Simple transmissions stability

Interconnected systems operated in synchronous operation (same frequency) if active power is lesser than limit value. Otherwise system gets out of synchronism (parallel operation stability disturbation)

Steady-stability – is the ability of system to remain in the synchronous operation during slow changes.

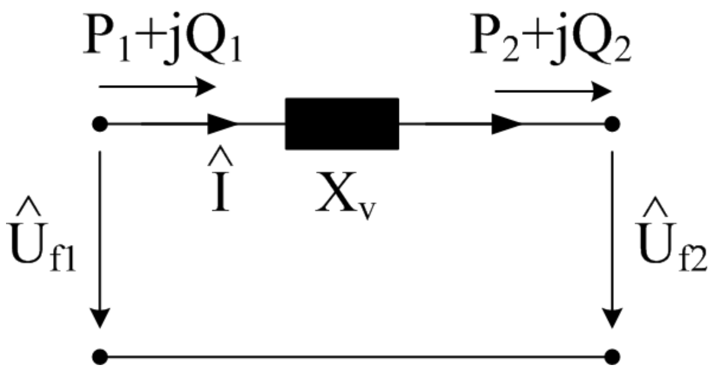
Transient stability – is the ability of system to remain in the synchronous operation during sudden large changes (loading, parameters). Electromechanical transient events → to have a sufficient reserve power during the steady-state operation.

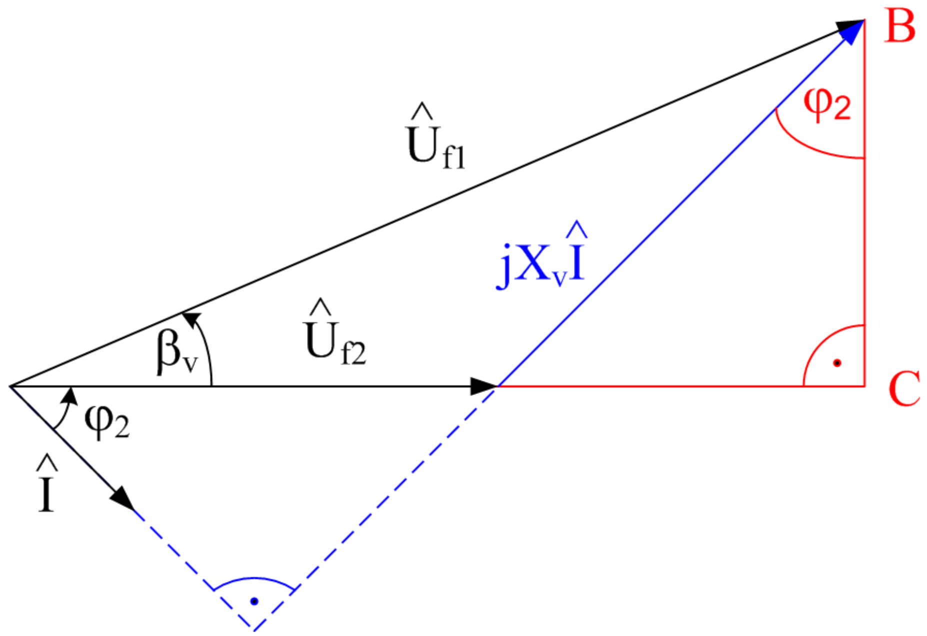
Importance: sources and systems collaboration keeping, AC power lines length limiting

Basic relations

Assumption: only longitudinal reactance considered

Transmission angle (power angle) β_v – between voltages at both transmission ends





Consumed active power (single phase)

$$P_f = U_{f2} I \cos\varphi_2 \qquad\qquad Q_f = U_{f2} I \sin\varphi_2$$

$$\overline{BC} \sim X_v I\cos\varphi_2 = U_{f1}\sin\beta_v \qquad X_v I\sin\varphi_2 + U_{f2} = U_{f1}\cos\beta_v$$

$$I\cos\varphi_2 = \frac{U_{f1}}{X_v}\sin\beta_v \qquad I\sin\varphi_2 = \frac{U_{f1}\cos\beta_v}{X_v} - \frac{U_{f2}}{X_v}$$

