The power lines

On fig. 1 is the substitute scheme of one phase power line with equally distributed parameters. The power line has length *l* (*R_1* is longitudinal resistance, *X_1* longitudinal inductance, *G_1* cross conductance and *B_1* cross susceptance, all is per kilometre length (for each phase)).

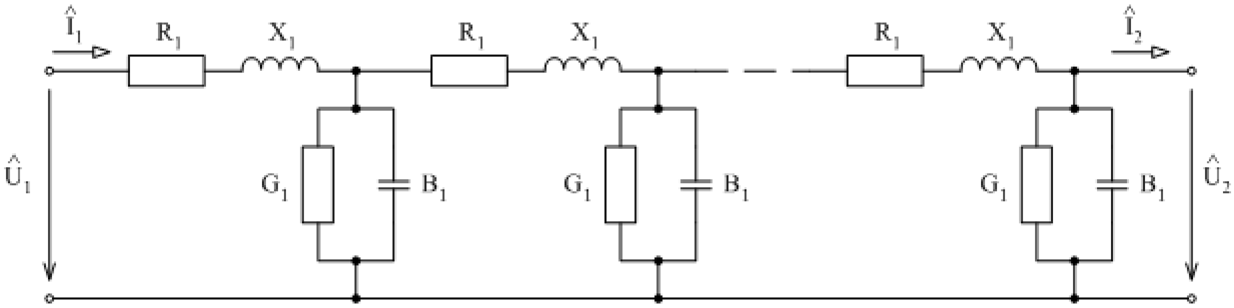

Fig. 1: The substitute scheme of one phase power line with equally distributed parameters

The symbols $\hat{U}_1$ and $\hat{I}_1$ are voltage and current phasors at the beginning of power line, $\hat{U}_2$ and $\hat{I}_2$ are phasors at the end of power line. The frequency of system is *f*. The basic equations expressing the relationship between the variables at the beginning and end of the power line. Their formula is:

$$\begin{pmatrix} \hat{U}_1 \\ \hat{I}_1 \end{pmatrix} = \begin{pmatrix} \cosh \hat{\gamma} l & \hat{Z}_v \sinh \hat{\gamma} l \\ \frac{1}{\hat{Z}_v} \sinh \hat{\gamma} l & \cosh \hat{\gamma} l \end{pmatrix} \cdot \begin{pmatrix} \hat{U}_2 \\ \hat{I}_2 \end{pmatrix} \tag{1}$$

where $\hat{Z}_v$ is wave impedance

$$\hat{Z}_v = \sqrt{\frac{R_1 + j \cdot X_1}{G_1 + j \cdot B_1}} \quad (\Omega) \tag{2}$$

and $\hat{\gamma}$ is propagation constant

$$\hat{\gamma} = \sqrt{(R_1 + j \cdot X_1) \cdot (G_1 + j \cdot B_1)} \quad (m^{-1}) \tag{3}$$

Propagation constant $\hat{\gamma}$ has complex character and can be written as $\hat{\gamma} = \alpha + j\beta$, where the real-part α is specific damping and imaginary part β is specific phase shift.

The longitudinal impedance $\hat{Z}_l$ and cross admittance $\hat{Y}_q$, per length *l* are:

$$\hat{Z}_l = (R_1 + j \cdot X_1) \cdot l \quad (\Omega) \tag{4}$$

$$\hat{Y}_q = (G_1 + j \cdot B_1) \cdot l \quad (S) \tag{5}$$

The figure 1 can be redrawn as shown (fig. 2). It is two-port π-network.

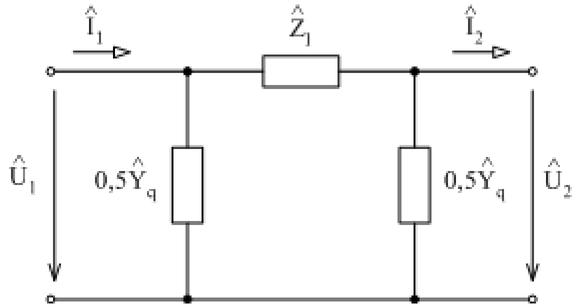

Fig. 2: Substitute π-network of long power line

The basic equation for the voltage and current at the beginning and end of power line:

