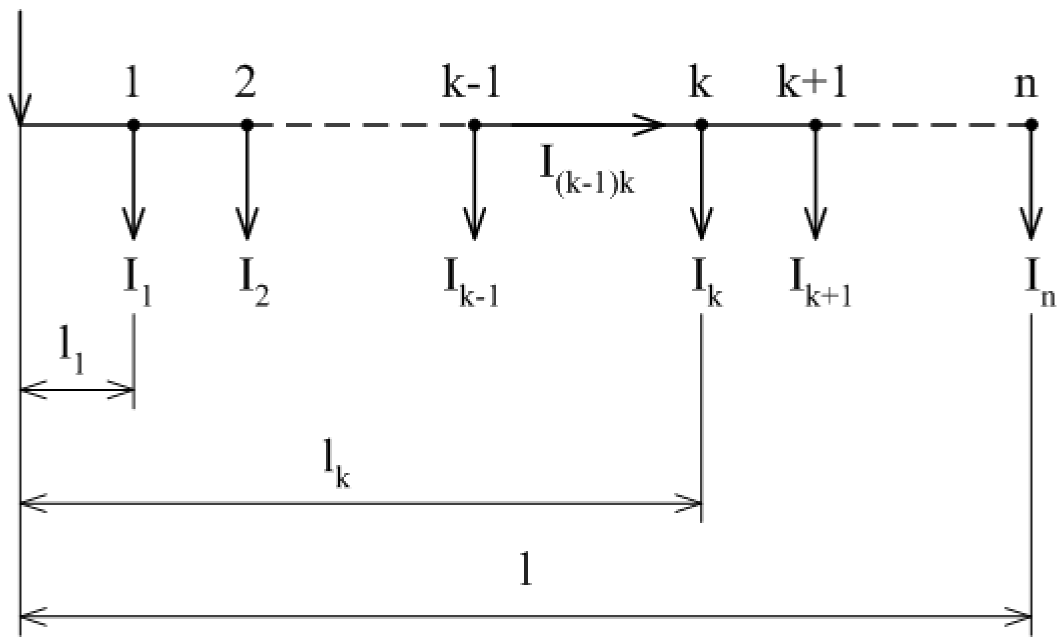
VOLTAGE DROPS IN POWER SYSTEM

Simple DC line (LV, MV)

Double-wire circuit. Assumption: constant cross-section and resistivity. El. traction, electrochemistry, light sources, long-distance transmission, power electronics.

Single loads supplied from one side



a) addition method

It adds voltage drops along the power line sections.
(Voltage drops are always in both conductors in the section.)

k^{th} section

$$U_{(k-1)} - U_k = \Delta U_{(k-1)k} = 2\frac{\rho}{S}\left(l_k - l_{(k-1)}\right)\cdot I_{(k-1)k} \quad (V; \Omega m, m^2, m, A)$$

Current in k^{th} section

$$I_{(k-1)k} = \sum_{y=k}^{n} I_y$$

Maximum voltage drop

$$\Delta U_n = \sum_{k=1}^{n} \Delta U_{(k-1)k} = 2\frac{\rho}{S}\sum_{k=1}^{n}\left(l_k - l_{(k-1)}\right)\cdot \sum_{y=k}^{n} I_y$$

b) superposition method

It adds voltage drops for individual discrete loads:

$$\Delta U_n = 2\frac{\rho}{S}\sum_{k=1}^{n} l_k I_k$$

$l_k I_k$ …current moments to the feeder

Relative voltage drop:

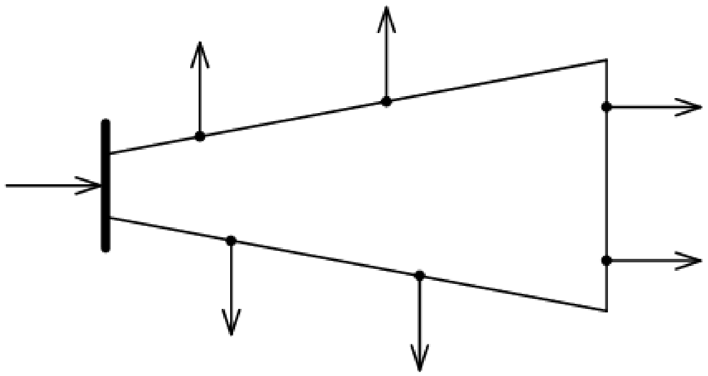
$$\varepsilon = \frac{\Delta U}{U_n} \quad (-;\, V,\, V)$$

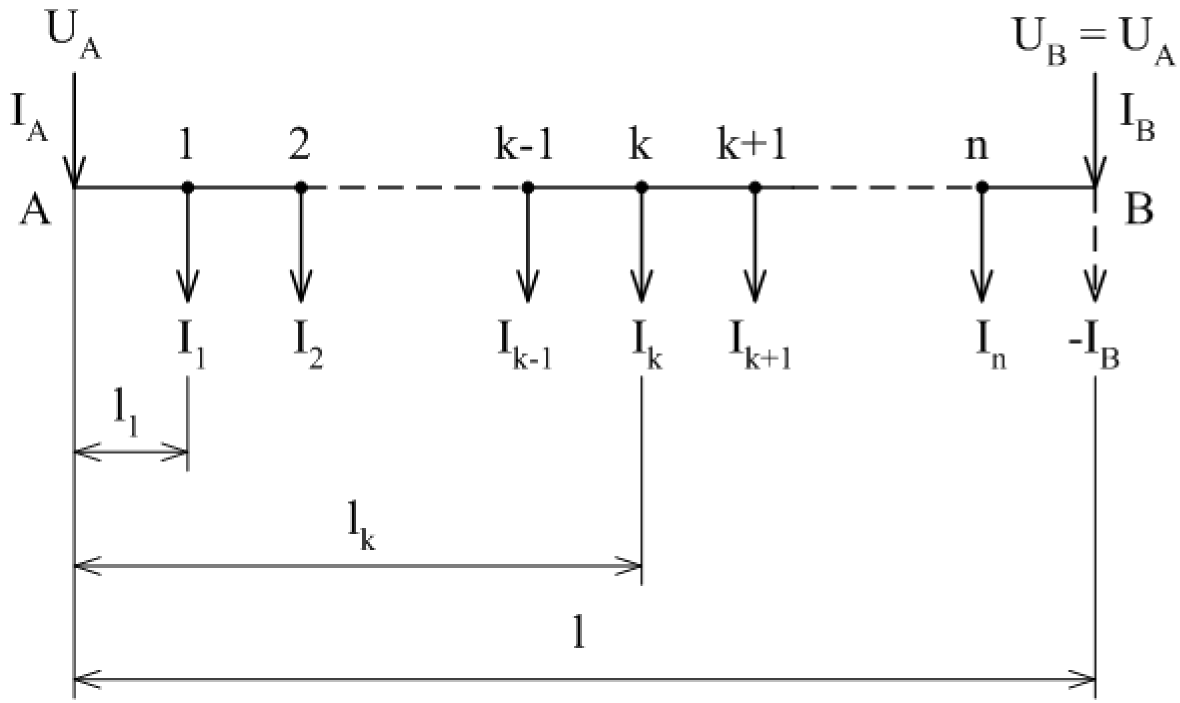
Note. Losses must be calculated only by means of the addition method!