Transmission power equation

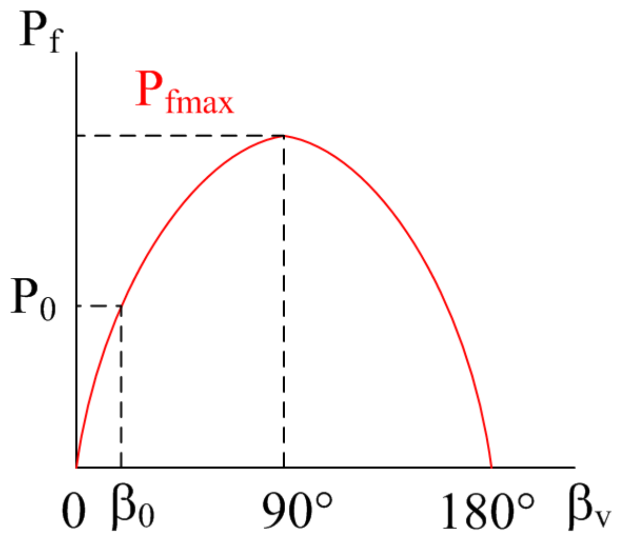
$$P_f = \frac{U_{f1}U_{f2}}{X_v}\sin\beta_v \qquad Q_f = \frac{U_{f1}U_{f2}\cos\beta_v}{X_v} - \frac{U_{f2}^2}{X_v}$$

$0^\circ \div 90^\circ$: stable area

$90^\circ \div 180^\circ$: unstable area

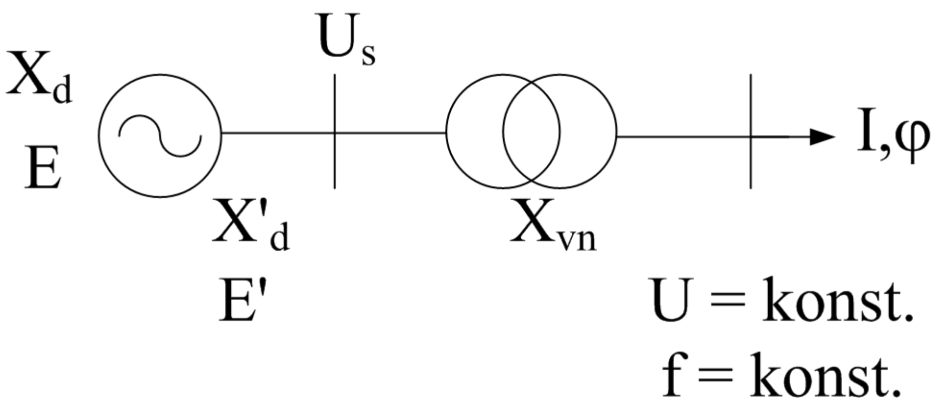
$\beta_v = 90^\circ$: steady-state stability limit

$$P_{f\max} = \frac{U_{f1}U_{f2}}{X_v}$$



In addition to power lines also reactance of generators, transformers … → higher power angle. Generators have the biggest influence (X).