$$\begin{pmatrix} \hat{U}_1 \\ \hat{I}_1 \end{pmatrix} = \begin{pmatrix} 1+\dfrac{\hat{Z}_l \cdot \hat{Y}_q}{2} & \hat{Z}_l \\ \hat{Y}_q + \dfrac{\hat{Z}_l \cdot \hat{Y}_q^2}{4} & 1+\dfrac{\hat{Z}_l \cdot \hat{Y}_q}{2} \end{pmatrix} \begin{pmatrix} \hat{U}_2 \\ \hat{I}_2 \end{pmatrix} = \begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix} \cdot \begin{pmatrix} \hat{U}_2 \\ \hat{I}_2 \end{pmatrix} \tag{6}$$

<u>Example 1</u>

Simply three phase power line with resistance $R_1 = 0{,}0715\ \Omega/km$, inductance $X_1 = 0{,}426\ \Omega/km$, conductance $G_1 = 0\ S/km$, susceptance $B_1 = 2{,}635\ \mu S/km$, the length of power line is $l = 400\ km$.

Calculate the phase voltage and current at the beginning for open end of power line. Calculate the charge power of power line. The voltage at the end is $U_2 = 220\ kV$.

Longitudinal impedance:

$$\hat{Z}_{l1} = R_1 + j \cdot X_1 = 0{,}0715 + j0{,}426 = 0{,}432 \cdot e^{j80{,}47^\circ}\ \ \Omega/km$$

Cross admittance:

$$\hat{Y}_{q1} = G_1 + j \cdot B_1 = 0 + j2{,}635 \cdot 10^{-6} = 2{,}635 \cdot 10^{-6} \cdot e^{j90^\circ}\ \ S/km$$

Wave impedance:

$$\hat{Z}_v = \sqrt{\frac{\hat{Z}_{l1}}{\hat{Y}_{q1}}} = 404{,}9 \cdot e^{-j4{,}764^\circ}\ \ \Omega = 403{,}5 - j33{,}63\ \Omega$$

Propagation constant:

$$\hat{\gamma} = \sqrt{\hat{Z}_{l1} \cdot \hat{Y}_{q1}} = 1{,}067 \cdot 10^{-3} \cdot e^{j85{,}24^\circ}\ \ km^{-1} = (0{,}0889 + j1{,}0632) \cdot 10^{-3}\ \ km^{-1}$$

Auxiliary mathematical derive:

$$\sinh(\alpha + j\beta) = \frac{1}{2}\left(e^{\alpha+j\beta} - e^{-(\alpha+j\beta)}\right) = \frac{1}{2}\left[e^{\alpha}(\cos\beta + j\sin\beta) - e^{-\alpha}(\cos\beta - j\sin\beta)\right] =$$
$$= \frac{1}{2}\cos\beta\left(e^{\alpha} - e^{-\alpha}\right) + \frac{1}{2}j\sin\beta\left(e^{\alpha} + e^{-\alpha}\right) = \sinh\alpha\cdot\cos\beta + j\cosh\alpha\cdot\sin\beta$$

Similar:

$$\cosh(\alpha + j\beta) = \frac{1}{2}\left(e^{\alpha+j\beta} + e^{-(\alpha+j\beta)}\right) = \frac{1}{2}\left[e^{\alpha}(\cos\beta + j\sin\beta) + e^{-\alpha}(\cos\beta - j\sin\beta)\right] =$$
$$= \frac{1}{2}\cos\beta\left(e^{\alpha} + e^{-\alpha}\right) + \frac{1}{2}j\sin\beta\left(e^{\alpha} - e^{-\alpha}\right) = \cosh\alpha\cdot\cos\beta + j\sinh\alpha\cdot\sin\beta$$

The phase voltage at the end of power line:

$$\hat{U}_{f2} = U_{f2} = \frac{220}{\sqrt{3}} \doteq 127\ \text{kV}$$

The voltage phasor at the beginning of power line with open end ($I_2 = 0$):

$$\hat{U}_{f10} = \hat{U}_{f2}\cosh\hat{\gamma}l = 127\cdot 0{,}9116\cdot e^{j0{,}92^\circ} = 115{,}77\cdot e^{j0{,}92^\circ}\ \text{kV} = U_{f10}\cdot e^{j\vartheta_0}$$

