Prvky a provoz elektroenergetických soustav (B1M15PPE)

ZS 2016/2017

B1M15PPE - Prvky a provoz elektroenergetických soustav (2+2s) - sudý a lichý týden podle Časového plánu															
hodina	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
čas	7:30 - 9:00		9:15 - 10:45		11:00 - 12:30		12:45 - 14:15		14:30 - 16:00		16:15 - 17:45		18:00 - 19:30		20:30
Pondělí							T2:A4-203b - Pře J. Švec, P. Pivoňka 1(18 stud.)								
Úterý															
Středa					T2:C2-82 - Cvi J. Švec 101(18 stud.)										
Čtvrtek															
Pátek															
Přednášky				Cvičení				Laboratoře				Ostatní			

Témata

- Elektrické obvody, rovnice, zákony, výkon, energie
- Parametry distribučních a přenosových vedení
- Parametry a význam transformátorů, tlumivek a kondenzátorů v ES
- Základní výpočty v sítích – ustálené stavy soustav
- Základní výpočty v sítích – poruchové stavy
- Dimenzování prvků ES, rozvodny
- Chránění a jištění, ochrany proti přepětí
- Řízení napětí a frekvence v ES
- Kvalita elektrické energie, PPDS, vliv zdrojů na provoz DS
- Další vybrané aspekty: Stabilita přenosu, tepelná bilance vodiče, phase-shift TRF

Literatura

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- klasifikovaný zápočet – samostatná práce + test

Current and Voltage

Current

$$i, I, i(t), \hat{I} \quad (A)$$

$$i = \frac{dq}{dt} \quad (A; C, s)$$

charge flow in time

Voltage

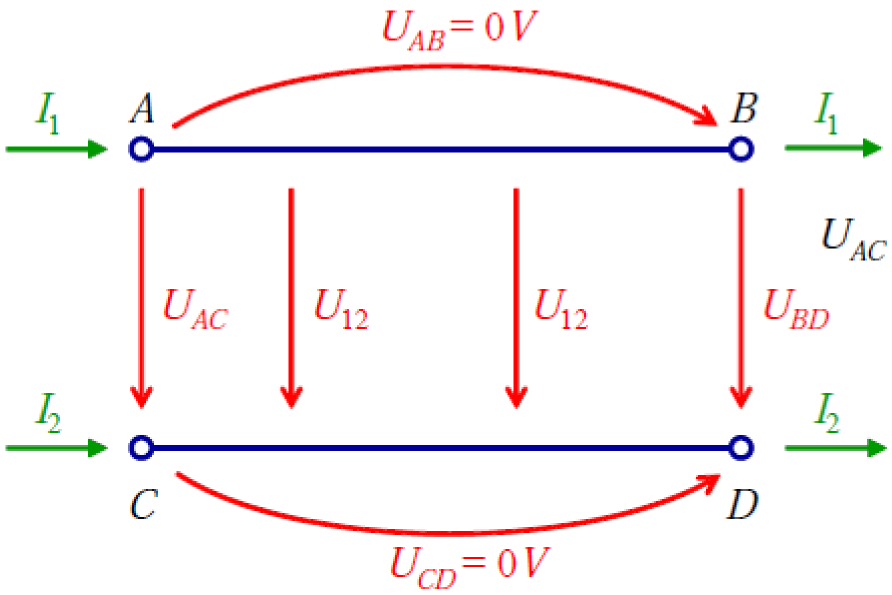
$$u, U, u(t), \hat{U} \quad (V)$$

$$u_{AB} = \frac{A}{q} \quad (V; J, C)$$

energy necessary to move unit charge

Current doesn't appear or disappear in el. circuits, it "flows in a round".
Current flow from A to B = I_{AB}, then current flow from B to A = $-I_{AB}$.
Voltage from A to B = U_{AB}, then voltage from B to A = $-U_{AB}$.