Turbo-alternator operating to a “hard” system



$$X_{dc} = X_d + X_{vn}$$

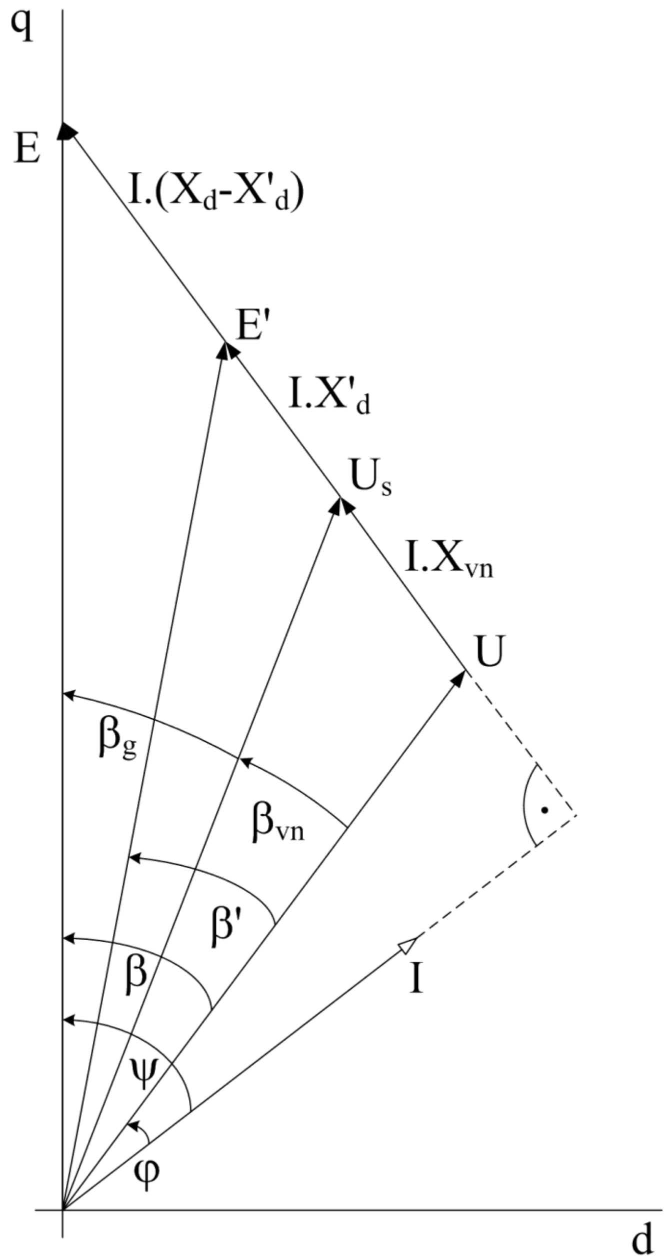
Alternator inside power

$$P = \frac{E \cdot U}{X_{dc}} \sin\beta$$

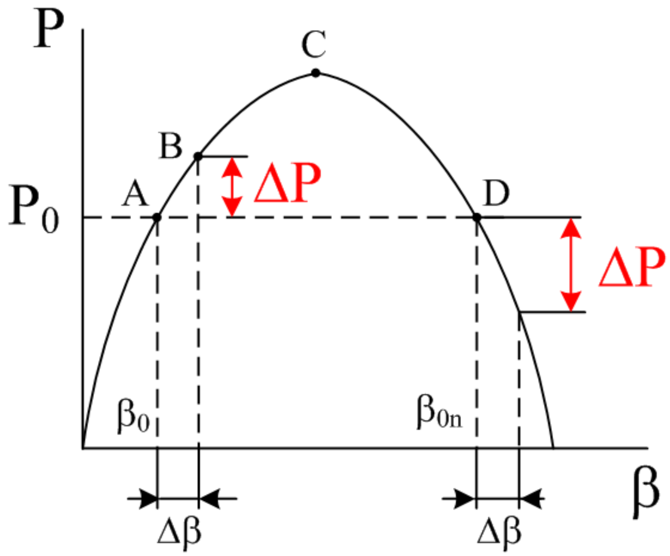
$$Q_i = \frac{E^2}{X_{dc}} - \frac{E \cdot U}{X_{dc}} \cos\beta$$

Valid for the hard system (f, U = konst., X = 0), constant excitation (E), smooth-core rotor (turbo alternator).

Stability influenced by transmitted power (P), excitation (E), transmission configuration (X). → longitudinal compensation (C), higher excitation



System steady-state stability



Stable state

$$P_0 = P_m$$

(electrical (machines) = mechanical (turbine), no losses)

$\omega_0 = konst.$ (system and machine)

Mechanical power of turbine is a constant – doesn’t depend on β but depend on ω (regulation P-f)

Point A: rotor acceleration $\rightarrow \Delta\beta \rightarrow (P_0+\Delta P) \rightarrow P_{el} > P_{mech} \rightarrow$ rotor retardation $\rightarrow$ stabilization $\rightarrow$ point A stable

Point D: rotor acceleration $\rightarrow \Delta\beta \rightarrow (P_0-\Delta P) \rightarrow P_{el} < P_{mech} \rightarrow$ rotor acceleration $\rightarrow$ loss of synchronism $\rightarrow$ point D unstable

Synchronization power

$$P_c = \frac{dP}{d\beta} = \frac{E \cdot U}{X_{dc}} \cos\beta$$

Stable area

$$P_c > 0$$

Steady-state stability limit

$$P_c = 0$$

Coefficient reserve power (usually above 20%)

$$k_P = \frac{P_m - P_0}{P_0}$$

Note: More machines → needs to watch P curve for each machine

Double-machine problem with between consumption (fault)

Superposition

$$\hat{I}_1 = \hat{I}_{11} - \hat{I}_{12} \qquad \hat{I}_2 = \hat{I}_{22} - \hat{I}_{21} \qquad \hat{I}_k = \hat{I}_{k1} + \hat{I}_{k2}$$

$$\hat{I}_{11} = \frac{\hat{U}_1}{\hat{Z}_{11}} \qquad \hat{I}_{22} = \frac{\hat{U}_2}{\hat{Z}_{22}} \qquad \hat{I}_{12} = \frac{\hat{U}_2}{\hat{Z}_{12}} \qquad \hat{I}_{21} = \frac{\hat{U}_1}{\hat{Z}_{21}}$$

$$\hat{Z}_{11} = \hat{Z}_1 + \frac{\hat{Z}_2 \hat{Z}_k}{\hat{Z}_2 + \hat{Z}_k}$$

$$\hat{Z}_{22} = \hat{Z}_2 + \frac{\hat{Z}_1 \hat{Z}_k}{\hat{Z}_1 + \hat{Z}_k}$$

$$\hat{Z}_{12} = \hat{Z}_{21} = \hat{Z}_1 + \hat{Z}_2 + \frac{\hat{Z}_1 \hat{Z}_2}{\hat{Z}_k}$$

$$\hat{S}_1 = \hat{U}_1\hat{I}_1^* = \frac{U_1^2}{\hat{Z}_{11}^*} - \frac{\hat{U}_1\hat{U}_2^*}{\hat{Z}_{12}^*}$$

$$\hat{S}_1 = \frac{U_1^2}{\hat{Z}_{11}^*} - \frac{U_1U_2}{\hat{Z}_{12}^*}e^{j\beta}$$

$$P_1 = \mathrm{Re}\{\hat{S}_1\} \qquad Q_1 = \mathrm{Im}\{\hat{S}_1\}$$

For R=0:

