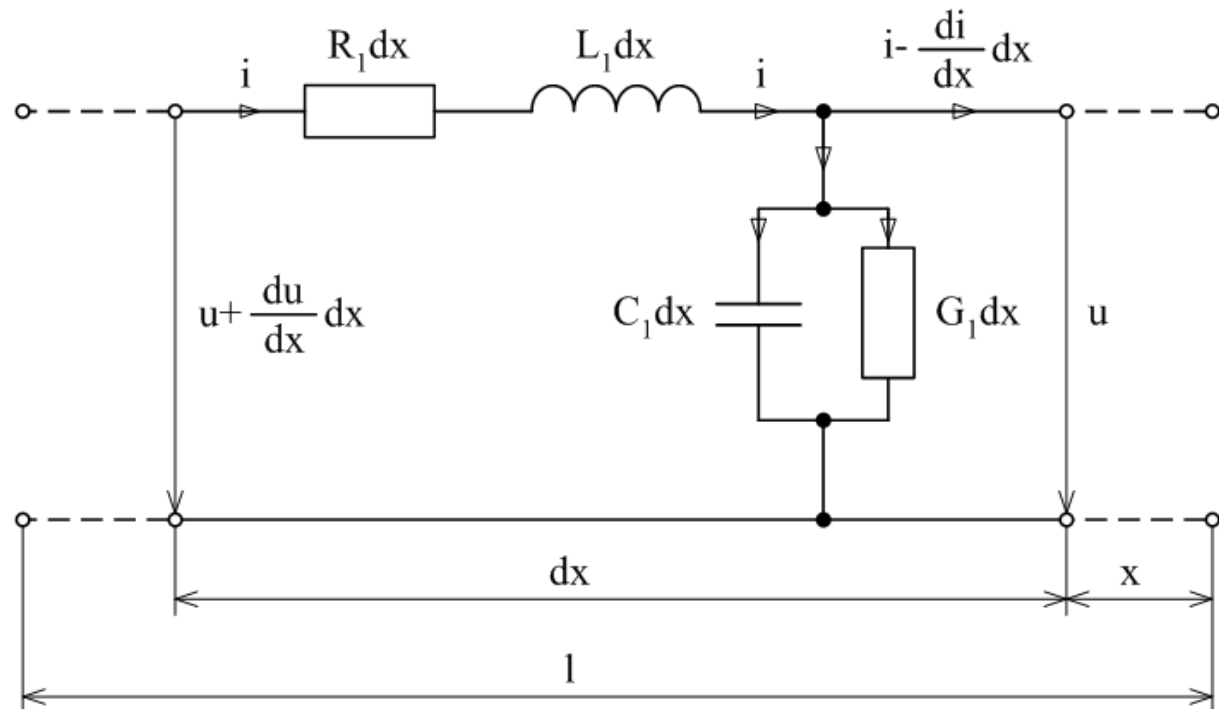# Three phase HV power lines

Transmission systems, international connections.
Aim: relations between sending and receiving ends conditions, losses, efficiency. RLGC

Line with equally distributed parameters

Homogeneous line - parameters R_1, L_1, G_1, C_1 are equally distributed along the total line length.

2nd Kirchhoff's law

$$u + \frac{\partial u}{\partial x}dx - u - R_1 dx \cdot i - L_1 dx \frac{\partial i}{\partial t} = 0$$

$$\frac{\partial u}{\partial x} = R_1 i + L_1 \frac{\partial i}{\partial t}$$

1st Kirchhoff's law

$$i - \frac{\partial i}{\partial x}dx - i + G_1 dx \cdot u + C_1 dx \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial i}{\partial x} = G_1 u + C_1 \frac{\partial u}{\partial t}$$

For AC voltage (current)

$$u(t) = \mathrm{Im}\left\{\sqrt{2}\,\hat{U}_f e^{j\omega t}\right\}$$

$$\frac{\partial u(t)}{\partial x} = \sqrt{2}\,\mathrm{Im}\left\{\frac{d\hat{U}_f}{dx}e^{j\omega t}\right\} \qquad \frac{\partial u(t)}{\partial t} = \sqrt{2}\,\mathrm{Im}\left\{j\omega\hat{U}_f e^{j\omega t}\right\}$$

After substitution

$$\frac{d\hat{U}_f}{dx} = \left(R_1 + j\omega L_1\right)\hat{I} = \hat{Z}_{l_1}\hat{I}$$

$$\frac{d\hat{I}}{dx} = \left(G_1 + j\omega C_1\right)\hat{U}_f = \hat{Y}_{q_1}\hat{U}_f$$

After derivation and substitution we get wave equations

$$\frac{d^2\hat{U}_f}{dx^2} = \hat{Z}_{l_1}\frac{d\hat{I}}{dx} = \hat{Z}_{l_1}\hat{Y}_{q_1}\hat{U}_f = \hat{\gamma}^2\hat{U}_f$$

$$\frac{d^2\hat{I}}{dx^2} = \hat{Y}_{q_1}\frac{d\hat{U}_f}{dx} = \hat{Z}_{l_1}\hat{Y}_{q_1}\hat{I} = \hat{\gamma}^2\hat{I}$$

Complex propagation constant

$$\hat{\gamma} = \sqrt{\hat{Z}_{l_1}\hat{Y}_{q_1}} \quad \left(km^{-1}; \Omega \cdot km^{-1}, S \cdot km^{-1}\right)$$

Note: For time domain it is telegraph equation (mathematically it is wave equation for R = G = 0)

$$\frac{\partial^2 u(t,x)}{\partial x^2} = R_1 G_1 u(t,x) + \left(R_1 C_1 + L_1 G_1\right)\frac{\partial u(t,x)}{\partial t} + L_1 C_1 \frac{\partial^2 u(t,x)}{\partial t^2}$$

General wave equations solution, i.e. second-order linear ordinary differential equation (char. equation $\hat{\lambda}^2 - \hat{\gamma}^2 = 0$) – progressive and reflected wave

$$\hat{U}_f = \hat{K}_1 e^{\hat{\gamma}x} + \hat{K}_2 e^{-\hat{\gamma}x}$$

$$\hat{I} = \frac{d\hat{U}_f}{dx}\frac{1}{\hat{Z}_{l_1}} = \frac{\hat{\gamma}}{\hat{Z}_{l_1}}\left(\hat{K}_1 e^{\hat{\gamma}x} - \hat{K}_2 e^{-\hat{\gamma}x}\right) = \sqrt{\frac{\hat{Y}_{q_1}}{\hat{Z}_{l_1}}}\left(\hat{K}_1 e^{\hat{\gamma}x} - \hat{K}_2 e^{-\hat{\gamma}x}\right)$$

$$\hat{I} = \frac{1}{\hat{Z}_v}\left(\hat{K}_1 e^{\hat{\gamma}x} - \hat{K}_2 e^{-\hat{\gamma}x}\right)$$

Surge (wave, characteristic) impedance

$$\hat{Z}_v = \sqrt{\frac{\hat{Z}_{l_1}}{\hat{Y}_{q_1}}} \quad \left(\Omega; \Omega \cdot km^{-1}, S \cdot km^{-1} \right)$$

Integration constants $\hat{K}_1, \hat{K}_2$ are calculated from boundary conditions. The line end (x = 0) index 2, the line beginning (x = l) index 1.