On power line is evident the Ferranti effect $U_{f10} < U_{f2}$!
The current phasor at the beginning of power line with open end:

$$\hat{I}_{10} = \frac{\hat{U}_{f2}}{\hat{Z}_v}\sinh\hat{\gamma}l = \frac{127}{404{,}9\cdot e^{-j4{,}764^\circ}}\cdot 0{,}4129\cdot e^{j85{,}53^\circ} = 129{,}51\cdot e^{j90{,}29^\circ}\ \text{A} = I_{10}\cdot e^{j\delta}$$

The phase shift between voltage and current at the beginning of power line:

$$\varphi_{10} = \delta - \vartheta_0 = 90{,}29^\circ - 0{,}92^\circ = 89{,}37^\circ$$

The power line with open end is almost as capacity load.
Three phase charge power:

$$\hat{S}_{10} = 3\hat{U}_{f10}\hat{I}^*_{10} = 3\cdot 115{,}77\cdot 10^3\cdot e^{j0{,}92^\circ}\cdot 129{,}51\cdot e^{-j90{,}29^\circ} = 44{,}6\cdot e^{-j89{,}37^\circ}\ \text{MVA}$$

$$P_{10} = 0{,}49\ \text{MW}\ ,\quad Q_{10} = 44{,}6\ \text{MVAr}\ \ \text{capacitive}$$

Example 2

Three phase transpose power line with nominal voltage 220 kV, length 400 km, and with R_1 = 0,0715 Ω/km, X_1 = 0,426 Ω/km, G_1 = 0 S/km, B_1 = 2,635 μS/km.

Calculate the phase voltage and current at the beginning when at the end is active load P_2 = 125 MW with cosφ = 1, voltage at the end is U_2 = 220 kV. Calculate the charge power of power line. The voltage at the end is U_2 = 220 kV. Solve the problem with π-network and symmetrical parameters.

Preliminary calculation:

$$\hat{U}_{f2} = U_{f2} = \frac{220}{\sqrt{3}} \doteq 127 \text{ kV}$$

$$\hat{Z}_l = (R_1 + j \cdot X_1) \cdot l = (0{,}0715 + j0{,}426) \cdot 400 = 28{,}6 + j170{,}4\ \Omega$$

$$\frac{\hat{Y}_q}{2} = \frac{1}{2}(G_1 + j \cdot B_1) \cdot l = \frac{1}{2}\, j2{,}635 \cdot 10^{-6} \cdot 400 = j0{,}527 \cdot 10^{-3} \text{ S}$$

The current phasor for the load:

$$\hat{I}_2 = \left(\frac{\hat{S}_2}{3\hat{U}_{f2}}\right)^* = \left(\frac{125 + j0}{3 \cdot 127}\right)^* = 328 \text{ A}$$

The voltage phasor at the beginning of power line:

$$\hat{U}_{f1} = \hat{A} \cdot \hat{U}_{f2} + \hat{B} \cdot \hat{I}_2 = 137{,}7 \cdot e^{j24{,}82^\circ} \text{ kV}$$

The current phasor at the beginning of power line:

$$\hat{I}_1 = \hat{C} \cdot \hat{U}_{f2} + \hat{D} \cdot \hat{I}_2 = 325{,}828 \cdot e^{j24{,}05^\circ} \text{ A}$$

The complex load at the beginning of power line:

$$\hat{S}_1 = 3\hat{U}_{f1}\hat{I}_1^* = 3 \cdot 137{,}7 \cdot 10^3 \cdot e^{j24{,}82^\circ} \cdot 325{,}828 \cdot e^{-j24{,}05^\circ} = 134{,}6 \cdot e^{j0{,}77^\circ} \text{ MVA}$$

$$P_1 = 134{,}59 \text{ MW}, \quad Q_1 = 1{,}81 \text{ MVAr} \quad \text{inductive}$$

The power factor at the beginning of power line:

$$\cos\varphi_1 = \cos 0{,}77^\circ = 0{,}9999 \text{ ind.}$$

π-network is the replacement of homogeneous line. The elements A, B, C, D of matrix include only few (1 to 2) initial part of Taylor series of hyperbolical function describe homogeneous line. For large power line lengths are generated considerable errors.

Next figures are showing the comparison of values solved in example 2 for changeable lengths of power line and for various network (π, T, Γ) compared to homogenous line.