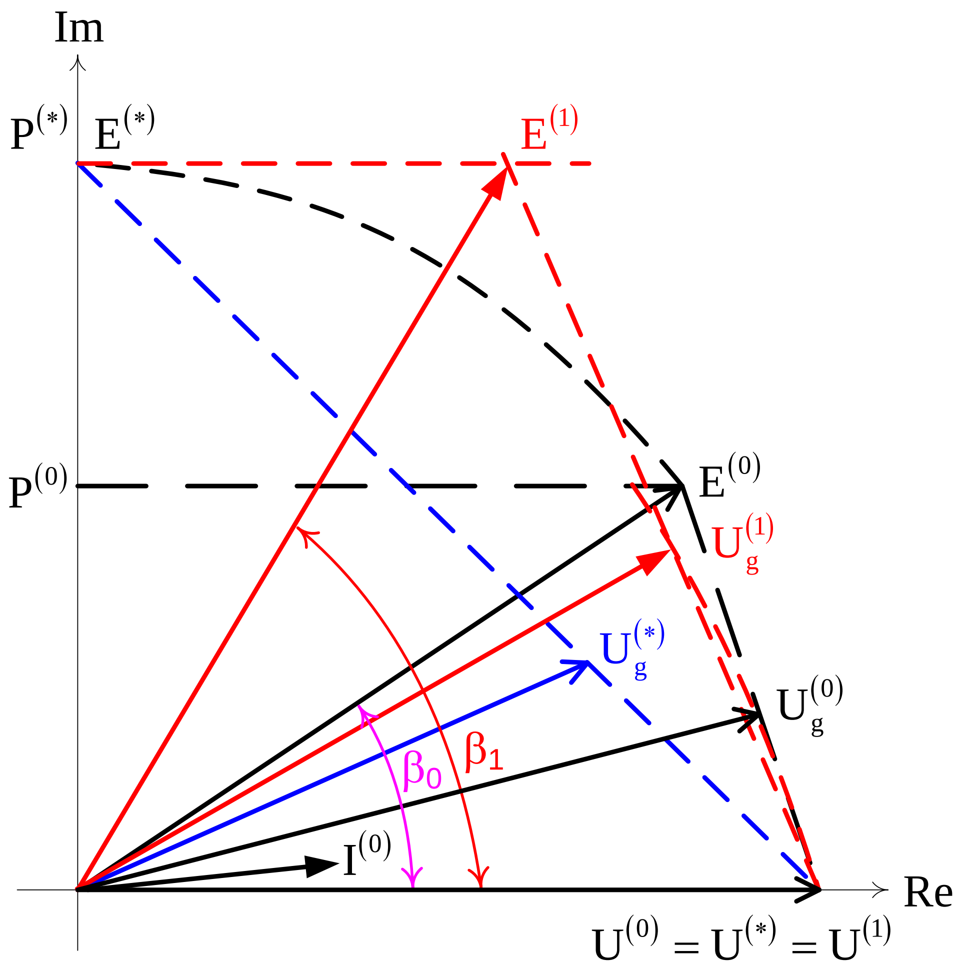
$$\hat{S}_1 = \frac{U_1^2}{-jX_{11}} - \frac{U_1U_2}{-jX_{12}}\left(\cos\beta + j\sin\beta\right)$$

$$P_1 = \frac{U_1U_2}{X_{12}}\sin\beta$$

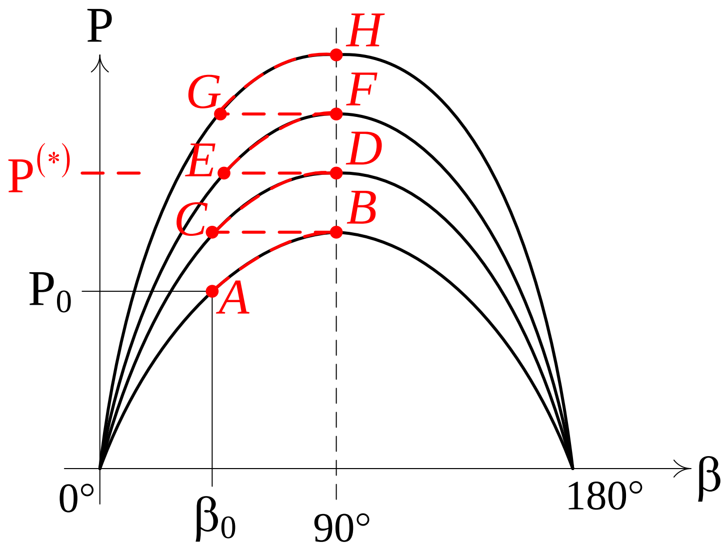
$$Q_1 = \frac{U_1^2}{X_{11}} - \frac{U_1U_2}{X_{12}}\cos\beta$$

