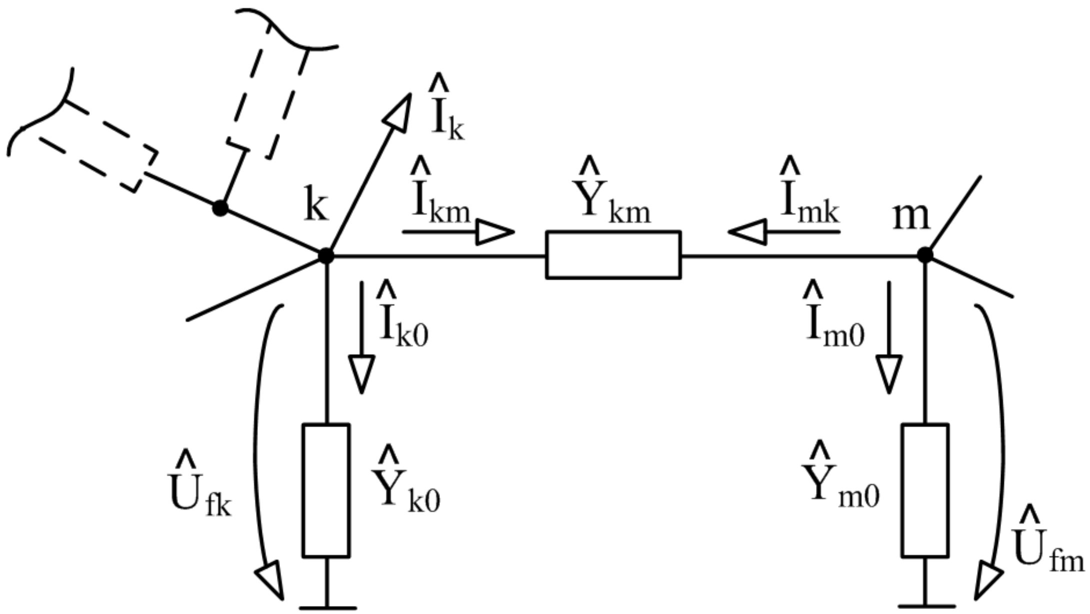
Calculation with series admittances

$$\hat{Y}_{km} = \hat{Z}_{km}^{-1} = \frac{1}{R_{km} + jX_{km}}$$

Node *k*:

$$\hat{I}_k + \sum_{\substack{m=1 \\ m \neq k}}^{n} \hat{I}_{km} + \hat{I}_{k0} = 0$$

$$\hat{I}_{k0} = \hat{U}_{fk} \hat{Y}_{k0}$$



Branch *k, m*:

$$\hat{I}_{km} = \left(\hat{U}_{fk} - \hat{U}_{fm}\right)\hat{Y}_{km}$$

After modification:

$$\hat{I}_k = -\sum_{\substack{m=1 \\ m \neq k}}^{n} \left(\hat{U}_{fk} - \hat{U}_{fm}\right)\hat{Y}_{km} - \hat{U}_{fk}\hat{Y}_{k0}$$

$$\hat{I}_k = -\hat{U}_{fk}\left(\sum_{\substack{m=1\\m\neq k}}^{n}\hat{Y}_{km} + \hat{Y}_{k0}\right) + \sum_{\substack{m=1\\m\neq k}}^{n}\hat{U}_{fm}\hat{Y}_{km}$$

Admittance matrix parameters set:

Nodal self-admittance (diagonal):

$$\hat{Y}_{(k,k)} = -\sum_{\substack{m=1\\m\neq k}}^{n}\hat{Y}_{km} - \hat{Y}_{k0}$$

Between nodes admittance (non-diagonal):

$$\hat{Y}_{(k,m)} = \hat{Y}_{(m,k)} = \hat{Y}_{km} \quad \text{pro } m \neq k$$

(for non-connected nodes $\hat{Y}_{(k,m)} = 0$)

Hence

$$\hat{I}_k = \sum_{m=1}^{n}\hat{Y}_{(k,m)}\hat{U}_{fm}$$

Matrix form:

$$\left(\hat{I}\right)=\left(\hat{Y}\right)\left(\hat{U}_{f}\right)$$

Set voltages at nodes 1 to *k* (x), currents at nodes *k+1* to *n* (y)

$$\begin{pmatrix}\left(\hat{I}_{x}\right)\\\left(\hat{I}_{y}\right)\end{pmatrix}=\begin{pmatrix}\left(\hat{Y}_{xx}\right) & \left(\hat{Y}_{xy}\right)\\\left(\hat{Y}_{xy}\right)^{T} & \left(\hat{Y}_{yy}\right)\end{pmatrix}\begin{pmatrix}\left(\hat{U}_{fx}\right)\\\left(\hat{U}_{fy}\right)\end{pmatrix}$$

Hence

$$\left(\hat{I}_{x}\right)=\left(\hat{Y}_{xx}\right)\left(\hat{U}_{fx}\right)+\left(\hat{Y}_{xy}\right)\left(\hat{U}_{fy}\right)$$

$$\left(\hat{I}_{y}\right)=\left(\hat{Y}_{xy}\right)^{T}\left(\hat{U}_{fx}\right)+\left(\hat{Y}_{yy}\right)\left(\hat{U}_{fy}\right)$$

Calculate $\left(\hat{I}_{x}\right),\left(\hat{U}_{fy}\right)$

$$\left(\hat{U}_{fy}\right)=\left(\hat{Y}_{yy}\right)^{-1}\left(\hat{I}_{y}\right)-\left(\hat{Y}_{yy}\right)^{-1}\left(\hat{Y}_{xy}\right)^{T}\left(\hat{U}_{fx}\right)$$

If some nodes are connected to the ground (through admittance), then admittance matrix is regular → to set all nodal current is enough.

$$\left(\hat{U}_f\right) = \left(\hat{Y}\right)^{-1}\left(\hat{I}\right)$$

Note 1: Similar for DC grid.

$$(I) = (G)(U)$$

Note 2: For power engineering – powers are set, than current are calculated.

$$\hat{I} = \left(\frac{\hat{S}}{\sqrt{3}\hat{U}}\right)^{*}$$

Results are not accurate if nominal voltages are used → iteration method.