For x = 0:

$$\hat{U}_{f2} = \hat{K}_1 + \hat{K}_2$$

$$\hat{I}_2 = \frac{1}{\hat{Z}_v} \left(\hat{K}_1 - \hat{K}_2 \right)$$

Hence

$$\hat{K}_1 = \frac{1}{2} \left(\hat{U}_{f2} + \hat{Z}_v \hat{I}_2 \right)$$

$$\hat{K}_2 = \frac{1}{2} \left(\hat{U}_{f2} - \hat{Z}_v \hat{I}_2 \right)$$

For x = l:

$$\hat{U}_{f1} = \hat{U}_{f2} \frac{e^{\hat{\gamma}l} + e^{-\hat{\gamma}l}}{2} + \hat{Z}_v \hat{I}_2 \frac{e^{\hat{\gamma}l} - e^{-\hat{\gamma}l}}{2}$$

$$\hat{I}_1 = \frac{\hat{U}_{f2}}{\hat{Z}_v} \frac{e^{\hat{\gamma}l} - e^{-\hat{\gamma}l}}{2} + \hat{I}_2 \frac{e^{\hat{\gamma}l} + e^{-\hat{\gamma}l}}{2}$$

Definition of hyperbolic function

$$\hat{U}_{f1} = \hat{U}_{f2} \cosh \hat{\gamma}l + \hat{Z}_v \hat{I}_2 \sinh \hat{\gamma}l$$

$$\hat{I}_1 = \frac{\hat{U}_{f2}}{\hat{Z}_v} \sinh \hat{\gamma}l + \hat{I}_2 \cosh \hat{\gamma}l$$

Matrix form

$$\begin{pmatrix} \hat{U}_{f1} \\ \hat{I}_1 \end{pmatrix} = \begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix} \begin{pmatrix} \hat{U}_{f2} \\ \hat{I}_2 \end{pmatrix}$$

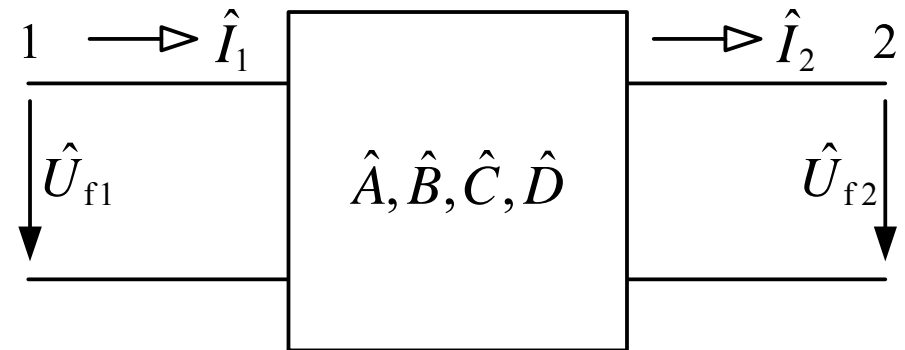where $\hat{A}\,(-), \hat{B}\,(\Omega), \hat{C}(S), \hat{D}(-)$ are Blondel's constants

it is valid $\hat{A} = \hat{D}, \hat{A}D - \hat{B}\hat{C} = 1$

$$\left(\cosh \hat{\gamma} l\right)^2 - \left(\sinh \hat{\gamma} l\right)^2 = 1$$
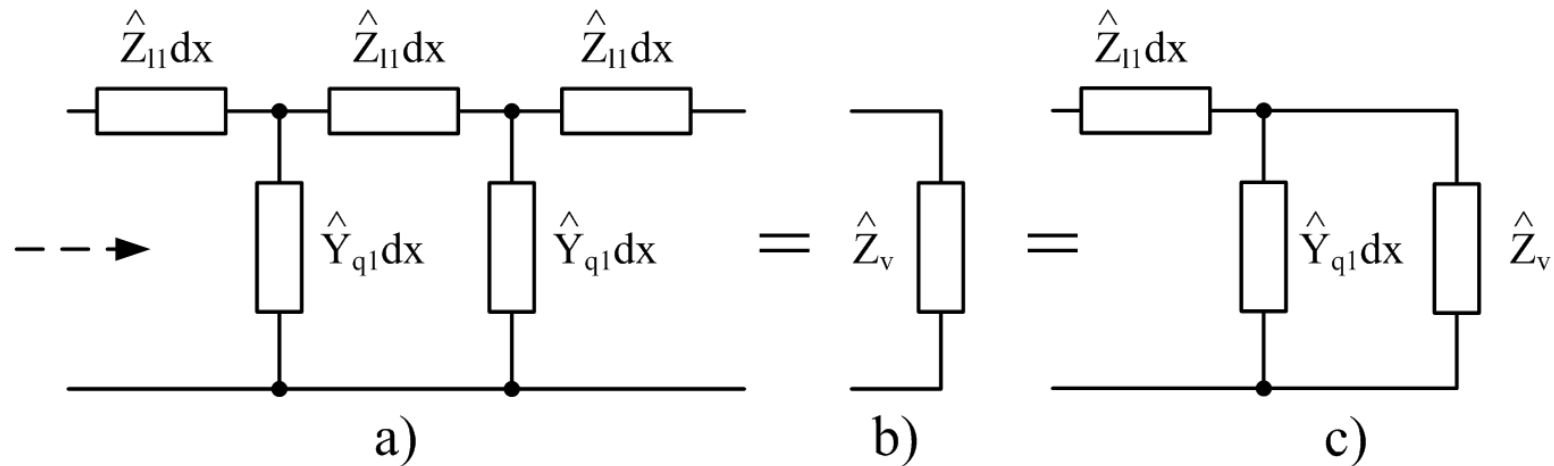
(symmetrical, passive two-port network)

Values at the line beginning are set

$$\begin{pmatrix} \hat{U}_{f2} \\ \hat{I}_2 \end{pmatrix} = \begin{pmatrix} \hat{D} & -\hat{B} \\ -\hat{C} & \hat{A} \end{pmatrix} \begin{pmatrix} \hat{U}_{f1} \\ \hat{I}_1 \end{pmatrix}$$

Surge impedance = impedance of the infinite long line

a)   b)   c)

Input impedance

$$\hat{Z}_v = \hat{Z}_{l_1} dx + \frac{\hat{Z}_v \cdot \left(\hat{Y}_{q_1} dx\right)^{-1}}{\hat{Z}_v + \left(\hat{Y}_{q_1} dx\right)^{-1}}$$

$$\hat{Z}_v^2 - \hat{Z}_{l_1} dx \cdot \hat{Z}_v - \hat{Z}_{l_1} dx \cdot \left(\hat{Y}_{q_1} dx\right)^{-1} = 0$$

$$\hat{Z}_v = \frac{\hat{Z}_{l_1} dx \pm \sqrt{\left(\hat{Z}_{l_1} dx\right)^2 + 4\hat{Z}_{l_1} dx \cdot \left(\hat{Y}_{q_1} dx\right)^{-1}}}{2}$$

Continuous distribution of parameters for dx → 0:

$$\hat{Z}_v = \sqrt{\frac{\hat{Z}_{1_1}}{\hat{Y}_{q_1}}}$$

Ideal line –R = 0 and G = 0. For higher voltage levels, the model does not allow to calculate losses.

$$\hat{\gamma} = \sqrt{(R_1 + jX_1)(G_1 + jB_1)} = j\sqrt{X_1 B_1} = j\beta$$

$$\hat{Z}_v = \sqrt{\frac{R_1 + jX_1}{G_1 + jB_1}} = \sqrt{\frac{L_1}{C_1}} = Z_v$$

