Three-phase symmetrical transmission system in Fig. 1 delivers three-phase power P = 100 MW with cos φ = 1 at the end to a large power grid. Calculate the transmission angle and check the system steady-state stability. Neglect resistances and shunt admittances of elements in the system (generator synchronous reactance x_d respects saturation and voltage control).

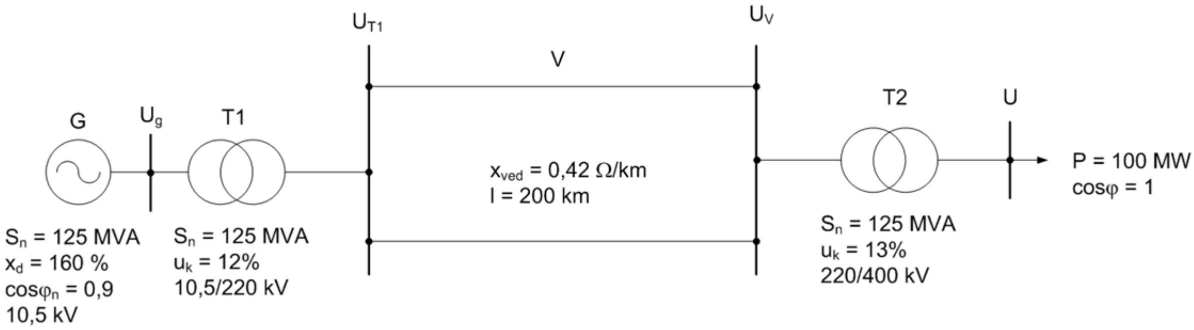

Fig. 1: *Circuit diagram*

Calculation in relative values considering base values:

- base power $S_v = 100$ MVA
- base voltage $U_v = 220$ kV (after T2)

Calculation to relative values:

- generator

$$x_g = \frac{x_d}{100} \cdot \frac{S_v}{S_g} \cdot \left(\frac{U_g}{U_v}\right)^2 \cdot \left(\frac{220}{10,5}\right)^2 = \frac{160}{100} \cdot \frac{100}{125} \cdot \left(\frac{10,5}{220}\right)^2 \cdot \left(\frac{220}{10,5}\right)^2 = 1,28$$

- transformer T1

$$x_{T1} = \frac{u_k}{100} \cdot \frac{S_v}{S_{T1}} \cdot \left(\frac{U_{T1}}{U_v}\right)^2 \cdot \left(\frac{220}{10,5}\right)^2 = \frac{12}{100} \cdot \frac{100}{125} \cdot \left(\frac{10,5}{220}\right)^2 \cdot \left(\frac{220}{10,5}\right)^2 = 0,096$$

- transformer T2

$$x_{T2} = \frac{u_k}{100} \cdot \frac{S_v}{S_{T2}} \cdot \left(\frac{U_{T2}}{U_v}\right)^2 \cdot \left(\frac{220}{400}\right)^2 = \frac{13}{100} \cdot \frac{100}{125} \cdot \left(\frac{400}{220}\right)^2 \cdot \left(\frac{220}{400}\right)^2 = 0,104$$

- power line

$$x_V = x_{ved}.l.\frac{1}{2} \cdot \frac{S_v}{U_v^2} = \frac{1}{2} \cdot 0,42.200 \cdot \frac{100.10^6}{(220.10^3)^2} = 0,087$$

Total reactance:

$$x_c = x_g + x_{T1} + x_v + x_{T2} = 1,567$$

Further we use the phasor diagram in Fig. 2 where we consider the load P = 100 MW with cos φ = 1 at the end (→ P = S_V). Voltage and current phasor (i = 1, not entirely accurate) are in phase at the end of transmission.