$$\Delta P_{(k-1)k} = 2\frac{\rho}{S}\left(l_k - l_{(k-1)}\right)\cdot I^2_{(k-1)k} \quad (W;\, \Omega m,\, m^2,\, m,\, A)$$

$$\Delta P = \sum_{k=1}^{n} \Delta P_{(k-1)k}$$

Single loads supplied from both sides – the same feeders voltages





- Ring grid, higher reliability of supply.
- Two one-feeder lines after a fault.
- Calculation of current distribution and voltage drops.

Consider I_B as a negative load:

$$\Delta U_{AB} = U_A - U_B = 0 = 2\frac{\rho}{S}\sum_{k=1}^{n} l_k I_k - 2\frac{\rho}{S} l I_B$$

Hence (moment theorem)

$$I_B = \frac{\sum_{k=1}^{n} l_k I_k}{l}$$

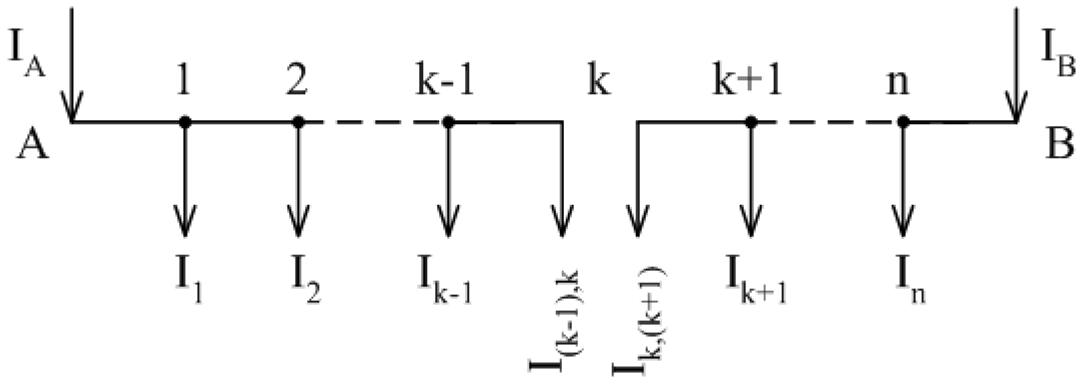
Analogous (current moments to other feeder)

$$I_A = \frac{\sum_{k=1}^{n} (l - l_k) I_k}{l}$$

Of course

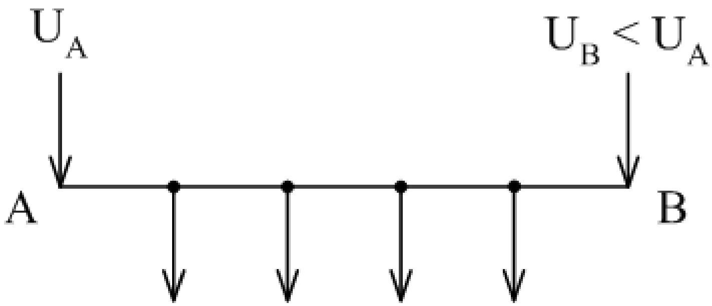
$$I_A + I_B = \sum_{y=1}^{n} I_y$$

Current distribution identifies the place with the biggest voltage drop = the place with feeder division → split-up into two one-feeder lines.



<u>Single loads supplied from both sides – different feeders voltages</u>

Two different sources, meshed grid.

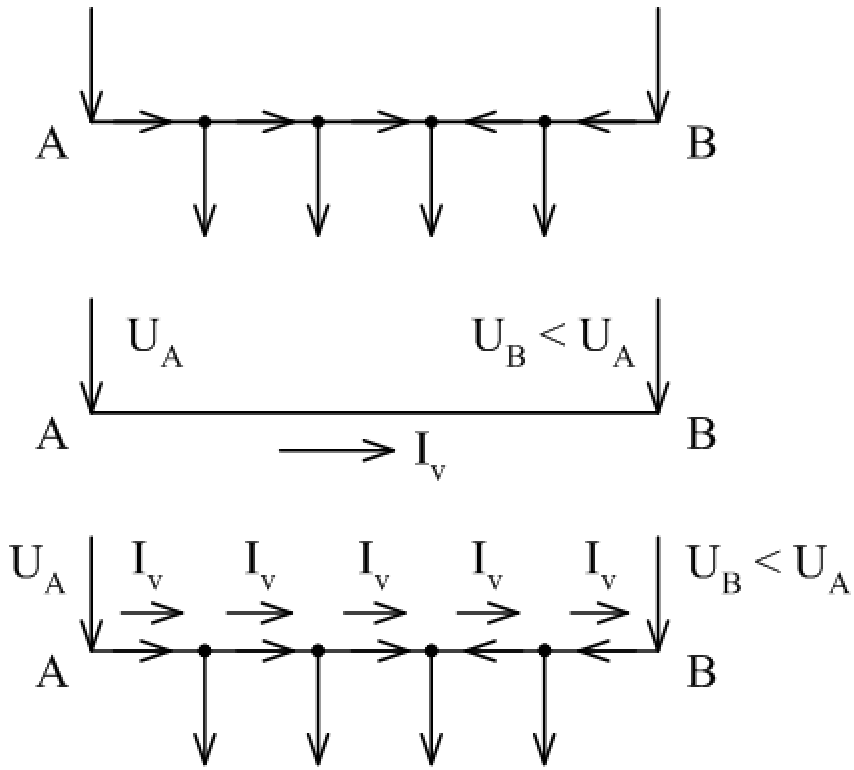


Superposition:

1) Current distribution with the same voltages.
2) Different voltages and zero loads → balancing current

$$I_v = \frac{U_A - U_B}{2\frac{\rho}{S}l}$$

3) Sum of the solutions 1+2



Further calculation is the same.