Change to trigonometric functions

$$\cosh(j\beta l) = \frac{e^{j\beta l} + e^{-j\beta l}}{2} = \cos\beta l$$

$$\sinh(j\beta l) = \frac{e^{j\beta l} - e^{-j\beta l}}{2} = j\sin\beta l$$

Hence

$$\frac{\hat{U}_{f1} = \hat{U}_{f2} \cos \beta l + jZ_v \hat{I}_2 \sin \beta l}{}$$

$$\frac{\hat{I}_1 = j\dfrac{\hat{U}_{f2}}{Z_v} \sin \beta l + \hat{I}_2 \cos \beta l}{}$$

$$\left(\cos \beta l\right)^2 - \left(j \cdot \sin \beta l\right)^2 = 1$$

Surge impedance load (SIL) – the line end is loaded by the surge impedance (for lines transmission capacity comparison).
As an imaginary infinitely long line → power is transmitted only by progressive wave, reflected one is zero.

$$\hat{I}_2 = \frac{\hat{U}_{f2}}{\hat{Z}_v}$$

$$\hat{U}_{odr} = \frac{1}{2}\left(\hat{U}_{f2} - \hat{Z}_v \hat{I}_2\right)e^{-\hat{\gamma}l} = \hat{K}_2 e^{-\hat{\gamma}l} = 0$$

$$\hat{S}_{p2} = 3\hat{U}_{f2}\hat{I}_2^* = 3\hat{U}_{f2}\left(\frac{\hat{U}_{f2}}{\hat{Z}_v}\right)^* = \frac{U_2^2}{\hat{Z}_v^*}$$

(Active part is much bigger than imaginary one → often in MW.)

$$\hat{U}_{f1} = \hat{U}_{f2}\left(\cosh\hat{\gamma}l + \sinh\hat{\gamma}l\right)$$

$$\hat{I}_1 = \hat{I}_2\left(\sinh\hat{\gamma}l + \cosh\hat{\gamma}l\right)$$

Phase between voltage and current does not change → reactive powers at L and C are equal.

$$\frac{\hat{U}_{f(x)}}{\hat{I}_{(x)}} = \frac{\hat{U}_{f2}}{\hat{I}_2} = \hat{Z}_v$$

Voltage and current magnitudes (and active power) decrease along the line.

$$\hat{U}_{f(x)} = \hat{U}_{f2}\left(\cosh\hat{\gamma}x + \sinh\hat{\gamma}x\right) = \hat{U}_{f2}\cdot e^{\hat{\gamma}x} = \hat{U}_{f2}\cdot e^{\alpha x}\cdot e^{j\beta x}$$

Overhead line

$$Z_v = (250 \div 400)\,\Omega \quad \text{pro} \quad (400 \div 22)\,kV$$

$$S_p = (1 \div 580)\,MW \quad \text{pro} \quad (22 \div 400)\,kV$$

Lower for cable lines $\quad Z_v = (50 \div 70)\,\Omega \quad \rightarrow$ higher S_p

Voltage and current magnitudes don't decrease for ideal line:

$$\hat{U}_{f1} = \hat{U}_{f2}\cos\beta l + j\hat{U}_{f2}\sin\beta l = \hat{U}_{f2}e^{j\beta l}$$

$$U_{f1} = U_{f2}$$

$$\hat{I}_1 = \hat{I}_2 e^{j\beta l}$$

<u>Open-circuit</u>

$$\hat{I}_2 = 0$$

$$\hat{U}_{f10} = \hat{U}_{f2} \cosh \hat{\gamma} l$$

$$\hat{I}_{10} = \frac{\hat{U}_{f2}}{\hat{Z}_v} \sinh \hat{\gamma} l$$

For ideal line

$$\hat{U}_{f10} = \hat{U}_{f2} \cos \beta l$$

$$\hat{I}_{10} = j \frac{\hat{U}_{f2}}{Z_v} \sin \beta l$$

It is valid $U_{f10} \leq U_{f2}$ $\rightarrow$ Ferranti effect
Line character is like capacity.

<u>Short-circuit</u>

$$\hat{U}_{f2} = 0$$

$$\hat{U}_{f1} = \hat{Z}_v \hat{I}_2 \sinh \hat{\gamma} l$$

$$\hat{I}_1 = \hat{I}_2 \cosh \hat{\gamma} l$$

For ideal line

$$\hat{U}_{f1} = j Z_v \hat{I}_2 \sin \beta l$$

$$\hat{I}_1 = \hat{I}_2 \cos \beta l$$

Voltage decreases from the beginning to the end.
Line character is like inductance.

line 1 x 400 kV with two ground wires

phase conductor: 3xACSR 450/52, ground wire: ACSR 185/31, l = 300 km

$$R_1 = 0{,}021 \, \Omega / km; \; X_1 = 0{,}293 \, \Omega / km; \; G_1 = 2 \cdot 10^{-8} \, S / km; \; B_1 = 3{,}9 \cdot 10^{-6} \, S / km$$

Voltage level ($U_2 = 400$ kV)