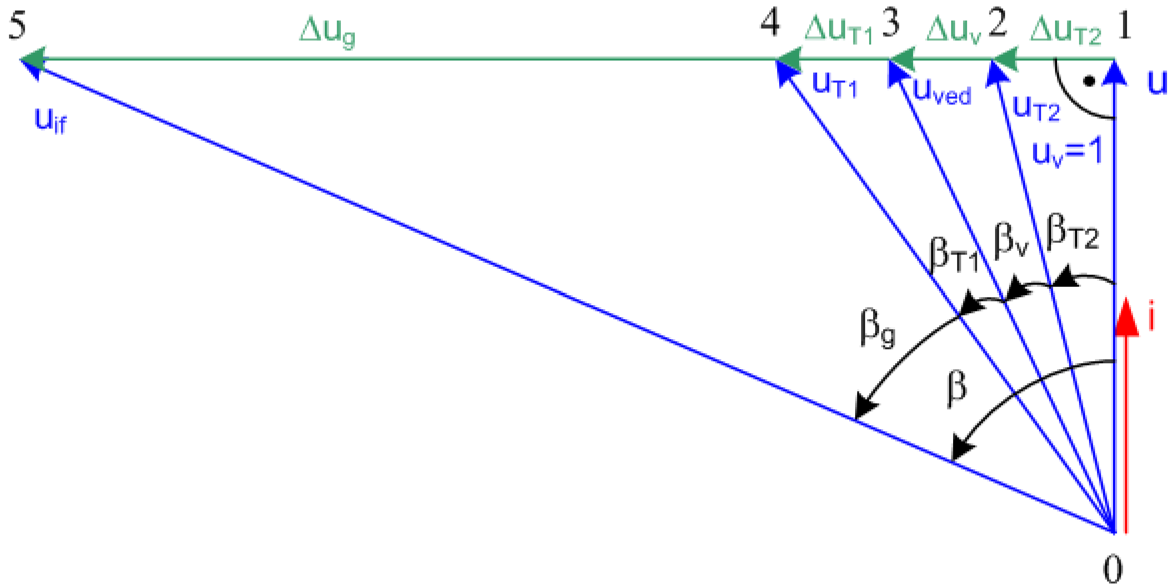

Fig. 2: *Phasor diagram*

We can see in Fig. 2 that the generator has the biggest influence on voltage phase shift. We must calculate first the voltage u to calculate the angle β. From triangle 0,1,2 we can calculate voltage u and angle β_{T2} because:

$$\Delta u_{T2} = x_{T2}.i = 0{,}104$$

Voltage drops on individual elements:

$$\Delta u_V = x_V.i = 0{,}087$$
$$\Delta u_{T1} = x_{T1}.i = 0{,}096$$
$$\Delta u_g = x_g.i = 1{,}28$$

Further:

$$u = \sqrt{u_v^2 - \Delta u_{T2}^2} = 0{,}995$$

Individual transmission angles according to goniometrical functions:

$$\sin\beta_{T2} = \frac{\Delta u_{T2}}{u_v} = \frac{0,104}{1} = 0,104$$

$\rightarrow \beta_{T2} = \underline{5,97°}$

$$tg(\beta_{T2} + \beta_v) = \frac{\Delta u_v + \Delta u_{T2}}{u} = \frac{0,087 + 0,104}{0,995} = 0,192$$

$\rightarrow \beta_{T2} + \beta_v = \underline{10,871°} \quad \rightarrow \beta_v = (\beta_{T2} + \beta_v) - \beta_{T2} = \underline{4,901°}$

$$tg(\beta_{T1} + \beta_{T2} + \beta_v) = \frac{\Delta u_{T2} + \Delta u_{ved} + \Delta u_{T1}}{u} = \frac{0,104 + 0,087 + 0,096}{0,995} = 0,287$$

$\rightarrow \beta_{T1} + \beta_{T2} + \beta_v = \underline{16,096°} \quad \rightarrow \beta_{T1} = (\beta_{T1} + \beta_{T2} + \beta_v) - (\beta_{T2} + \beta_v) = \underline{5,225°}$

$$tg(\beta_g + \beta_{T1} + \beta_{T2} + \beta_v) = \frac{\Delta u_g + \Delta u_{T1} + \Delta u_{ved} + \Delta u_{T2}}{u} = \frac{0,104 + 0,087 + 0,096 + 1,28}{0,995} = 1,5755$$

$\rightarrow \beta_g + \beta_{T1} + \beta_{T2} + \beta_v = \underline{57,597°} \quad \rightarrow \beta_g = (\beta_g + \beta_{T1} + \beta_{T2} + \beta_v) - (\beta_{T1} + \beta_{T2} + \beta_v) = \underline{41,\ 5°}$

Desired total transmission angle $\beta = \beta_g + \beta_{T1} + \beta_{T2} + \beta_v = \underline{57,597°}$.

For steady-state stability area:

$$\beta < \beta_{max} = 90\ °$$

It is valid, the transmission is steady-state stable.

Let us verify by means of the simplified steady-state stability equation. According to Fig. 2:

$$u_{if} = \frac{u}{\cos\beta} = \frac{0,995}{\cos 57,597°} = \underline{1,857}$$

Inserting to the equation

$$p = \frac{u_{if}.u}{x_c}\sin\beta = \frac{1,859.0,995}{1,567}\sin 57,597° \approx \underline{1}$$

than $P = p.S_v = 1.100 = \underline{100\ MW}$ with $\cos\varphi = 1$.

The maximal power which could be transmitted:

$$p_{max} = \frac{u_{if}.u}{x_c}\sin 90° = \frac{1,857.0,995}{1,567}\sin 90° = \underline{1,18}; \quad P_{max} = p_{max}.S_v = 1,18.100 = \underline{118\ MW}$$

Nominal generator active power:

$$P_g = S_g.\cos\varphi_n = 125.0,9 = \underline{112,5\ MW}$$