Or directly:

$$U_A - U_B = 2\frac{\rho}{S}\sum_{k=1}^{n} l_k I_k - 2\frac{\rho}{S} l I_B$$

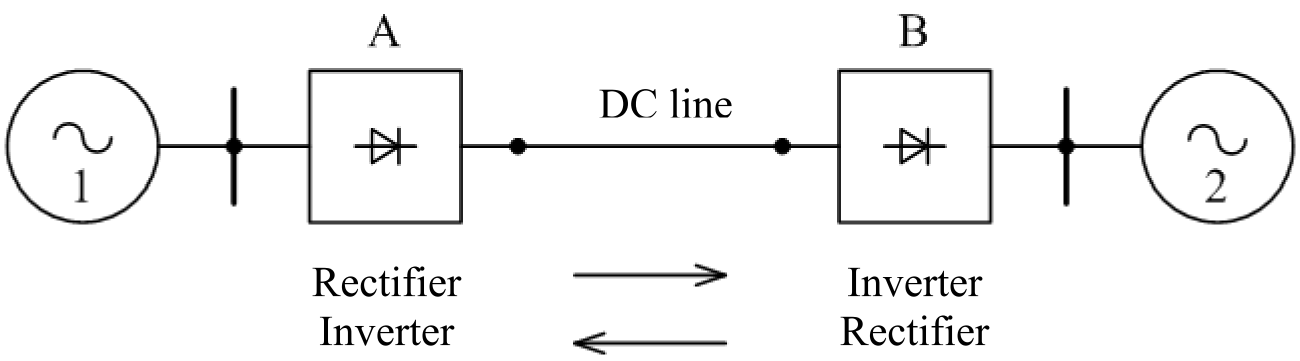
$$I_B = \frac{2\frac{\rho}{S}\sum_{k=1}^{n} l_k I_k}{2\frac{\rho}{S}l} - \frac{U_A - U_B}{2\frac{\rho}{S}l}$$

Direct current transmission HV

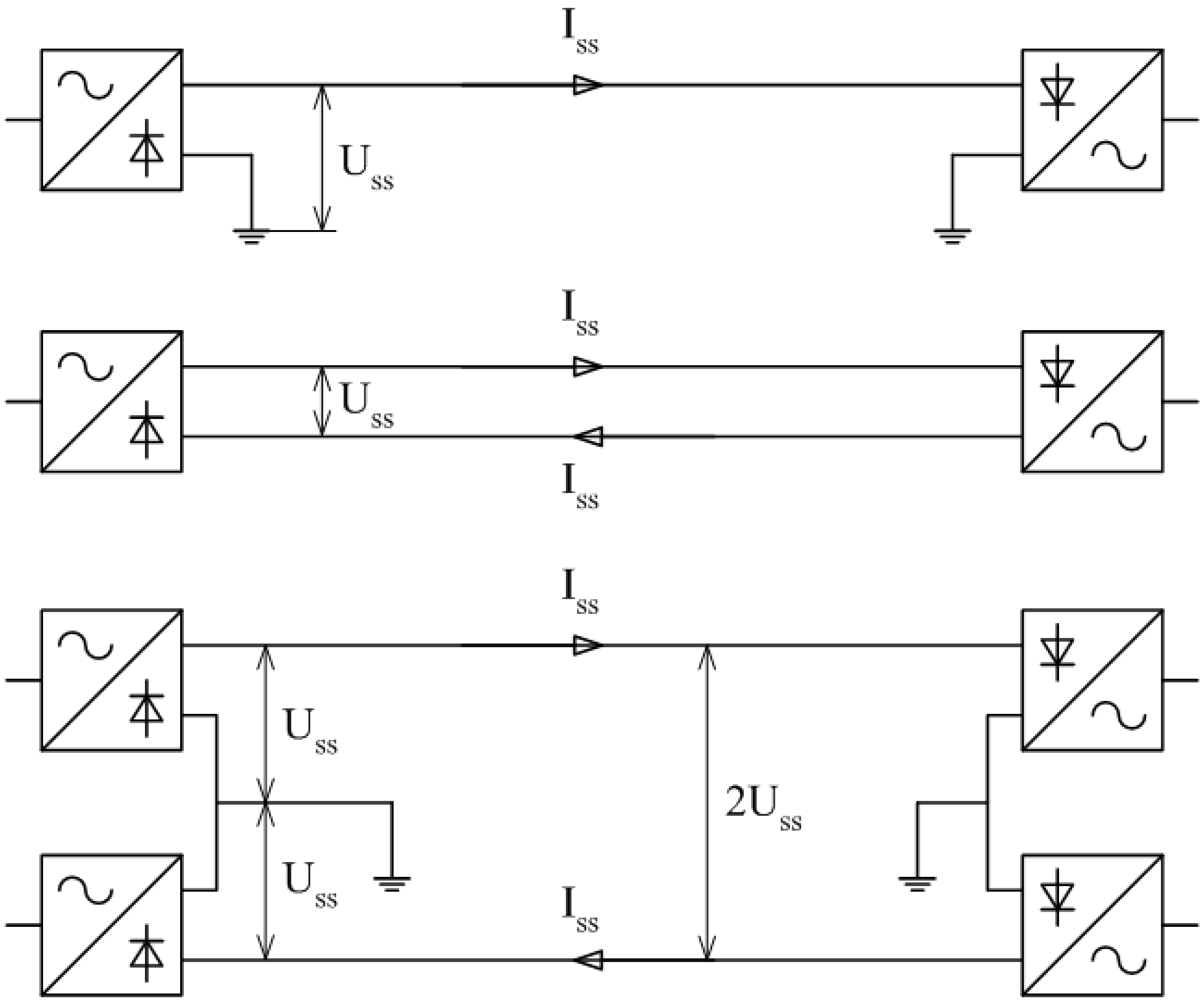
Long distance transmission or local stations (AC-DC-AC).
Reasons: transmission stability, short-circuit conditions, parameters compensation, power losses, economic aspects, power systems connection.