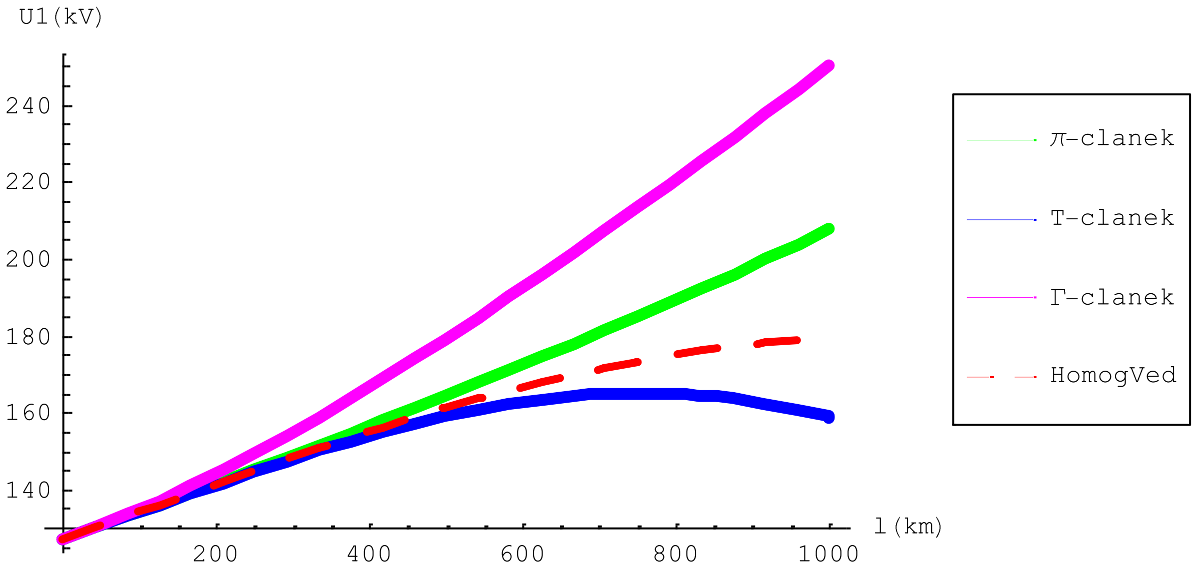
U1(kV)
240
220
200
180
160
140
200
400
600
800
1000
l(km)
π-clanek
T-clanek
Γ-clanek
HomogVed

Fig. 3

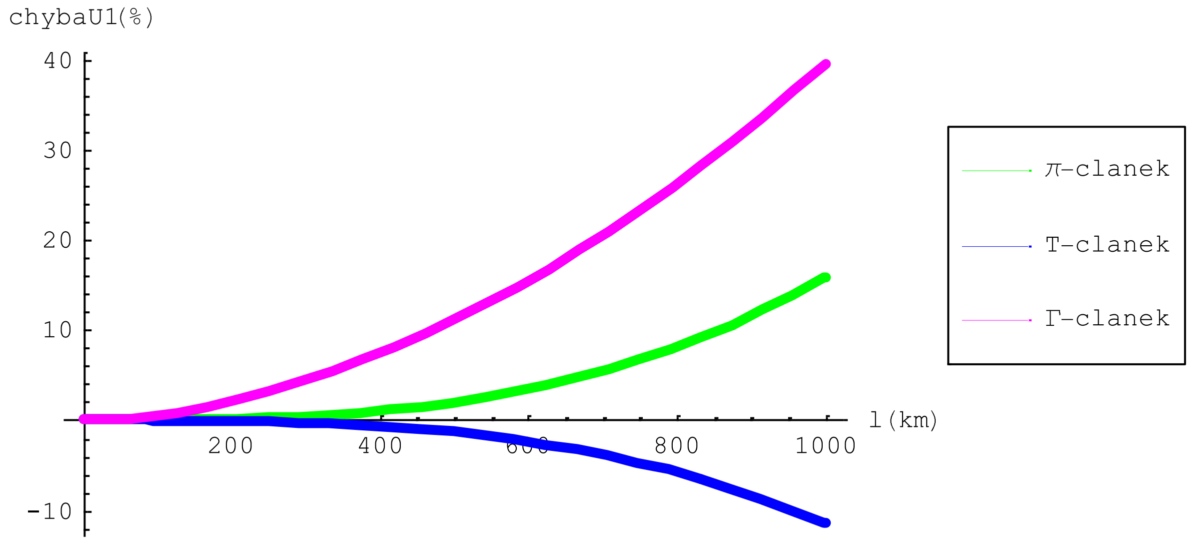
chybaU1(%)
40
30
20
10
-10
200
400
600
800
1000
l(km)
π-clanek
T-clanek
Γ-clanek