Regulation of excitation

$$P = \frac{E \cdot U}{X_{dc}} \sin\beta = k \cdot E \cdot \sin\beta$$



External characteristic →



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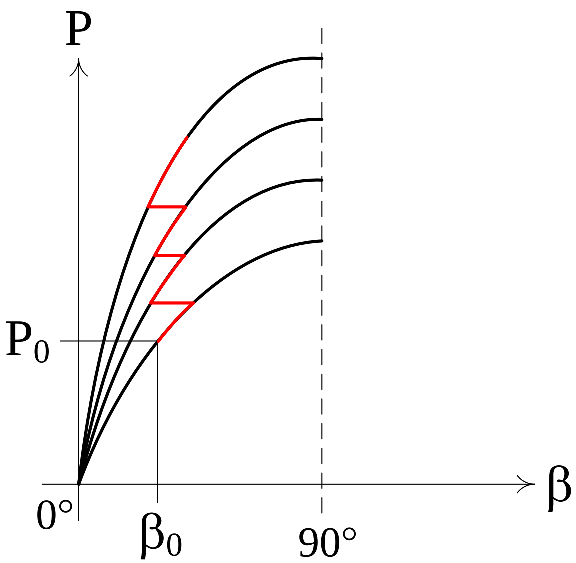

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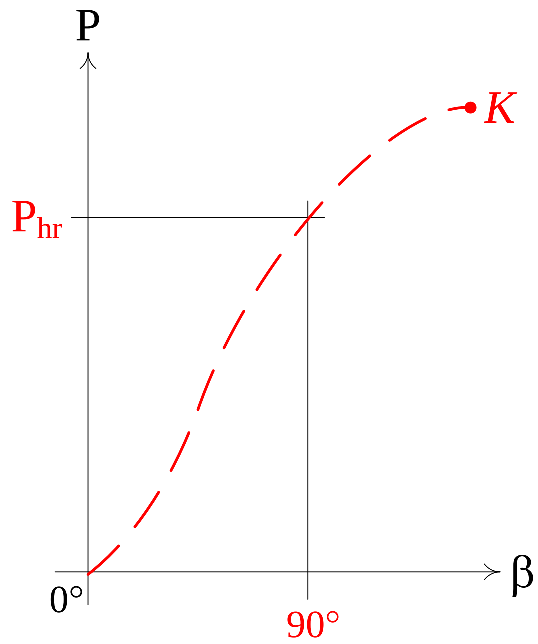