Possibility of smaller insulation distances and higher transmission capacity than for AC systems.
Control by DC voltage.

Principle

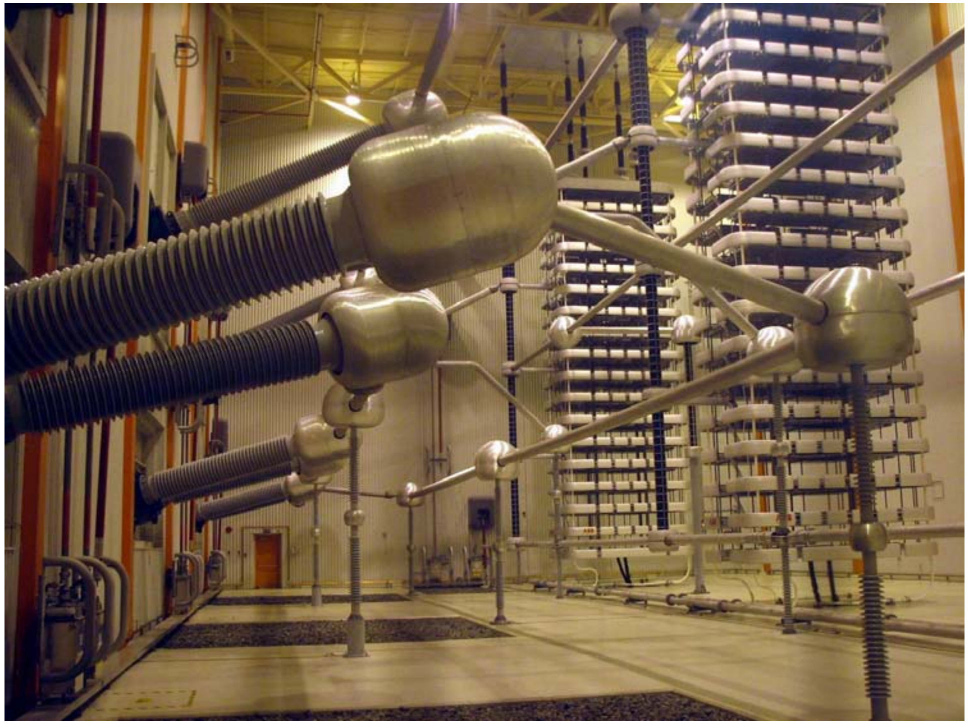


Configurations:



a) Ground as return conductor. Ground resistance does not depend on line length but on ground connection. Suitable for unoccupied regions (corrosion, EMC).
b) Two wires line. Compared to 3ph less material, lighter towers.
c) Two convertors in series. Low balancing ground current caused by unbalance. During a fault it transforms to a), half power.

450 kV DC Canada

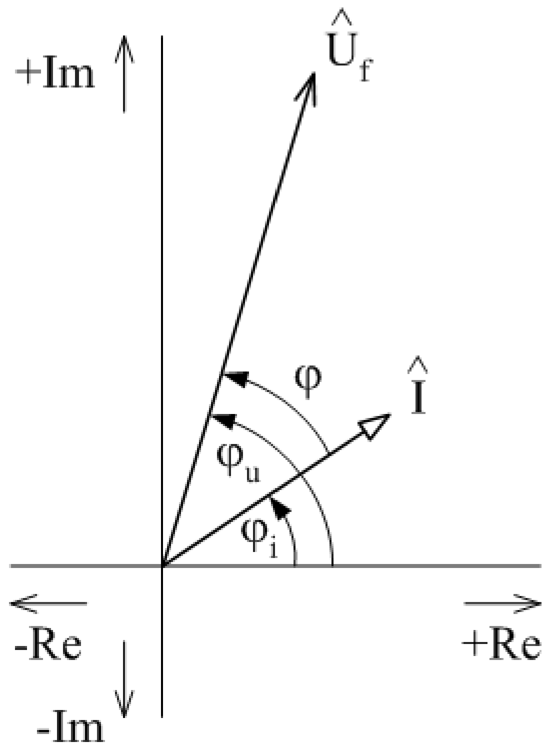


Complex power in AC grids

3 phase
$$P = 3U_f I\cos\varphi = \sqrt{3}UI\cos\varphi \quad (W)$$
$$Q = 3U_f I\sin\varphi = \sqrt{3}UI\sin\varphi \quad (VAr)$$
$$S = 3U_f I = \sqrt{3}UI = \sqrt{P^2 + Q^2} \quad (VA)$$

Complex (1 phase)
$$\hat{S}_f = P_f \pm jQ_f = U_f I(\cos\varphi \pm j\sin\varphi) = S_f e^{\pm j\varphi}$$
Sign according a convention.

Inductive load
$$\hat{U}_f = U_f e^{j\varphi_u}, \; \hat{I}_f = I e^{j\varphi_i}$$
Complex conjugated current
$$\hat{S}_f = \hat{U}_f \hat{I}^* = U_f I e^{j(\varphi_u - \varphi_i)} = U_f I e^{j\varphi}$$
$$\hat{S}_f = \hat{U}_f \hat{I}^* = P_f \pm jQ_f \begin{matrix} \text{IND} \\ \text{CAP} \end{matrix}$$

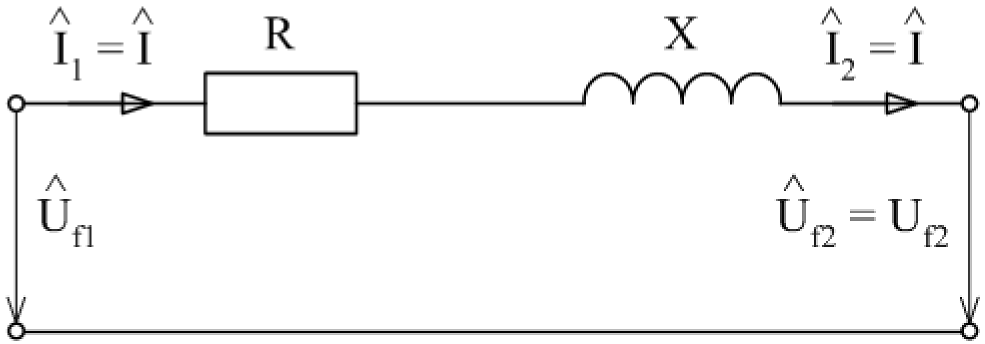


3 phase power lines LV, MV

Series parameters are applied, for LV X→0.