Fig. 4

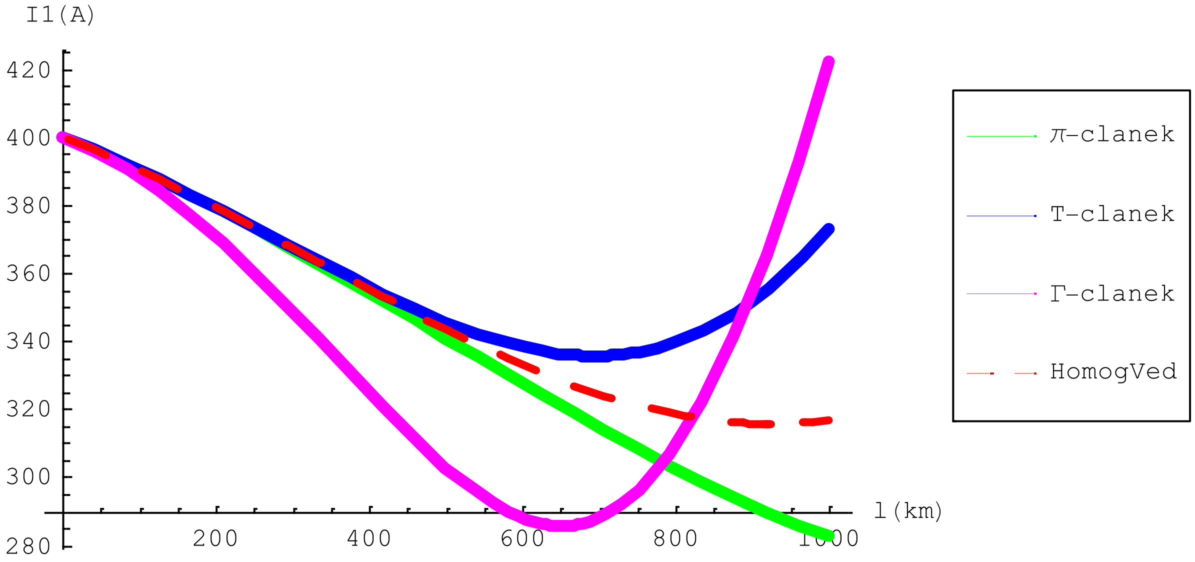
I1(A)
420
400
380
360
340
320
300
280
200
400
600
800
1000
l(km)
π–clanek
T–clanek
Γ–clanek
HomogVed

Fig. 5

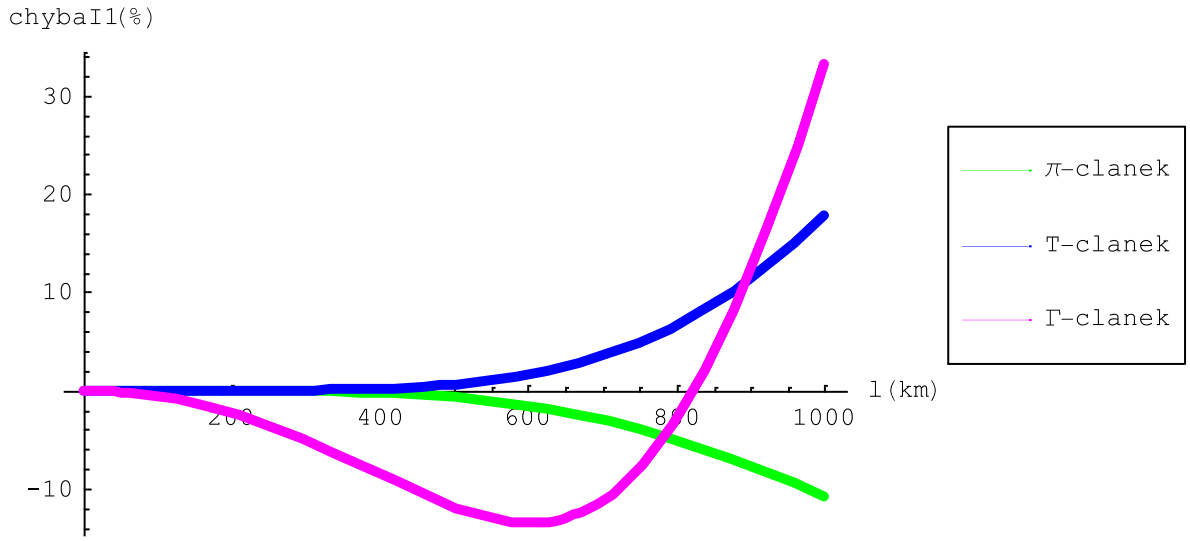
chybaI1(%)
30
20
10
-10
200
400
600
800
1000
l(km)
π–clanek
T–clanek
Γ–clanek