U1(kV)
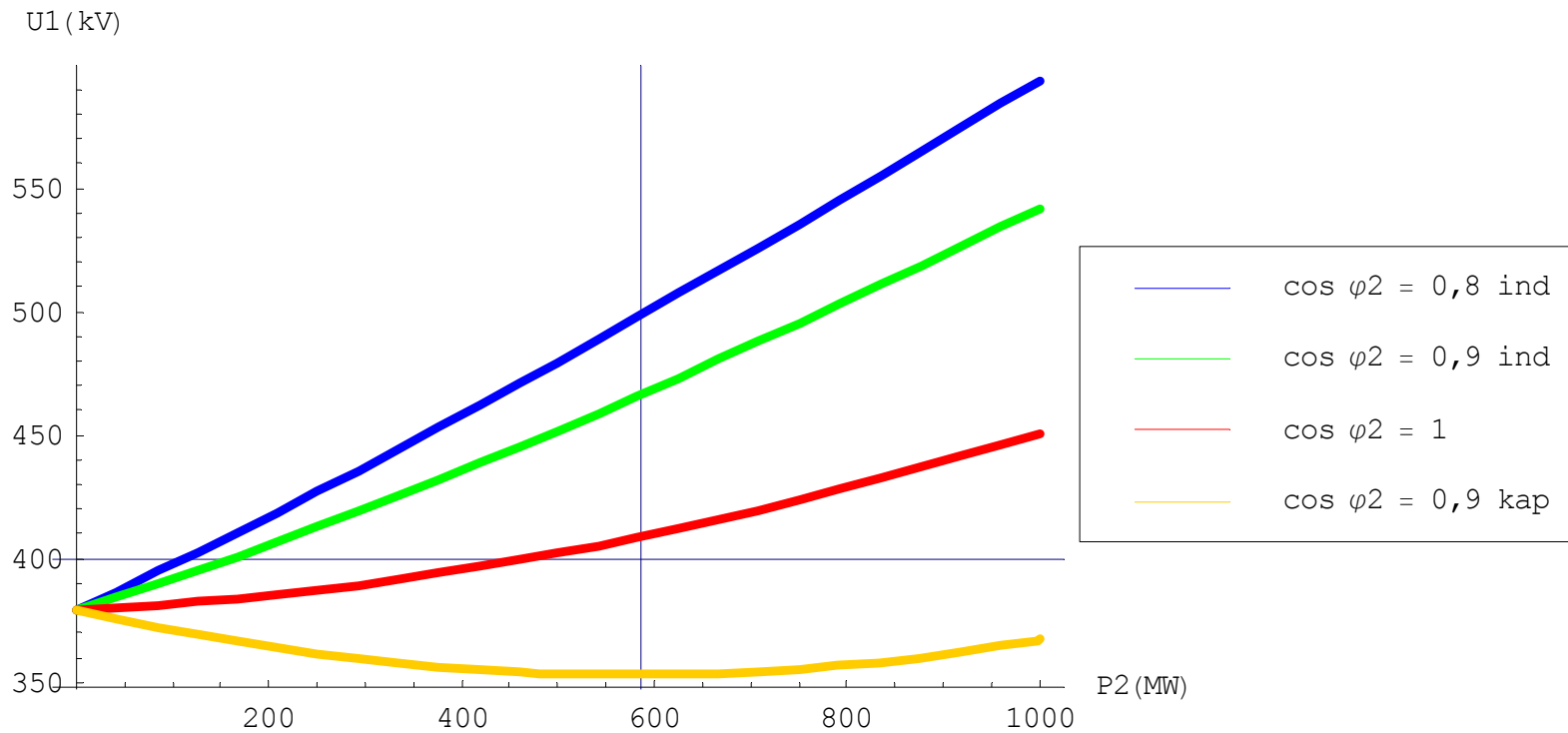
U1 < Un: Ferranti effect

U1 ~ Un for Sp area and cos φ = 1

Transmission power factor

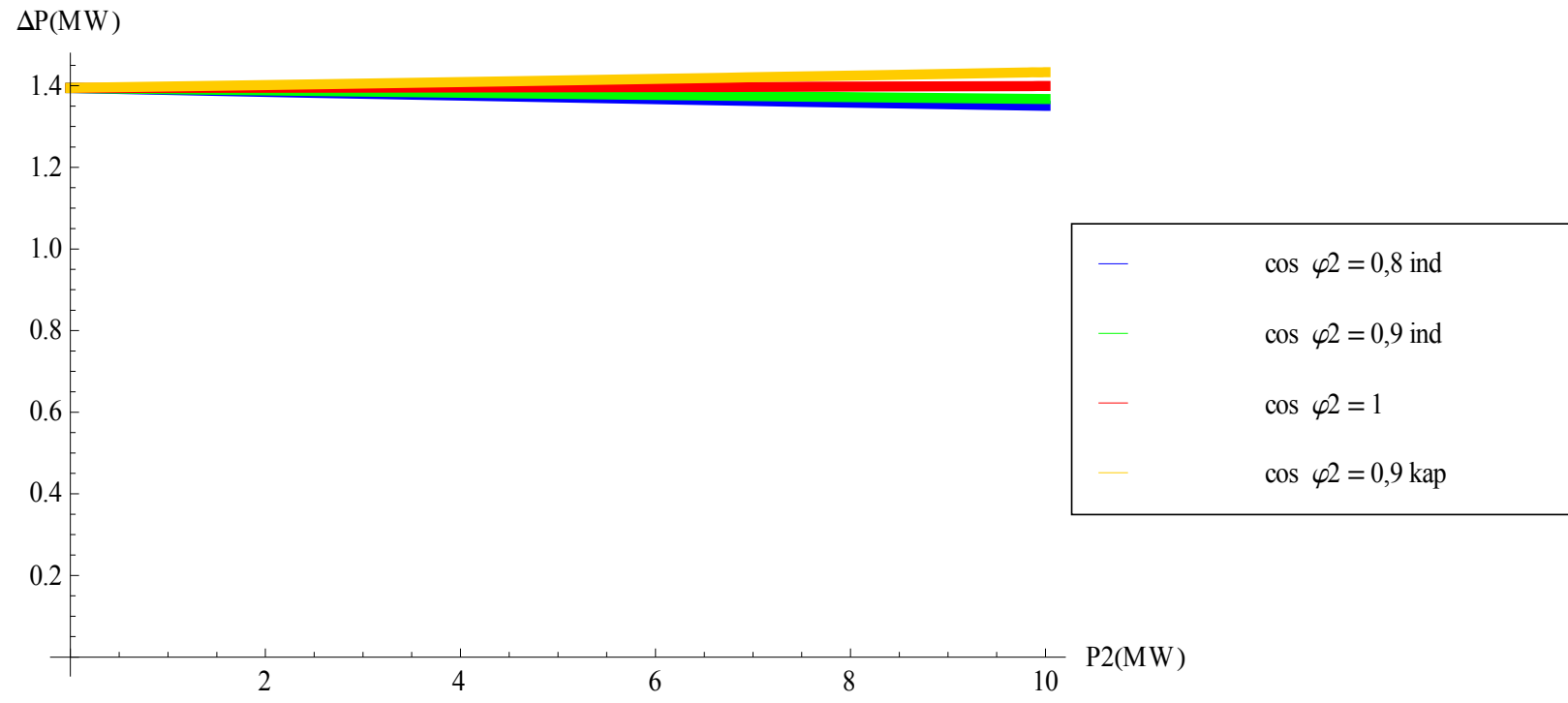$$\cos \varphi_1 = \frac{P_1}{S_1}$$
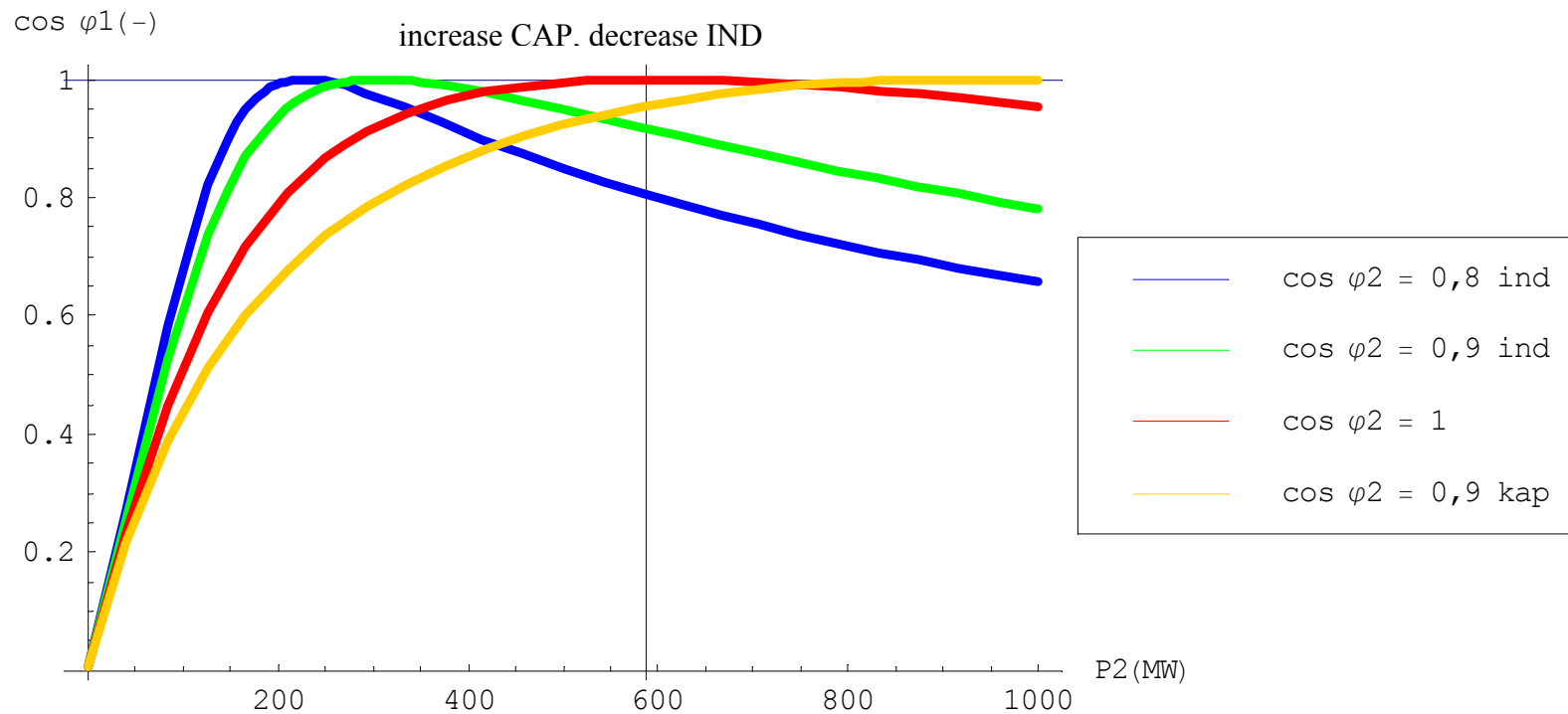
open-circuit → line is like capacitive load
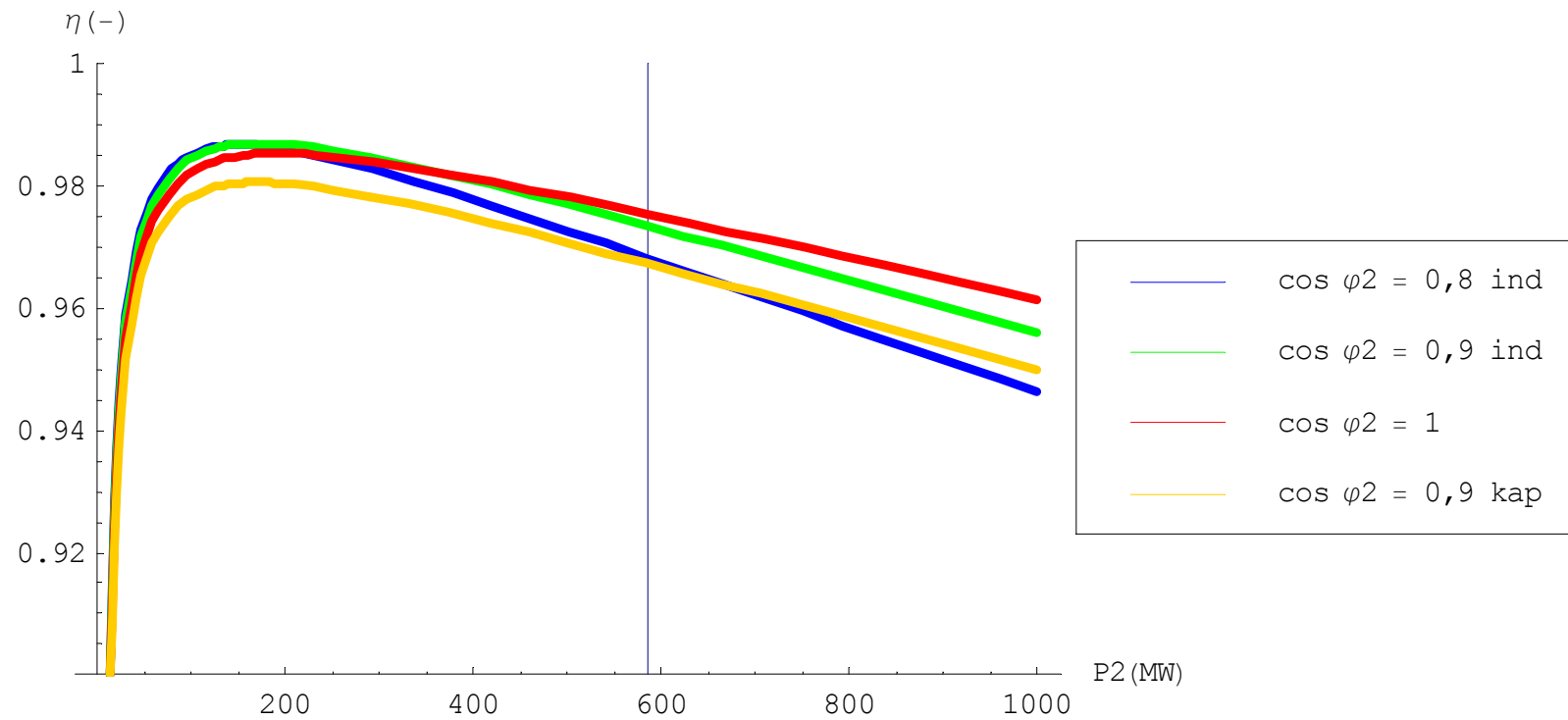higher power → line „self-compensation"