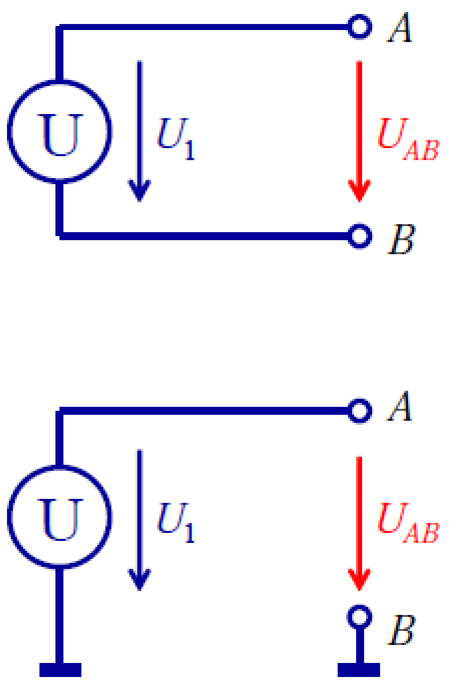
Ideal conductors



$$U_{AC} == U_{12} == U_{BD}$$

Electrical Sources

Ideal voltage source



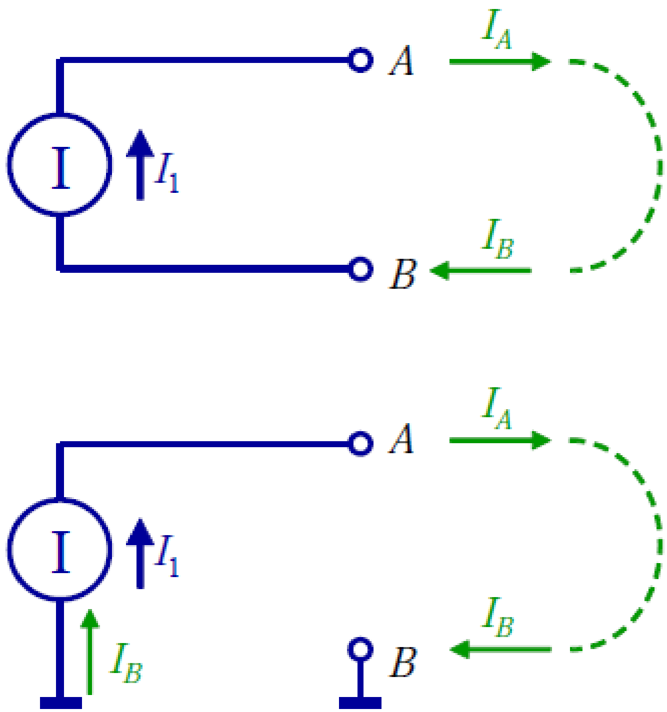
$U_1 == U_{AB}$

$U_1 == U_{AB}$

$U = const.$

$U \neq f(I)$

Ideal current source



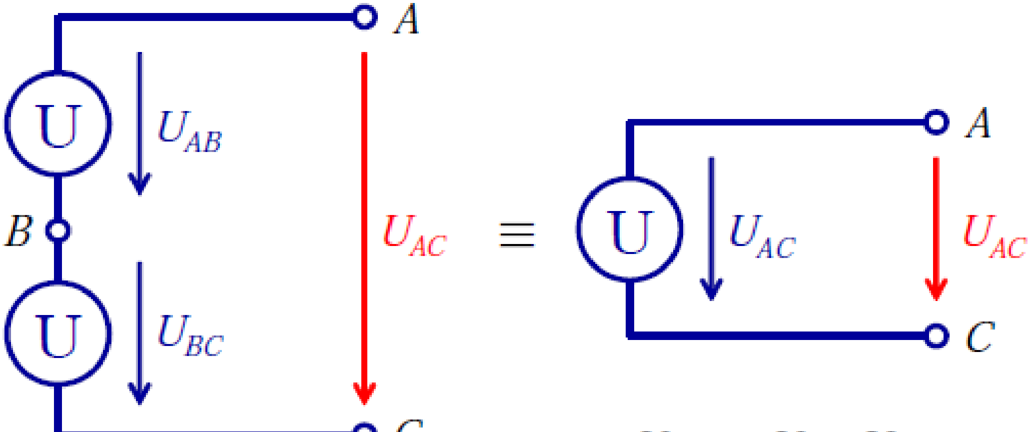
$I_1 == I_A == I_B$

$I = const.$