3 phase power line MV, 1 load at the end

Symmetrical load → 1 phase diagram, operational parameters.



Complex voltage drop

$$\Delta\hat{U}_f = \hat{Z}_1\hat{I} = (R + jX)(I_č \mp jI_j) \begin{matrix} \text{IND} \\ \text{CAP} \end{matrix}$$

$$\Delta\hat{U}_f = RI_č \pm XI_j + j(XI_č \mp RI_j) \begin{matrix} \text{IND} \\ \text{CAP} \end{matrix}$$

magnitude phase

Phasor diagram (input U_{f2}, I, φ_2)
(angle υ usually small, up to 3°)

Imagin. part neglecting and modifications

$$\Delta U_f = \frac{R 3 U_f I_č \pm X 3 U_f I_j}{3U_f} = \frac{RP \pm XQ}{3U_f}$$

Percentage voltage drop

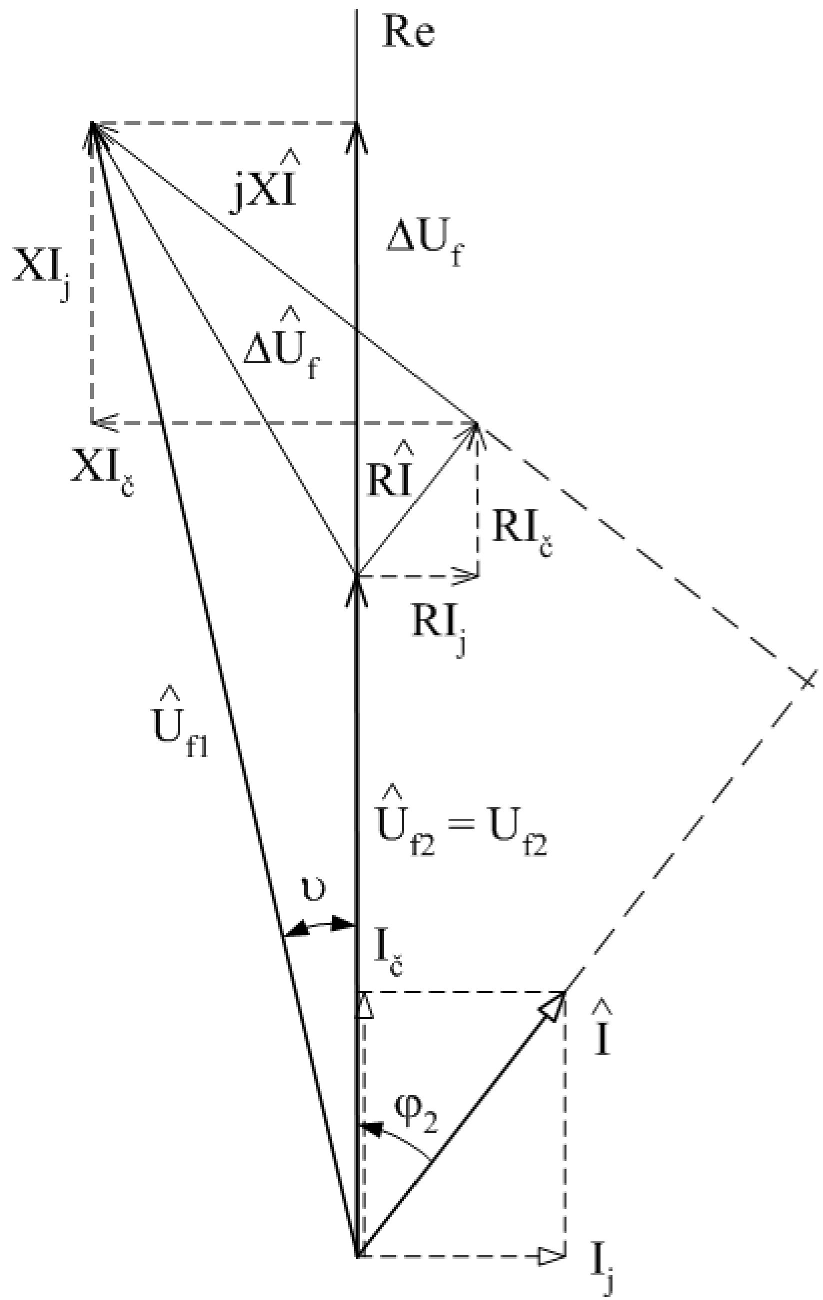
$$\varepsilon = \frac{\Delta U_f}{U_f} = \frac{RP \pm XQ}{3U_f^2} = \frac{RP \pm XQ}{U^2}$$

3 phase active power losses

$$\Delta \hat{S} = 3\Delta \hat{U}_f \hat{I}^* = 3\hat{Z}_1 \hat{I} \cdot \hat{I}^* = 3\hat{Z}_1 I^2 =$$