Fig. 6

Ferranti effect

The Ferranti effect is an increase in voltage occurring at the receiving end of a long transmission line, to the voltage at the sending end. This occur when the power line is energized but there is a no load at the end of power line. The situation is in the figure 7. The power line is shown as a T-network. The active part of cross admittance is neglected (G = 0), open end of power line and through the cross part of T-network will flow only capacity current. Due the open end the current is zero ($I_2 = 0$) so on the right side of longitudinal impedance is no voltage drop and on the cross admittance will be the voltage U_2. Through the left side of longitudinal impedance will flow the current:

$$I_1 = I_B = B_k \cdot l \cdot U_2 \quad (7)$$

The current I_1 effect voltage drop on a half of longitudinal impedance $\frac{Z_k}{2}$ and on cross admittance Y_k.

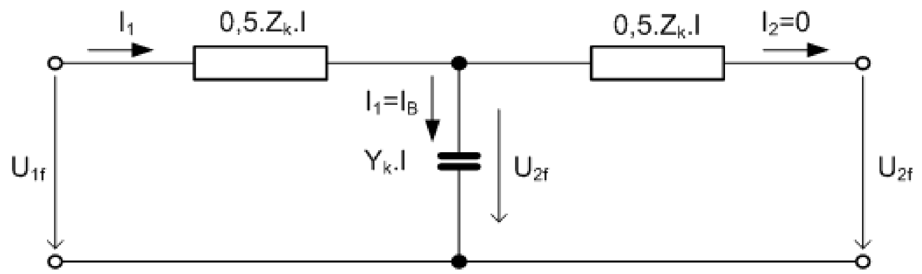

Fig. 7: The substitute scheme of T-network for open end of power network

Instead voltage drop occur in the power line the negative voltage drop. The difference between the voltage at the beginning and end is approximately (when neglected resistance R_k):

$$\mathbf{U}_2 - \mathbf{U}_1 = I_B \cdot \frac{X}{2} = 0{,}5.B_k.l.U.X_k.l \qquad (8)$$

For copper and aluminium power line ($\mu_r \doteq 1$) is the equation (8) simplified:

$$\mathbf{U}_2 - \mathbf{U}_1 = 0{,}55.U.l^2.10^{-6} \quad (kV;kV,km) \qquad (9)$$

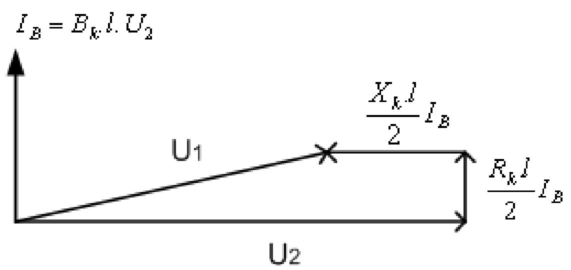

Fig. 8: Phasor diagram for power line with open end

Example: Calculate the voltage at the beginning when the power line lost his load at the end. The power line is 3 x 220 kV with length 500 km.

$\mathbf{U}_2 - \mathbf{U}_1 = 0{,}55.U.l^2.10^{-6} = 0{,}55.220.500.10^{-6} =$ **30,25 kV**

The voltage at the end increase approximately to the 250 kV.