Energy Consumption Measurements Based on Numerical Integration

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***Abstract* — This paper deals with the possibility of energy consumption measurement using numerical integration. In this paper is described method based on Simpson integration and its comparison with standard analytical method of evaluating of energy consumption. The result of this article is a comparison of the error of energy of the analytical and numerical methods, depending on the number of samples required.**

Keywords* — *Energy Consumption, Numerical Integration, Energy Efficiency

I. Introduction

Whether the hypothesis of anthropogenic global warming is valid or not, the policy of energy consumption reduction has been and will be a fact.

There are many papers about reducing energy consumption. Various models and mathematical apparatuses can be used, for example, for methods using predictive control [1]. In addition, there are real-time control systems that can detect and determine energy costs [2]. Advanced systems using neural networks [3] and it is also necessary to pay attention to the energy consumption of the increasing connection of renewable resources. There are many methods using numerical integration, for example Simpson integration rule [4].

So, having been focused (successfully, because new profitable branches of industry have been established and operate) on heavy industry, transport, buildings, authorities (formal and/or real) are to spread their energy ideology into the area being in progress: small, usually DC fed (for example appliances using USB standard, chipboards in the things of the internet of things) devices.

For there is no control without measurements, the need of energy consumption measurement of devices described above exists. Of course, the danger of regulations leads us to the idea of low-consumption device for measuring of consumption of low consumption devices.

Main energy loses in measuring device are caused by CMOS pairs switching, so if we reduce the needed sample rate, we can reduce the energy consumption. Of course, analytical, nature-based integration is possible, but it must have been re-set and analytical multiplications of signals persist to be a problem.

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II. Model Situation

A. Typical situation

DC device in the stand-by regime, current peaks only charges internal batteries or capacitors etc.

Current time profile is as Fig. 1.

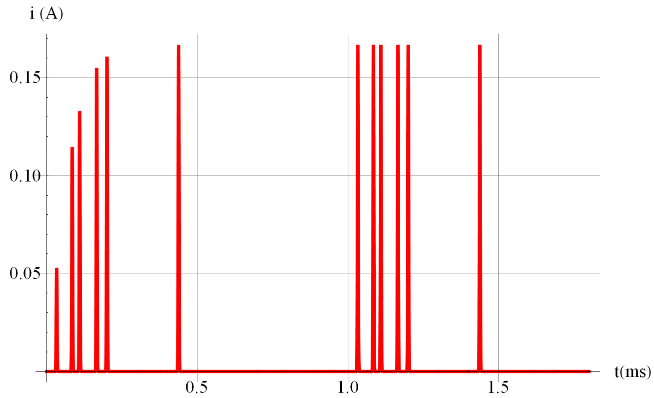

Fig. 1. Assumed current time profile.

Using an operational amplifier with constant k (Ω) and a shunt resistor, voltage $k \cdot i(t)$ is available.

The goal is a numerically found value of:

$$\Delta E = \int_{t=t1}^{t2} u_z(t) \cdot i(t) \cdot \mathrm{d}t = \int_{t=t1}^{t2} p(t) \cdot \mathrm{d}t \qquad (1)$$

Let's study the circuit according to Fig. 2.

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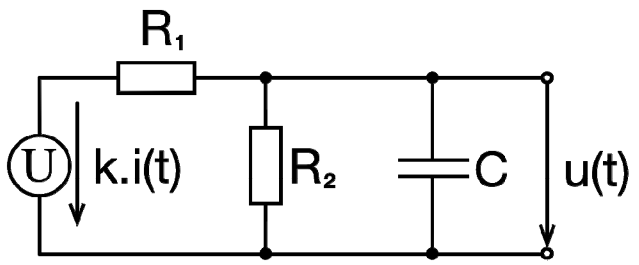

Fig. 2. Assumed input circuit for current measurement.