Machine with salient poles

$$I \cdot X_q \cdot \cos\varphi = E_q \cdot \sin\beta$$

$$E_q = E - I_d \cdot \left(X_d - X_q\right)$$

$$I_d \cdot X_d = E - U_g \cos\beta$$

Modification

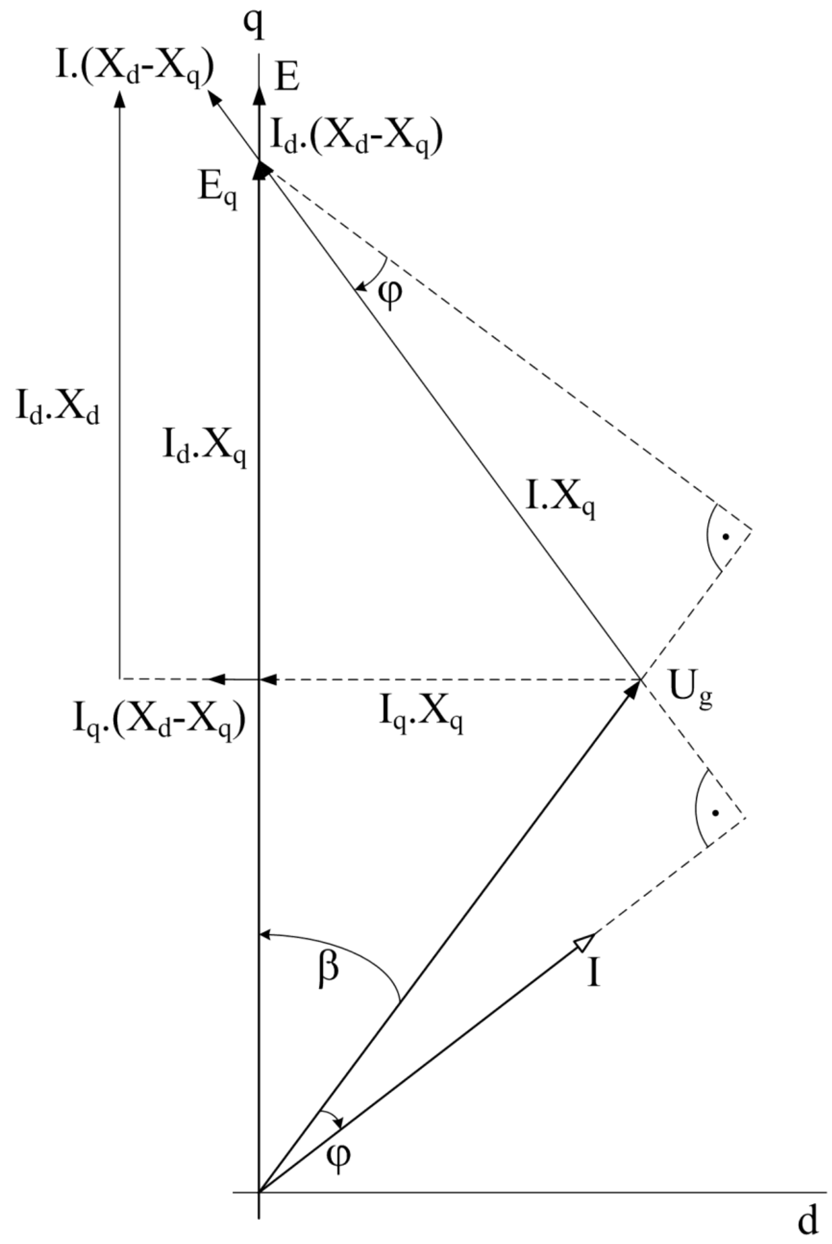
$$I_d = \frac{E}{X_d} - \frac{U_g}{X_d} \cos\beta$$

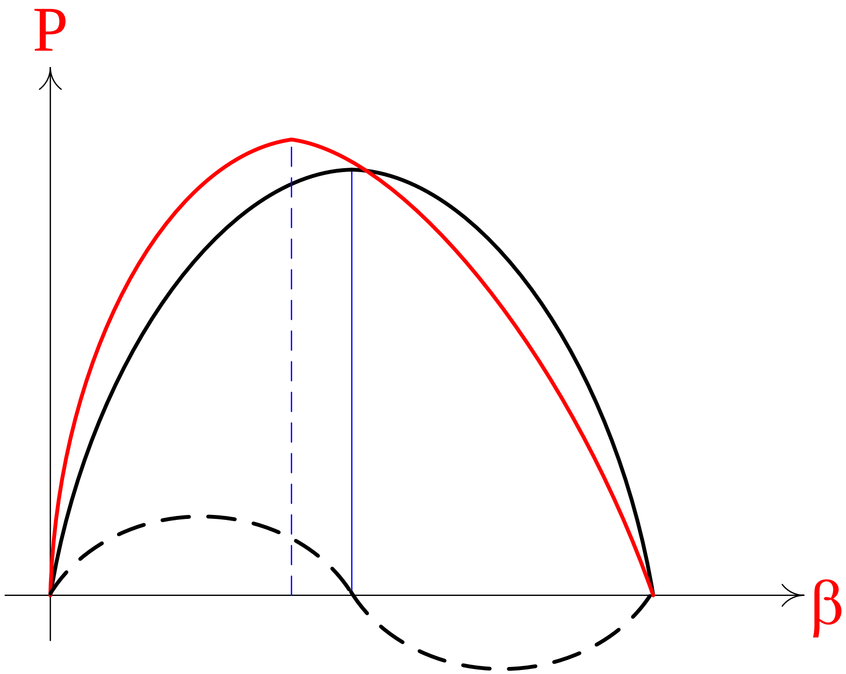
$$E_q = E \cdot \frac{X_q}{X_d} + U_g \cos\beta \cdot \frac{X_d - X_q}{X_d}$$

Generator power

$$P = U_g \cdot I \cdot \cos\varphi$$

$$P = \frac{E \cdot U_g}{X_d} \sin\beta + \frac{U_g^2}{2} \cdot \frac{X_d - X_q}{X_d X_q} \sin 2\beta$$