Line losses
= open-circuit ~ U^2 + load ~ I^2

$\Delta P(MW)$

P2(MW)

cos $\varphi 2 = 0{,}8$ ind

cos $\varphi 2 = 0{,}9$ ind

cos $\varphi 2 = 1$

cos $\varphi 2 = 0{,}9$ kap
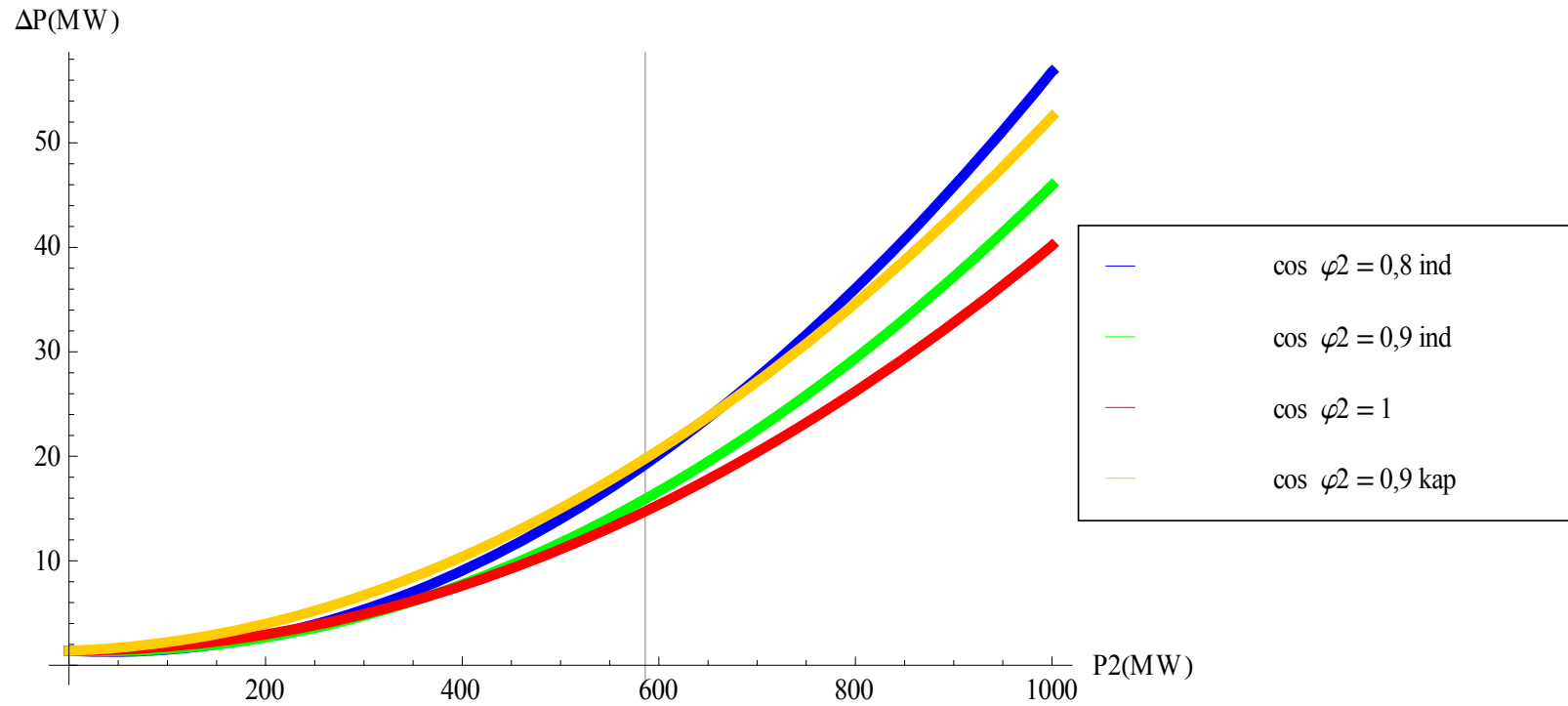
Transmission efficiency