The formula for this circuit:

$$\frac{k \cdot i(t) - u(t)}{R_1} = C \cdot \frac{\mathrm{d}u(t)}{\mathrm{d}t} + \frac{u(t)}{R_2} \tag{2}$$

So, after simple treatment can be obtained *i(t)*:

$$i(t) = \frac{R_1}{k} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \cdot u(t) + \frac{R_1 \cdot C}{k} \cdot \frac{\mathrm{d}u(t)}{\mathrm{d}t} \tag{3}$$

Denoting *a* and *b*:

$$a = \frac{1}{k} \cdot \left(1 + \frac{R_1}{R_2} \right) \quad (S) \tag{4}$$

$$b = \frac{R_1 \cdot C}{k} \quad (\Omega \cdot s) \tag{5}$$

Than *i(t)* is denote:

$$i(t) = a \cdot u(t) + b \cdot \frac{\mathrm{d}u(t)}{\mathrm{d}t} \tag{6}$$

B. *Study case*

Two notes should be added:

- Nothing new, each circuit state variable has to be a linear form the others.
- At the first glance there is no profit: numerical integration of *i(t)* is of the same sample-rate problem as numerical integration of *du(t)/dt* and he problem of value *du(t)/dt* assessment

$$\Delta E = \int_{t=t1}^{t2} p(t) \cdot \mathrm{d}t = a \cdot \int_{t=t1}^{t2} u_z(t) \cdot u(t) \cdot \mathrm{d}t + \\ + b \cdot \int_{t=t1}^{t2} u_z(t) \cdot \frac{\mathrm{d}u}{\mathrm{d}t} \mathrm{d}t = a \cdot I_1 + b \cdot I_2 \tag{7}$$

Integral I_1 will be evaluated numerically and integration by parts will be used for I_2:

$$I_2 = \int_{t=t1}^{t2} u_z(t) \cdot \frac{\mathrm{d}u}{\mathrm{d}t} \mathrm{d}t = \left[u_z(t) \cdot u(t) \right]_{t=t_1}^{t_2} - \\ \int_{t=t1}^{t2} u(t) \cdot \frac{\mathrm{d}u_z}{\mathrm{d}t} dt = \left[u_z(t) \cdot u(t) \right]_{t=t_1}^{t_2} - I_3 \tag{8}$$

According to calculus there must exist at least one value $t_\xi \in (t_1, t_2)$ such as:

$$I_3 = \int_{t=t1}^{t2} u(t) \cdot \frac{\mathrm{d}u_z}{\mathrm{d}t} \mathrm{d}t = u(t = t_\xi) \cdot \int_{t=t1}^{t2} \frac{\mathrm{d}u_z}{\mathrm{d}t} \cdot \mathrm{d}t = \\ = u(t_\xi) \cdot (u_z(t2) - u_z(t1)) \tag{9}$$

Unfortunately, theory does not give us any information on how to find t_ξ and $u(t_\xi)$.

In the case of an ideal voltage source $u_z(t) = const.$, however the integral I_3 is zero.

For non – ideal voltage source $u_z(t) \neq const.$ we have studied two situation:

$$t_\xi = \frac{t_1 + t_2}{2} \tag{10}$$

And:

$$t_\xi = Mean(u(t))\Big|_{t \in (t_1, t_2)} \tag{11}$$

III. Numerical Experiments

There are many possible currents as functions of time. Combination of analytical functions and generated pseudorandom numbers has been used in order to avoid periodical interference between samples and current profile.

Current time dependence is according to Fig. 1. In fact, combination of *cosh*, *cos* and *tanh* functions periodical steep but smooth and continuous are generated.

Exact definition of current time dependance is:

$$g(t, j) = \tanh\left(\frac{t}{0.1 \cdot T} \right) \cdot \frac{\cosh\left(ns \cdot \left(\sin\left(\frac{\pi}{T} \cdot t + \frac{\pi}{2} \cdot rn(j) \right) \right)^{132} \right)}{\cosh(ns)} \tag{12}$$

Where:

$T = 1\,ms$, $ns = 480\,(-)$

and current is

$$i(t) = \frac{1}{np} \cdot \sum_{j=1}^{np} g(t, j) \qquad (13)$$

Where:

$np = 6$

$rn(j)$ is j-th term of vector.