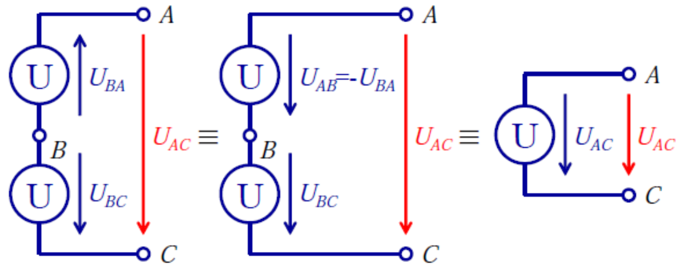
$I \neq f(U)$

Voltage sources connection

- in series



$$U_{AC} = U_{AB} + U_{BC}$$



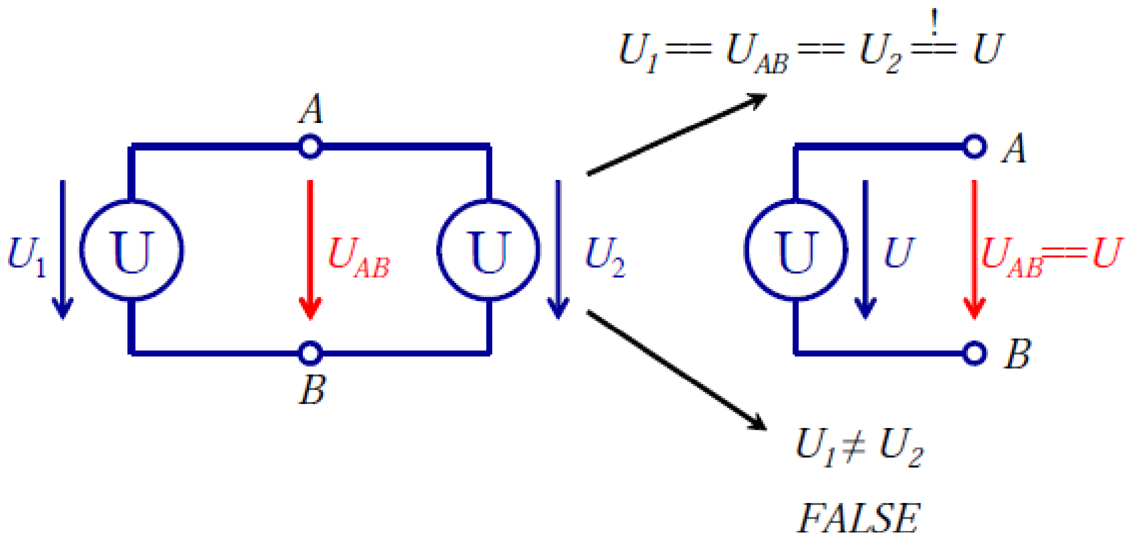
$$U_{AC} = U_{AB} + U_{BC} = -U_{BA} + U_{BC} = U_{BC} - U_{BA}$$

2nd Kirchhoff's law

$$\sum_{k=1}^{n} u_k = 0$$

Voltage sum in the circuit closed loop equals zero.