$$\eta = \frac{P_2}{P_1}$$

maximum for low powers
for higher powers a flat curve

Hyperbolic functions series
(Taylor series at x = 0)

$$f(x)_{x_0} = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k = f(x_0) + f'(x_0) \cdot (x - x_0) + \frac{f''(x_0)}{2} \cdot (x - x_0)^2 + \ldots$$

$$\cosh \hat{\gamma} l = 1 + \frac{(\hat{\gamma} l)^2}{2} + \frac{(\hat{\gamma} l)^4}{24} + \ldots = 1 + \frac{\hat{Z}_{l_1} \hat{Y}_{q_1}}{2} l^2 + \ldots$$

$$\sinh \hat{\gamma} l = \hat{\gamma} l + \frac{(\hat{\gamma} l)^3}{6} + \ldots = \sqrt{\hat{Z}_{l_1} \hat{Y}_{q_1}} \, l + \frac{\left(\hat{Z}_{l_1} \hat{Y}_{q_1}\right)^{3/2}}{6} l^3 + \ldots$$
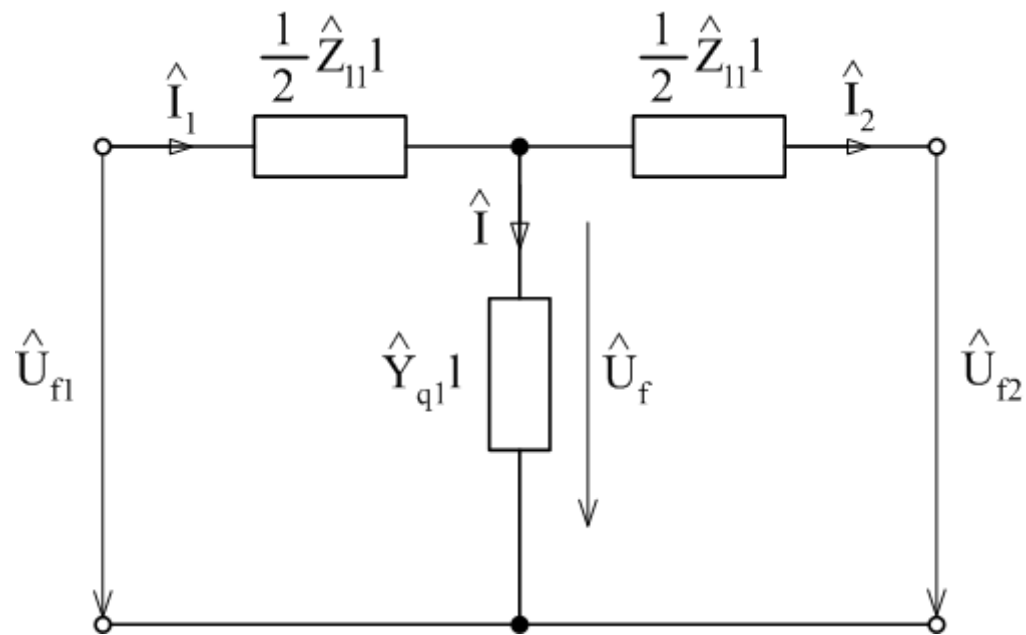
$$\hat{Z}_v \sinh \hat{\gamma} l = \sqrt{\frac{\hat{Z}_{l_1}}{\hat{Y}_{q_1}}} \sinh \hat{\gamma} l = \hat{Z}_{l_1} l + \hat{Z}_{l_1} l \frac{\hat{Z}_{l_1} \hat{Y}_{q_1}}{6} l^2 + \ldots$$

$$\frac{1}{\hat{Z}_v} \sinh \hat{\gamma} l = \sqrt{\frac{\hat{Y}_{q_1}}{\hat{Z}_{l_1}}} \sinh \hat{\gamma} l = \hat{Y}_{q_1} l + \hat{Y}_{q_1} l \frac{\hat{Z}_{l_1} \hat{Y}_{q_1}}{6} l^2 + \ldots$$

Line with lumped parameters

For ordinary calculations (meshed grids) it is possible to use substitution networks with a good accuracy (according to line length).

T-network – short lines, transformers; it adds another node (substitution diagram for overhead lines up to 200 km, cable lines up to 80 km)

Voltage and current at the line beginning

$$\hat{U}_{f1} = \hat{U}_{f2} + \frac{1}{2}\hat{Z}_{l_1}1\cdot\hat{I}_2 + \frac{1}{2}\hat{Z}_{l_1}1\cdot\hat{I}_1 \qquad\qquad \hat{I}_1 = \hat{I}_2 + \hat{I}$$

Voltage and current for the cross branch

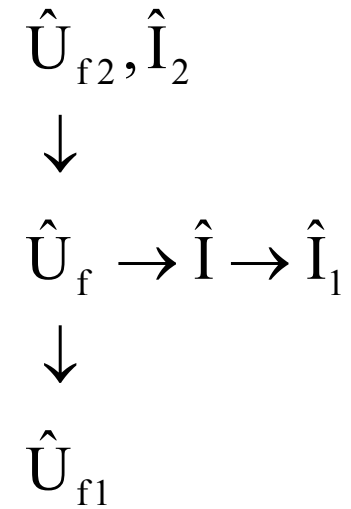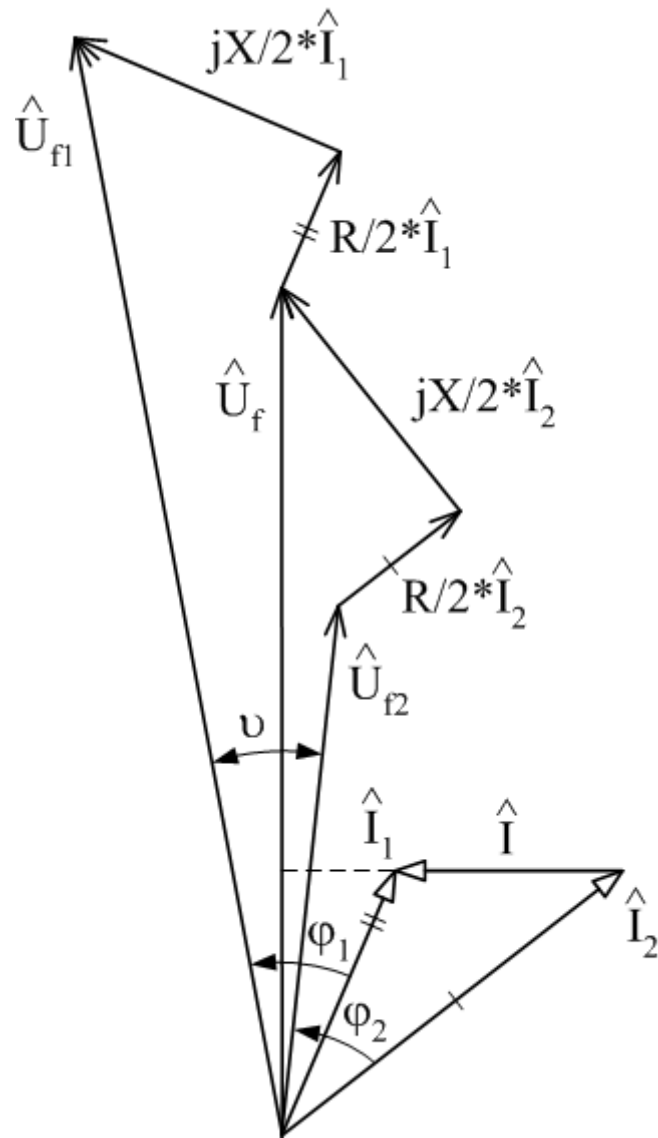$$\hat{U}_f = \hat{U}_{f2} + \frac{1}{2}\hat{Z}_{l_1}1\cdot\hat{I}_2$$