{0.66869, 0.8312, 0.7818, 0.1246, 0.9345, 0.60025} (-)

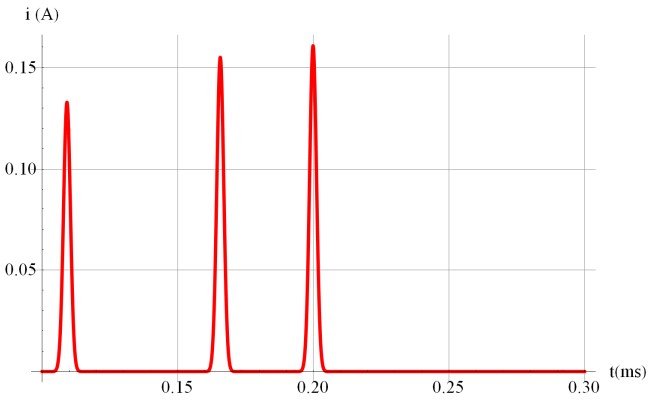

Fig. 3. On current peak.

Power source has been simulated according to Fig. 4.

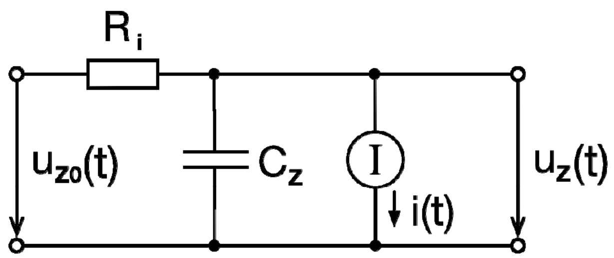

Fig. 4. Power supply circuit.

In future we want to analyze situations with $u_z0(t) \neq const.$, in the treatment below $u_z0(t) = const.$ is used.

Power circuit can be described by equation

$$\frac{uz0(t) - u_z(t)}{R_i} = C_Z \cdot \frac{\mathrm{d}u_z(t)}{\mathrm{d}t} + i(t) \qquad (14)$$

For ease of energy consumption evaluation, we add another equation:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = i(t) \cdot u_z(t) \qquad (15)$$

hence:

$$\Delta E = \int_{t=t1}^{t2} u_z(t) \cdot i(t) \cdot \mathrm{d}t = E(t2) - E(t1) \qquad (16)$$

The relevant equations were solved using Wolfram Mathematica software. Attention was paid to setting the solver parameters, especially the time step size.

Having found u_z, u and E in form of "InterpolatingFunction" object, we generated data for simulation of real measurement process.

Values t_i, $u_{zi} = u_z(t = t_i)$, $u_i = u(t = t_i)$ area available like arrays of samples, $i = 1, 2, \ldots, n-1, n$, $t_i = t_1 + (i-1) \cdot \Delta t$, $t2 = t_n$, $\Delta t = \frac{1}{sr}$, $sr \in \mathbf{N}$, where sr is sample rate.

Sample rate is the number of equal length intervals into which 1 second is divided.

Although values of quantities at other times can be obtained by means of interpolation, we do not include them in our research.

Several methods of numeric integration have been studied (trapezoidal and Simpson's rule and Gaussian formulae up to 6th order), but there is nearly no profit to use higher order methods.

In next Simpson's rule is used for numerical integration.

So, according to the treatment above, the total time is divided to sub – intervals and integrals evaluated as:

$$\int_{t_i}^{t_{i+2}} f(t) \cdot \mathrm{d}t \approx \frac{\Delta t}{3} \cdot \left(f(t_i) + 4 \cdot f(t_{i+1}) + f(t_{i+2}) \right) \qquad (17)$$

Integral I_3 is evaluated using

$$t_\xi = t_{i+1} \qquad (18)$$

$$u(t_\xi) = \frac{1}{3} \left(u(t_i) + u(t_{i+1}) + u(t_{i+2}) \right) \qquad (19)$$

Equation (18) is represented by blue line, (19) by green line, see Fig. 6.