$$= 3(R + jX)I^2 = 3RI^2 + j3XI^2$$

$$\Delta P = 3RI^2 = 3R\left(I_č^2 + I_j^2\right) \quad (W; \Omega, A)$$

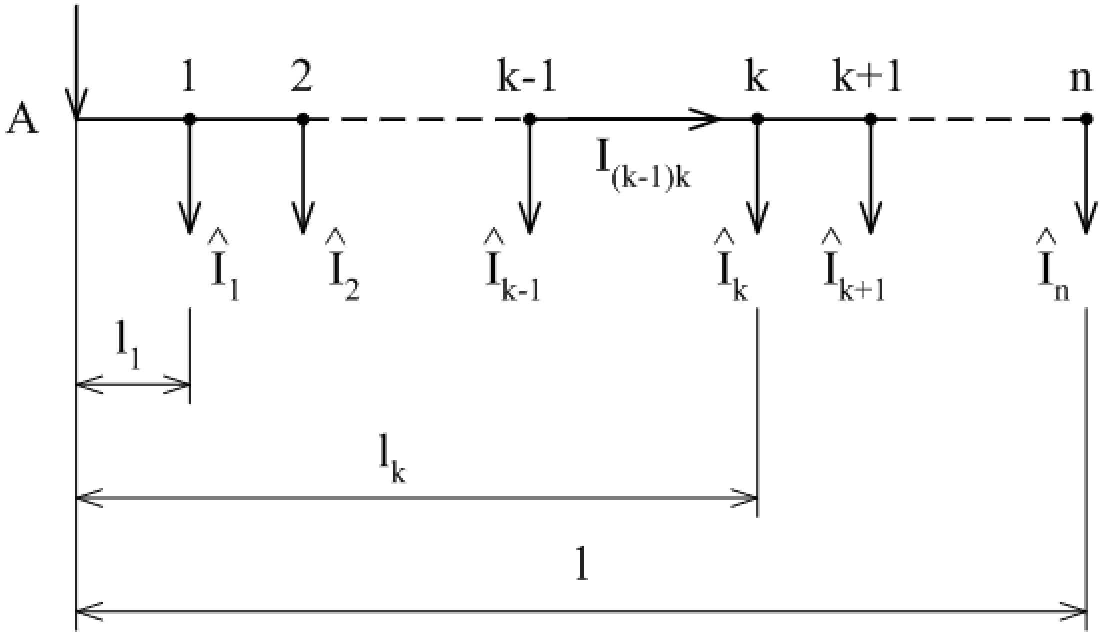


! Even the reactive current causes active power losses!
→ reactive power compensation

3 phase MV power line supplied from one side

Constant series impedance

$$\hat{Z}_{l_1} = R_1 + jX_1 \quad (\Omega / km)$$



Voltage drop at the end (needn't be the highest one, it depends on load character)

$$\Delta \hat{U}_{fAn} = \hat{Z}_{l_1} \sum_{k=1}^{n} l_k \hat{I}_k$$

After imaginary part neglecting

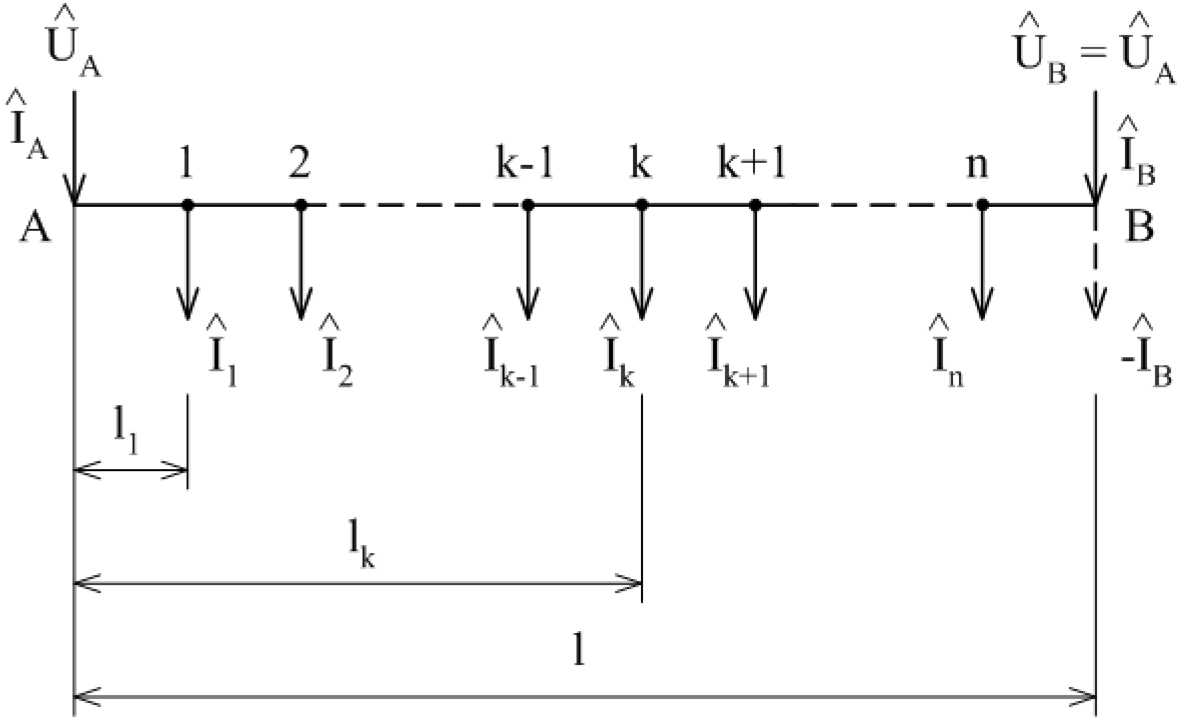
$$\Delta U_{fAn} \doteq R_1 \sum_{k=1}^{n} l_k I_{čk} \pm X_1 \sum_{k=1}^{n} l_k I_{jk} \quad \begin{matrix} \text{IND} \\ \text{CAP} \end{matrix}$$

$$\Delta U_{fAn} \doteq \frac{R_1 \sum_{k=1}^{n} l_k P_k \pm X_1 \sum_{k=1}^{n} l_k Q_k}{3U_f} \quad \begin{matrix} \text{IND} \\ \text{CAP} \end{matrix}$$

Voltage drop up to the point X (superposition)

$$\Delta \hat{U}_{fAX} = \hat{Z}_{l_1} \sum_{k=1}^{X} l_k \hat{I}_k + \hat{Z}_{l_1} l_{AX} \sum_{k=X+1}^{n} \hat{I}_k$$

3 phase MV power line supplied from both sides



Calculation as for DC line (feeder is a negative load, zero voltage drop).

$$\Delta \hat{U}_{AB} = 0 = \hat{Z}_{l_1} \sum_{k=1}^{n} l_k \hat{I}_k - \hat{Z}_{l_1} l \cdot \hat{I}_B$$

Moment theorems

$$\hat{I}_B = \frac{\sum_{k=1}^{n} l_k \hat{I}_k}{l} \qquad \hat{I}_A = \frac{\sum_{k=1}^{n} (l - l_k) \hat{I}_k}{l} \qquad \hat{I}_A + \hat{I}_B = \sum_{y=1}^{n} \hat{I}_y$$

(In principle it is the current divider for each load.)

Active and reactive current sign change could be in different nodes → maximum voltage drop should be checked in all grid points.

Meshed grids MV

NO!