$$\hat{I} = \hat{Y}_{q_1}1\cdot\hat{U}_f = \hat{U}_{f2}\hat{Y}_{q_1}1 + \hat{I}_2\frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{2}1^2$$
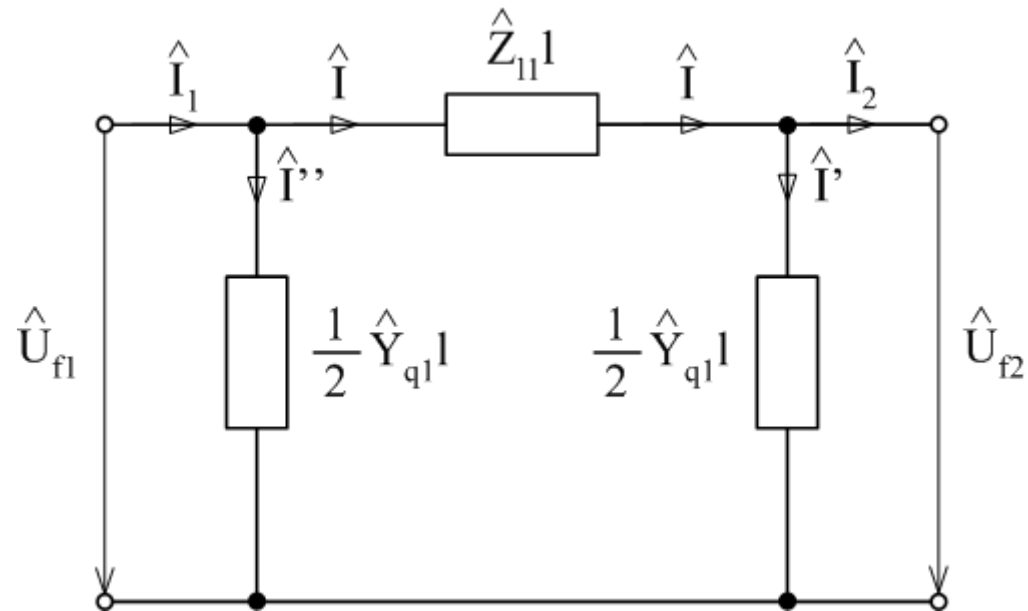
Hence (Blondel's constants relations)

$$\hat{U}_{f1} = \hat{U}_{f2}\left(1 + \frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{2}1^2\right) + \hat{I}_2\hat{Z}_{l_1}1\left(1 + \frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{4}1^2\right)$$

$$\hat{I}_1 = \hat{U}_{f2}\hat{Y}_{q_1}1 + \hat{I}_2\left(1 + \frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{2}1^2\right)$$

Phasor diagram (G = 0)

π-network – longer lines, more accurate (substitution diagram for overhead lines up to 250 km, cable lines up to 100 km)

Voltage and current at the line beginning

$$\hat{U}_{f1} = \hat{U}_{f2} + \hat{Z}_{l_1}l \cdot \hat{I} = \hat{U}_{f2} + \hat{Z}_{l_1}l \cdot \left(\hat{I}_2 + \hat{I}'\right)$$

$$\hat{I}_1 = \hat{I}_2 + \hat{I}' + \hat{I}''$$

Currents for cross branches
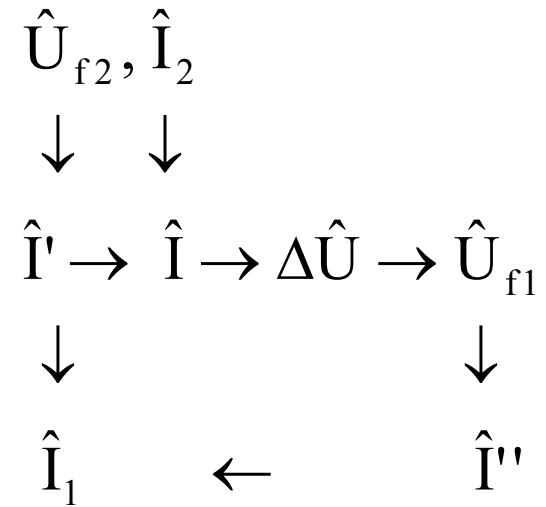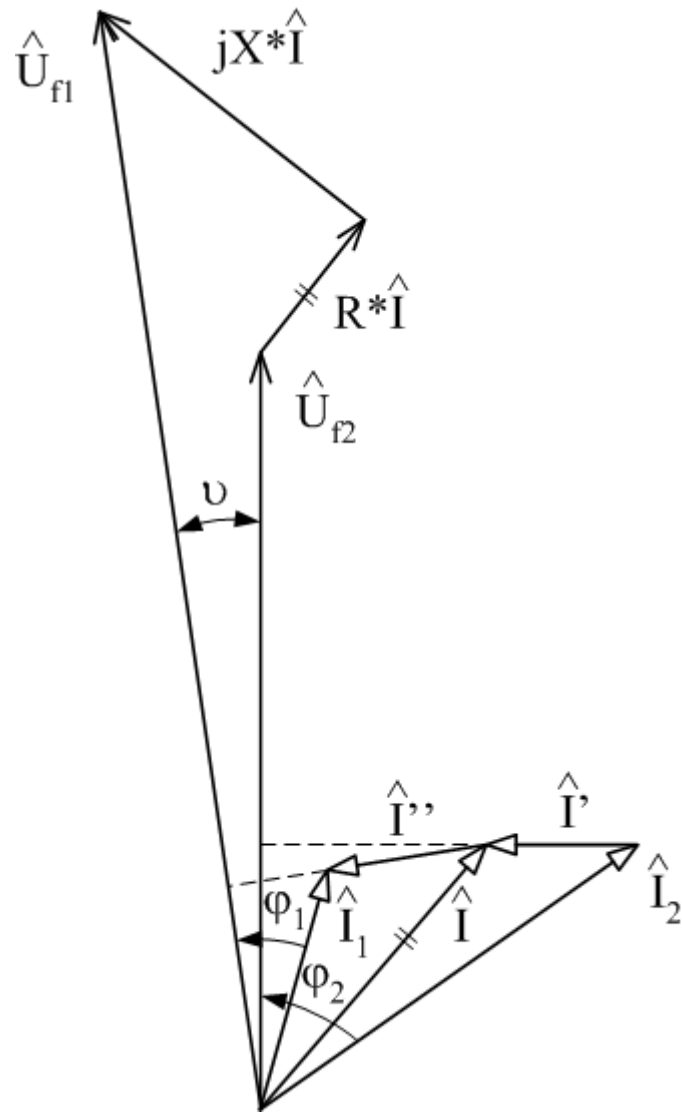
$$\hat{I}' = \frac{1}{2}\hat{Y}_{q_1} l \hat{U}_{f2}$$