IV. Results

Of course detailed analysis of the accuracy can be made, for example relative error (20) is a dimensionless value so it should be function of dimensionless quantities. However, a potential designer of a measuring device can easily perform a precision analysis for a particular situation using this article.

$$err = \frac{E_{true} - E_{measured}}{E_{true}} \cdot 100\ (\%) \qquad (20)$$

Where:

$E_{measured}$ was got as a precise integration of (13)

Used values $u_z0 = 5\ V$, $R_i = 15.6\ V$, $Cz = 100\ \mu F$, $C = 5.2\ \mu F$, $R_1 = 3\ k\Omega$, $R_2 = 2.5\ k\Omega$.

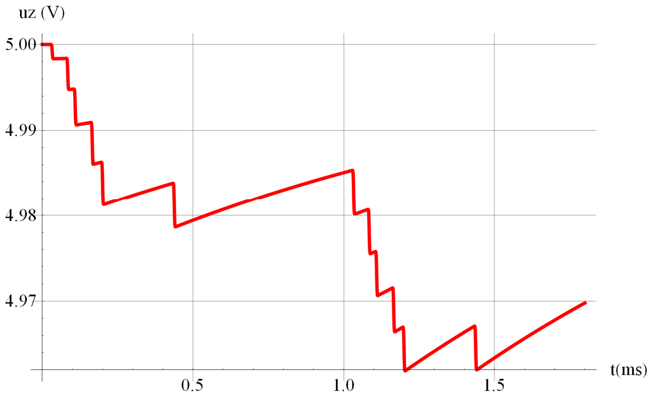

Fig. 5. Source voltage.

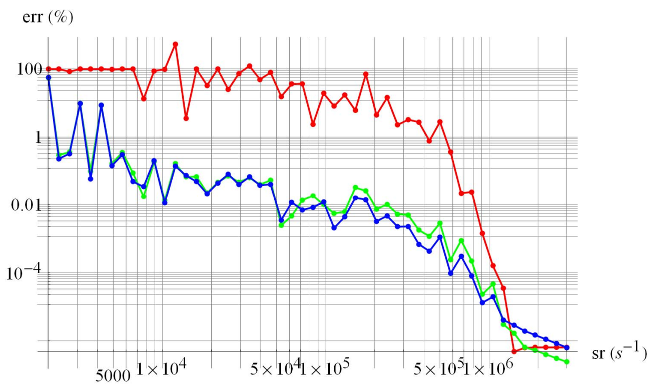

Fig. 6. Numerical simulations results.

Red line shows error of energy consumption obtained by direct numerical integration of $\int_{t=t1}^{t2} u_z(t) \cdot i(t) \cdot \mathrm{d}t$, blue line proposed method using (18) and green line proposed method using (19).

Exact energy has been assessed using "NDSolve" command in Wolfram Mathematica software. "MaxStepSize" value was reduced until the calculated energy ceased to change.

Fig. 6 shows us, that proposed method can give energy with good accuracy using nearly 100 times lower sample rate. It is also seen that with the increasing sampling rate the error starts to grow from a certain point in time, that's because rounding off of small numbers.

These advantages of the proposed method the autors don't know, but of course that is the task for future research.

V. Conclusion

This paper summarizes the issue of power sampling per time and numerical calculation of energy consumption. The simulated waveforms show the dependence of the total error of energy calculation while comparing the analytical and numerical methods. The numerical model based on the Simpson Integration Rule calculation is characterized by a relatively small error for the low number of samples per second. Using this could accelerate the algorithm for calculating energy consumption using sampling methods; there is no need for a large number of samples to be processed. The graph clearly shows that the numerical method has close to none significance while compared to the analytical, particularly because of the large number of samples and also due to the fact that the analytical method shows zero error. The main advantages of using the numerical method are lower number of required samples and shorter calculation times.

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