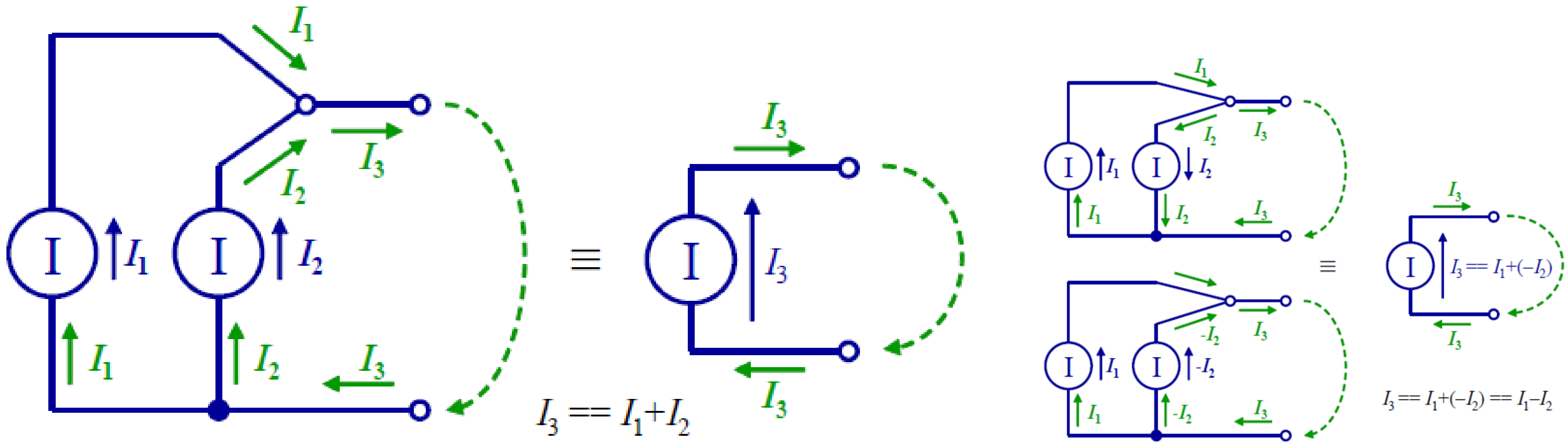
- in parallel



for real sources possible

Current sources connection

- in parallel

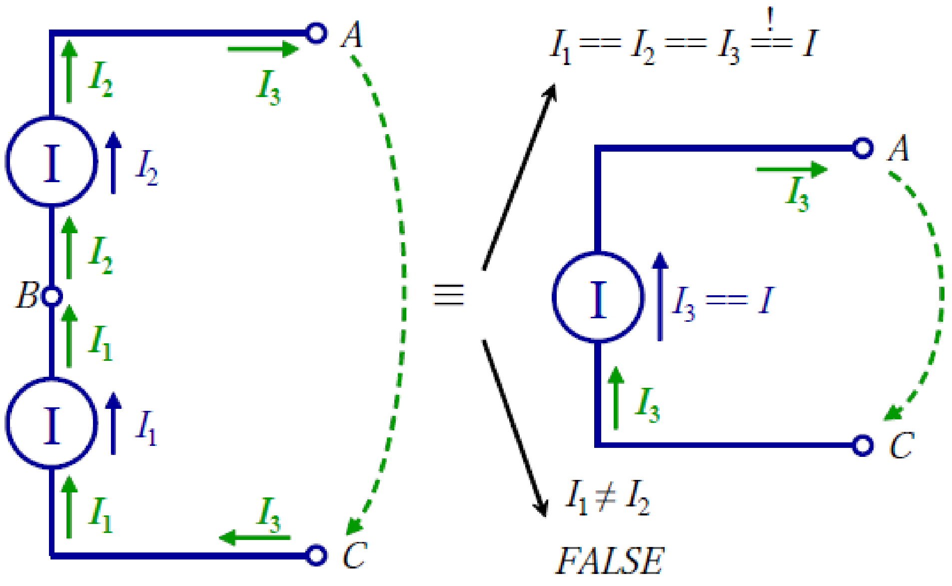


1st Kirchhoff's law

$$\sum_{k=1}^{n} i_k = 0$$

Current sum in the circuit bus (node) equals zero.

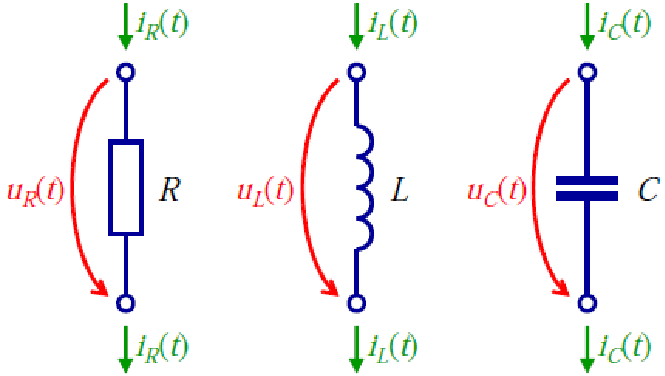
• in series



for real sources possible

Ideal voltage source mustn't be in short-circuit.
Ideal current source mustn't be in open-circuit.