$$\hat{I}'' = \frac{1}{2}\hat{Y}_{q_1} l \hat{U}_{f1}$$
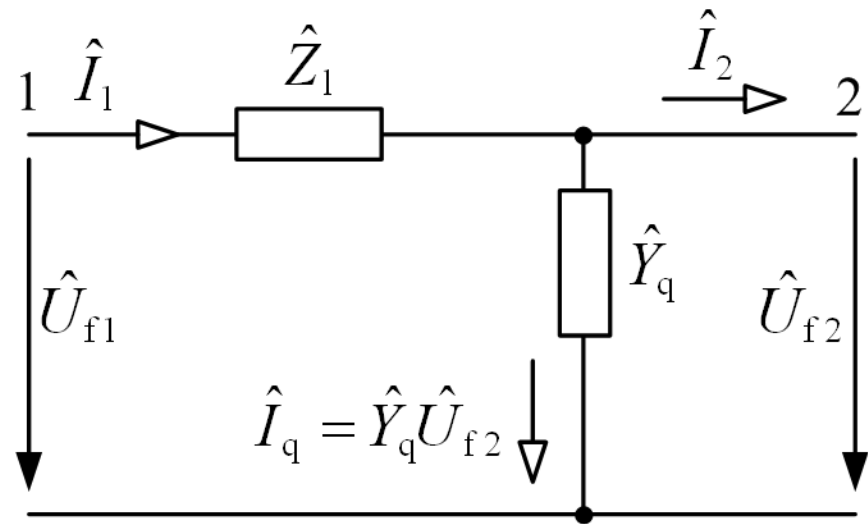
After modification (Blondel's constants relations)
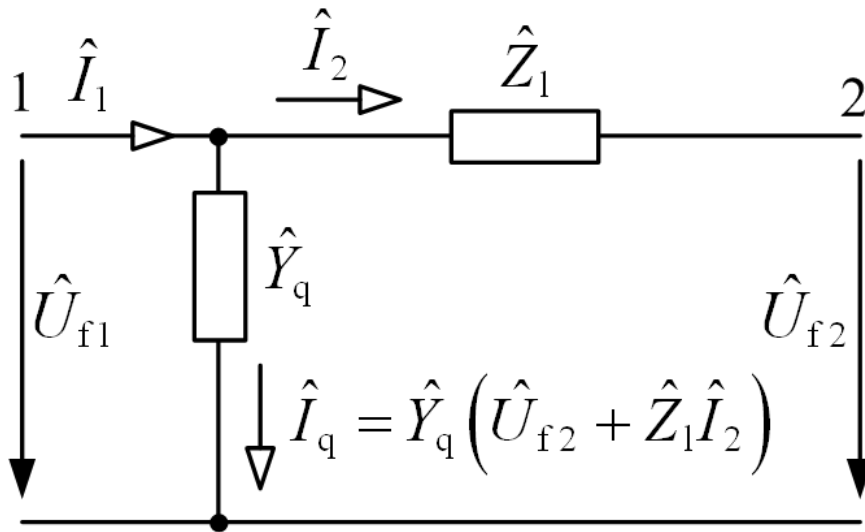
$$\hat{U}_{f1} = \hat{U}_{f2}\left(1 + \frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{2}l^2\right) + \hat{I}_2\hat{Z}_{l_1}1$$

$$\hat{I}_1 = \hat{U}_{f2}\hat{Y}_{q_1}l\left(1 + \frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{4}l^2\right) + \hat{I}_2\left(1 + \frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{2}l^2\right)$$

Phasor diagram (G = 0)

$\hat{U}_{f1}$

$jX*\hat{I}$

$R*\hat{I}$

$\hat{U}_{f2}$

υ

$\hat{I}''$

$\hat{I}'$

$\hat{I}_2$

φ_1

$\hat{I}_1$

$\hat{I}$

φ_2

$\hat{U}_{f2}, \hat{I}_2$

$\downarrow \quad \downarrow$

$\hat{I}' \rightarrow \hat{I} \rightarrow \Delta\hat{U} \rightarrow \hat{U}_{f1}$

$\downarrow \qquad\qquad\qquad \downarrow$

$\hat{I}_1 \qquad \leftarrow \qquad \hat{I}''$

$$\hat{I}_q = \hat{Y}_q\left(\hat{U}_{f2} + \hat{Z}_l\hat{I}_2\right)$$

$$\hat{I}_q = \hat{Y}_q\hat{U}_{f2}$$

$$\hat{U}_{f1} = \hat{U}_{f2} + \hat{I}_2\hat{Z}_{l_1}1$$
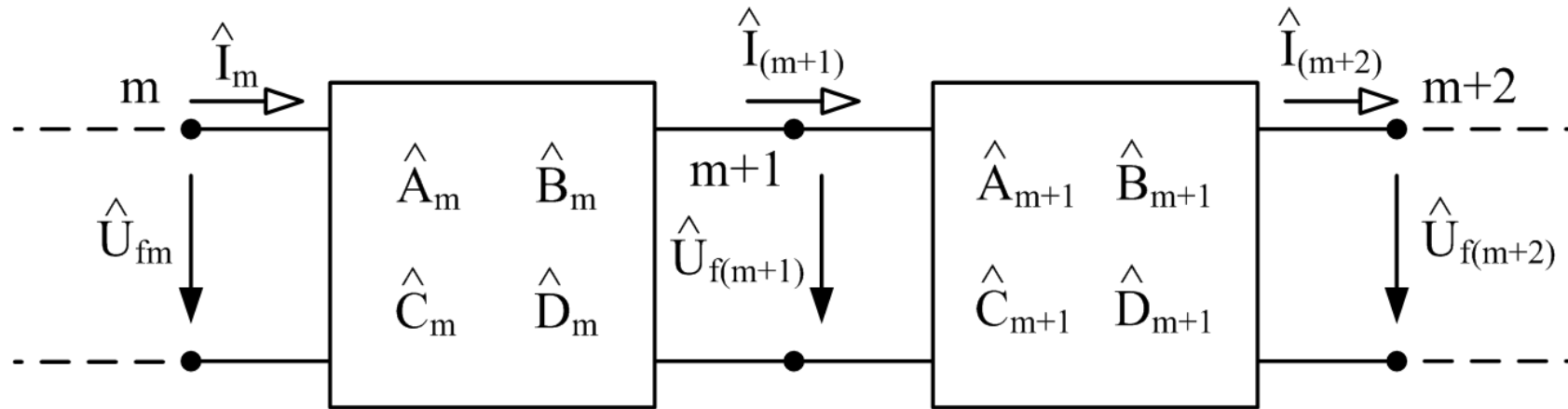
$$\hat{I}_1 = \hat{U}_{f2}\hat{Y}_{q_1}1 + \hat{I}_2\left(1 + \hat{Z}_{l_1}\hat{Y}_{q_1}1^2\right)$$

$$\hat{U}_{f1} = \hat{U}_{f2}\left(1 + \hat{Z}_{l_1}\hat{Y}_{q_1}1^2\right) + \hat{I}_2\hat{Z}_{l_1}1$$

$$\hat{I}_1 = \hat{U}_{f2}\hat{Y}_{q_1}1 + \hat{I}_2$$

it is valid $\hat{A} \neq \hat{D}, \hat{A}D - \hat{B}\hat{C} = 1$ (unbalanced, passive two-port network)

Longer lines → cascade connection of networks for shorter sections (additional nodes)

$$\begin{pmatrix} \hat{U}_{fm} \\ \hat{I}_m \end{pmatrix} = \begin{pmatrix} \hat{A}_m & \hat{B}_m \\ \hat{C}_m & \hat{D}_m \end{pmatrix} \begin{pmatrix} \hat{U}_{f(m+1)} \\ \hat{I}_{(m+1)} \end{pmatrix}$$

$$\begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix} = \prod_{m=1}^{n} \begin{pmatrix} \hat{A}_m & \hat{B}_m \\ \hat{C}_m & \hat{D}_m \end{pmatrix}$$