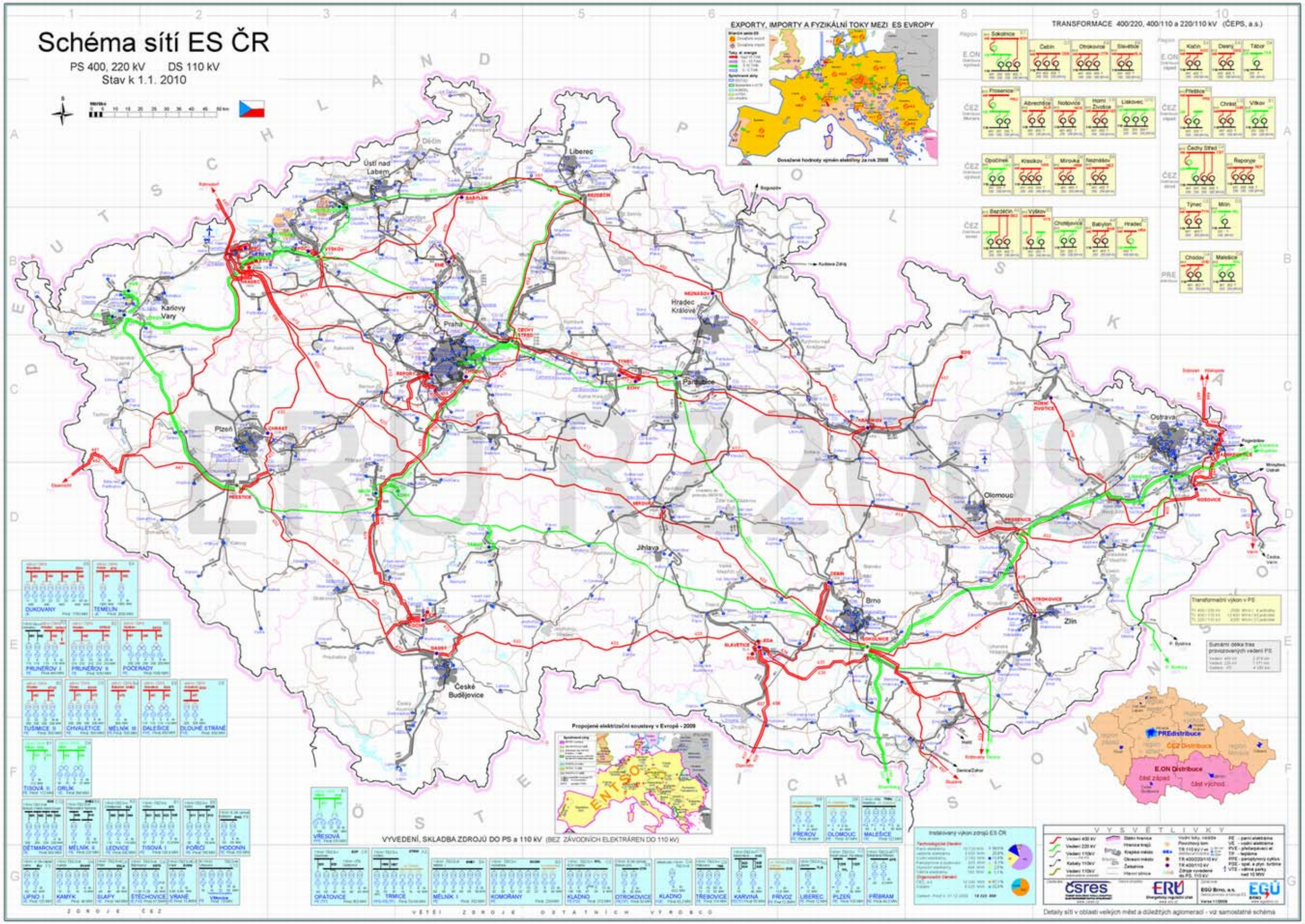


Schéma sítí ES ČR
PS 400, 220 kV DS 110 kV
Stav k 1.1. 2010
EXPORTY, IMPORTY A FYZIKÁLNÍ TOKY MEZI ES EVROPY
TRANSFORMACE 400/220, 400/110 a 220/110 kV (ČEPS, a.s.)
Praha
Plzeň
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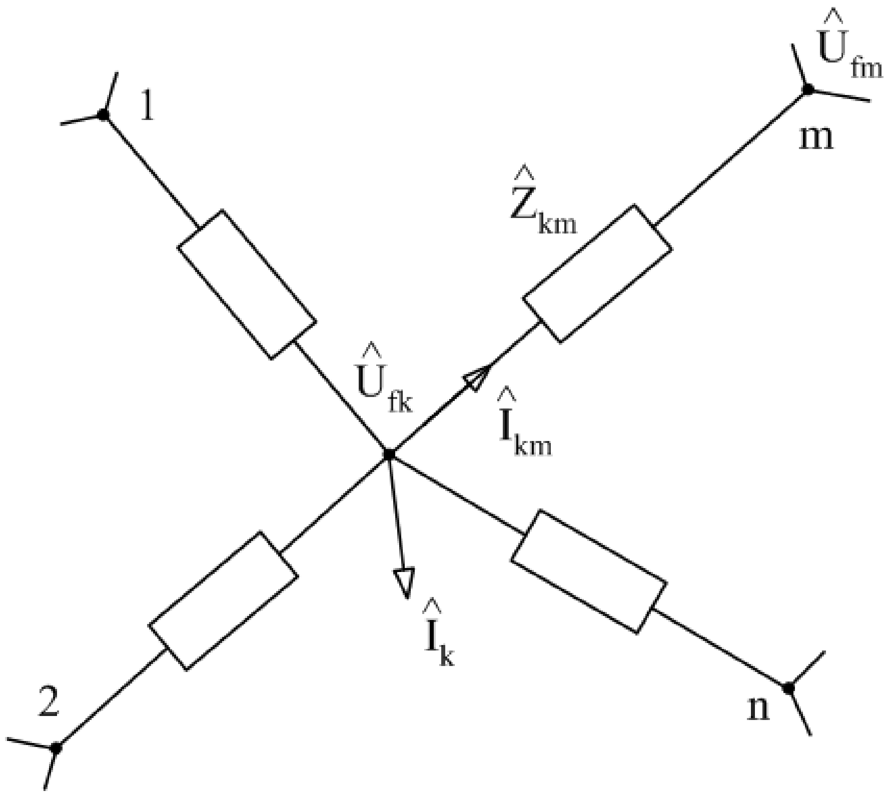
VYVEDENÍ, SKLADBA ZDROJŮ DO PS a 110 kV (BEZ ZÁVODNÍCH ELEKTRÁREN DO 110 kV)
PREdistribuce
ČEZ Distribuce
E.ON Distribuce
V Y S V Ě T L I V K Y

YES!