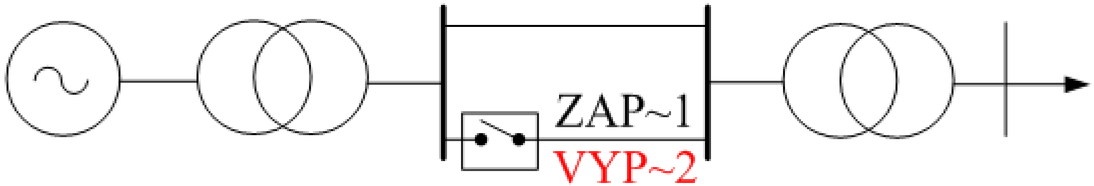


Generator to the network over series impedance

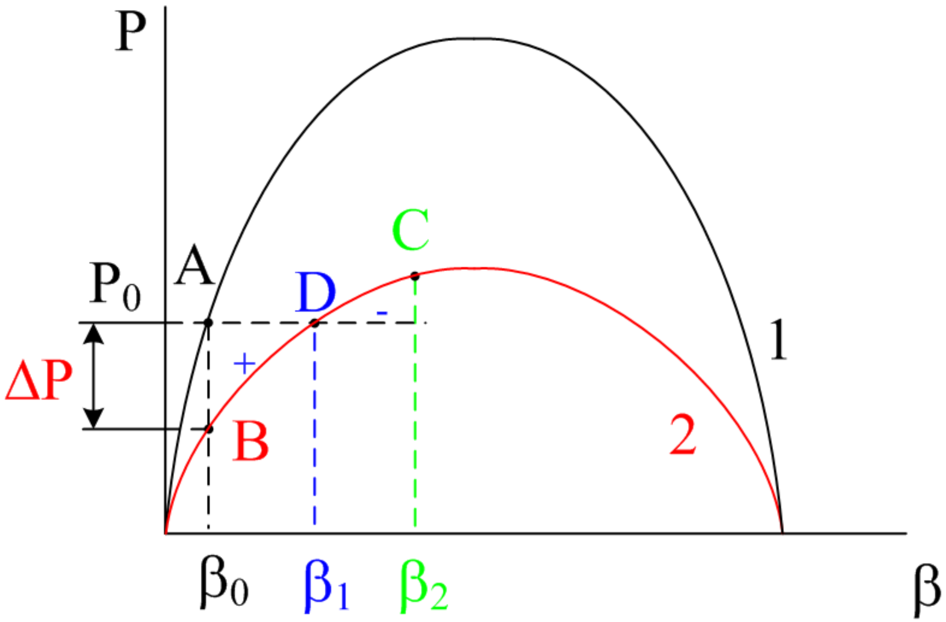
$$P = \frac{E \cdot U_g}{X_d + X_{vn}} \sin\beta + \frac{U_g^2}{2} \cdot \frac{X_d - X_q}{(X_d + X_{vn})(X_q + X_{vn})} \sin 2\beta$$

Sytem transient stability

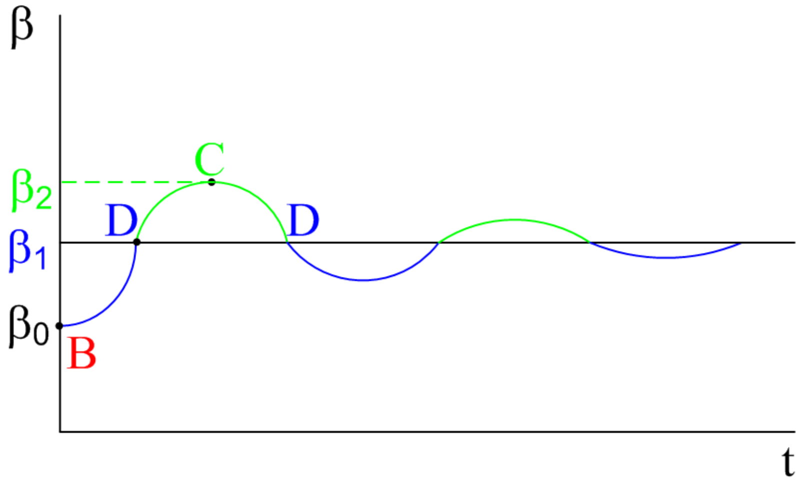
Parameters changes in the system → impact power changes → possible loss of stability.



$X_2 > X_1$



- Point A before the changes
- Inertia $\longrightarrow \beta_0$ doesn't change by step $\longrightarrow$ generator power drops by ΔP
- $P_m - P_{el}$ = accelerating power
- In the point D $\omega > \omega_0 \longrightarrow \beta$ grows up to the point C, gradual retardation
- Stabilization after several swings in the point D (a few seconds period)



The point C is too far (ΔP accelerating) $\longrightarrow$ loss of synchronism.

Behavior solution $\beta = f(t)$

Motion equation

$$W_k = \frac{1}{2} J\omega^2$$

$$P_a = P_m - P_e = \frac{dW_k}{dt} = \frac{1}{2} J 2\omega \frac{d\omega}{dt}$$

$P_a > 0$ - accelerating power

$P_a < 0$ - retarding power

Swing equation

$$P_m - \frac{E \cdot U}{X_{dc}} \sin\beta(t) = J\omega(t) \frac{d\omega(t)}{dt}$$

$$\omega(t) = \omega_0 + \frac{d\beta(t)}{dt}$$

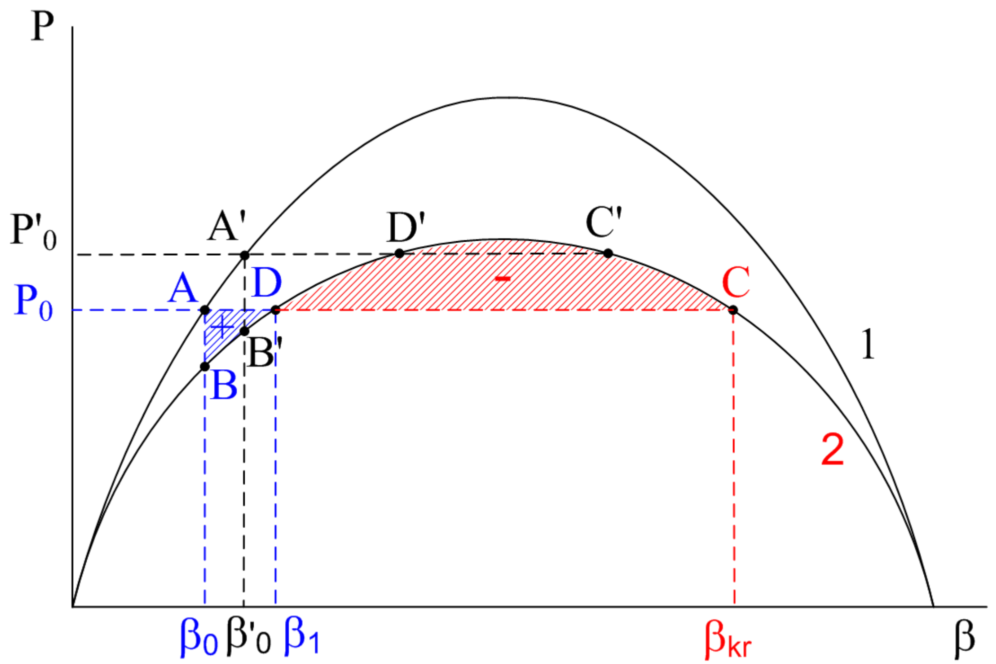
$$\beta(0) = \beta_0 \ ; \quad \omega(0) = \omega_0$$

Inertia moment from start-up time

$$P = M\omega = J\varepsilon\omega \cong J\frac{\Delta\omega}{\Delta t}\omega = \frac{J\omega^2}{T}$$

$$J = \frac{T_m P_n}{\omega_0^2}$$

Equal areas method



Accelerating energy (area +)

$$A_a = \int_{\beta_0}^{\beta_1} (P_0 - P_{el}) d\beta$$

Retarding energy (area -)

$$A_r = \int_{\beta_1}^{\beta_{kr}} (P_{el} - P_0) d\beta$$

For keeping in synchronism

$$A_a \leq A_r$$

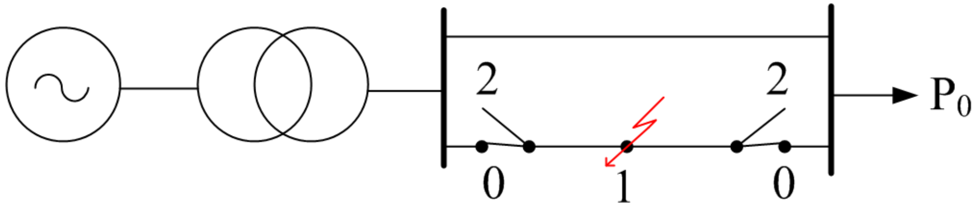
Transient stability

- limit at $A_a = A_r$, there is no strict angle limit
- during swing the angle can be $\beta > 90^\circ$

Auto-reclosing (AR)

During failure short disconnecting and after a while repeated reclosing of the faulted section.

- temporary failure disappeared → operation
- permanent failure → final disconnecting



A_0 initial state
A_1 - B_1 short-circuit
B_1 short-circuit disconnecting
B_2 - C_2 disconnected power line
C_2 AR (successful)
C_3 - D_3 swinging on the original characteristic

AR disconnecting time ~ between angles β_1 and β_2 (compromise between stability and arc burning).