Node voltage method

Grid with *n* nodes. Set series branch parameters $\hat{Z}_{km}$, load currents (nodal currents) $\hat{I}_k$, min. 1 node voltage $\hat{U}_{fk}$ (between the node and the ground).



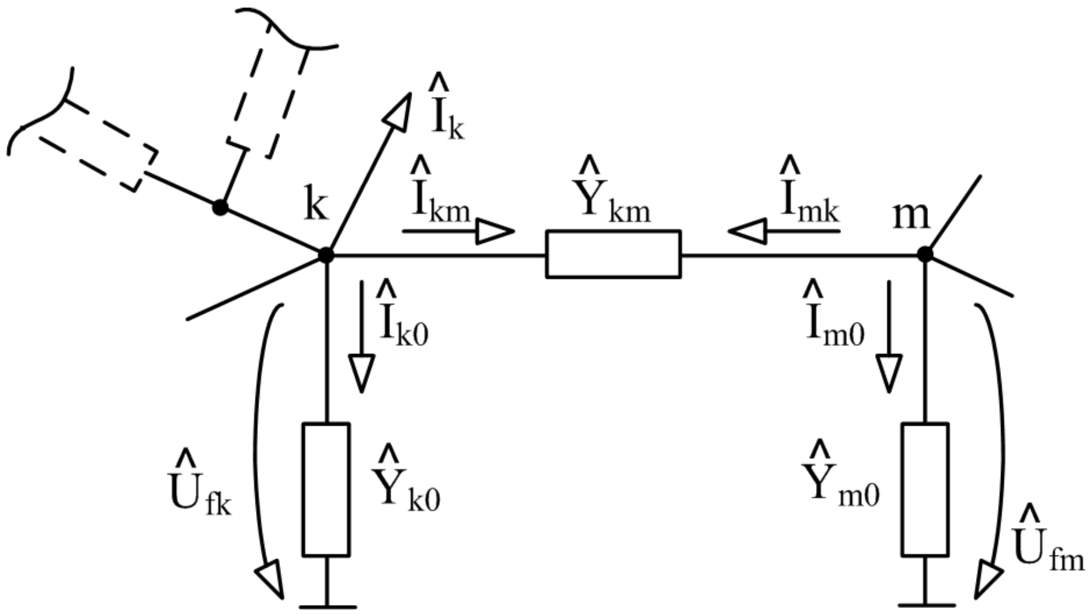
Calculation with series admittances

$$\hat{Y}_{km} = \hat{Z}_{km}^{-1} = \frac{1}{R_{km} + jX_{km}}$$

Node *k*

$$\hat{I}_k + \sum_{\substack{m=1 \\ m \neq k}}^{n} \hat{I}_{km} + \hat{I}_{k0} = 0$$

$$\hat{I}_{k0} = \hat{U}_{fk} \hat{Y}_{k0}$$



Branches *k, m*

$$\hat{I}_{km} = \left(\hat{U}_{fk} - \hat{U}_{fm}\right)\hat{Y}_{km}$$

After modifications:

$$\hat{I}_k = -\sum_{\substack{m=1 \\ m \neq k}}^{n} \left(\hat{U}_{fk} - \hat{U}_{fm}\right)\hat{Y}_{km} - \hat{U}_{fk} \hat{Y}_{k0}$$

$$\hat{I}_k = -\hat{U}_{fk}\left(\sum_{\substack{m=1\\m\neq k}}^{n}\hat{Y}_{km} + \hat{Y}_{k0}\right) + \sum_{\substack{m=1\\m\neq k}}^{n}\hat{U}_{fm}\hat{Y}_{km}$$

Admittance matrix parameters definition:

Nodal self-admittance (diagonal element)

$$\hat{Y}_{(k,k)} = -\sum_{\substack{m=1\\m\neq k}}^{n}\hat{Y}_{km} - \hat{Y}_{k0}$$

Between nodes admittance (non-diagonal element)

$$\hat{Y}_{(k,m)} = \hat{Y}_{(m,k)} = \hat{Y}_{km} \quad \text{for } m \neq k$$

(for non-connected nodes $\hat{Y}_{(k,m)} = 0$)

Hence

$$\hat{I}_k = \sum_{m=1}^{n}\hat{Y}_{(k,m)}\hat{U}_{fm}$$

Matrix form

$$(\hat{I}) = (\hat{Y})(\hat{U}_f)$$

Set voltages at nodes 1 to *k* (x), currents at nodes *k+1* to *n* (y)

$$\begin{pmatrix} (\hat{I}_x) \\ (\hat{I}_y) \end{pmatrix} = \begin{pmatrix} (\hat{Y}_{xx}) & (\hat{Y}_{xy}) \\ (\hat{Y}_{xy})^T & (\hat{Y}_{yy}) \end{pmatrix} \begin{pmatrix} (\hat{U}_{fx}) \\ (\hat{U}_{fy}) \end{pmatrix}$$

Hence

$$(\hat{I}_x) = (\hat{Y}_{xx})(\hat{U}_{fx}) + (\hat{Y}_{xy})(\hat{U}_{fy})$$

$$(\hat{I}_y) = (\hat{Y}_{xy})^T(\hat{U}_{fx}) + (\hat{Y}_{yy})(\hat{U}_{fy})$$

Calculate $(\hat{I}_x), (\hat{U}_{fy})$

$$(\hat{U}_{fy}) = (\hat{Y}_{yy})^{-1}(\hat{I}_y) - (\hat{Y}_{yy})^{-1}(\hat{Y}_{xy})^T(\hat{U}_{fx})$$

If some nodes are connected to the ground (through an admittance), then the admittance matrix is regular → to set all nodal current is enough.

$$\left(\hat{U}_f\right) = \left(\hat{Y}\right)^{-1}\left(\hat{I}\right)$$

Note 1: Similar for DC grid.

$$(I) = (G)(U)$$

Note 2: For power engineering – powers are set, currents are calculated from the powers.

$$\hat{I} = \left(\frac{\hat{S}}{\sqrt{3}\hat{U}}\right)^*$$

Results are not precise if nominal voltages are used → iteration methods.