Basic Elements of Electric Circuits



- resistance R (Ω)

$$u_R(t) = R \cdot i_R(t)$$ (Ohm's law)

- inductance L (H = Ω·s)

$$u_L(t) = L \cdot \frac{di_L(t)}{dt}$$

$$i_L(t) = \frac{1}{L}\int_0^t u_L(\tau)d\tau + i_L(0)$$ (continuous current)

- capacity C (F = s/ Ω)

$$i_C(t) = C \cdot \frac{du_C(t)}{dt}$$

$$u_C(t) = \frac{1}{C}\int_0^t i_C(\tau)d\tau + u_C(0)$$ (continuous voltage)

note: time constants

$$\tau = R \cdot C$$

$$\tau = \frac{L}{R}$$

$$\tau = \sqrt{L \cdot C}$$

Electric Circuit Description (Circuit Equations)

Bus voltage method: bus voltages, branch currents, 1st Kirchohoff's laws for buses, elements equations, initial conditions

Example (2 buses, 4 elements) – 6 equations, 6 unknown variables

$$i_{R1}(t) + i_L(t) - i_C(t) = 0$$

$$i_L(t) + i_{R2}(t) = 0$$

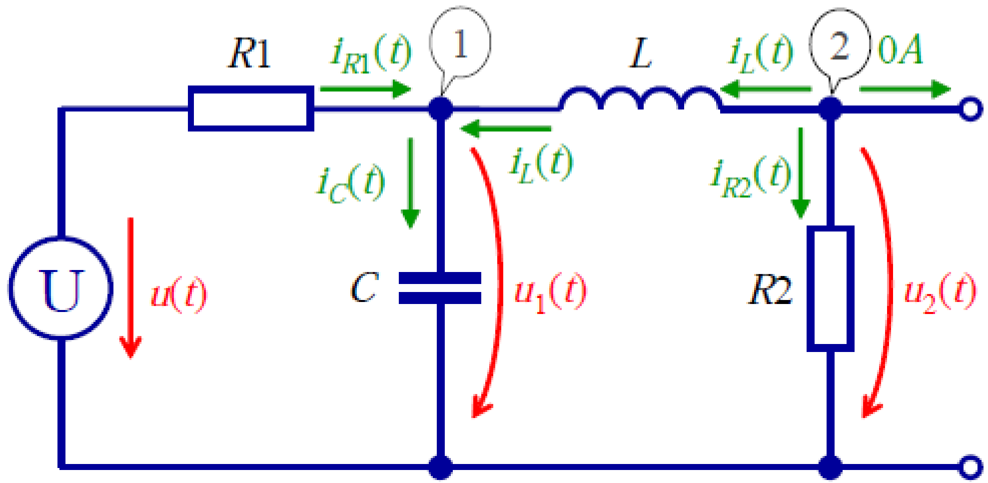
$$u(t) - u_1(t) = R_1 \cdot i_{R1}(t)$$

$$u_2(t) - u_1(t) = L \cdot \frac{di_L(t)}{dt}$$

$$i_C(t) = C \cdot \frac{du_1(t)}{dt}$$

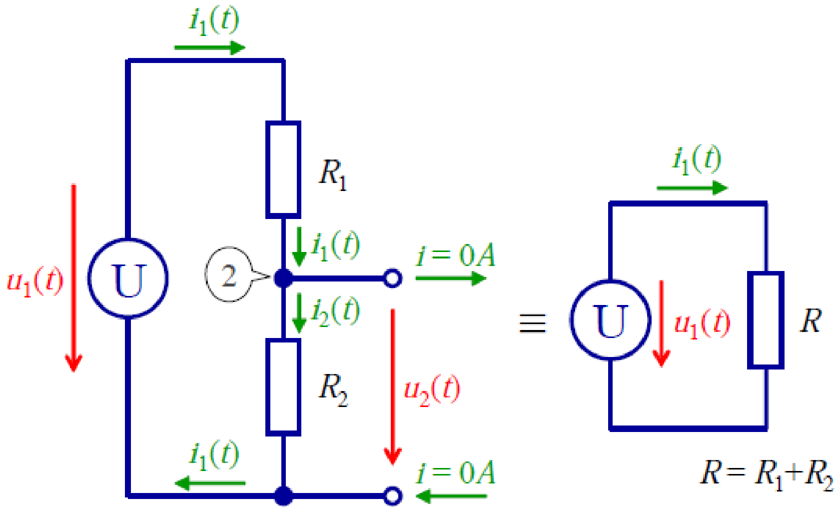
$$u_2(t) = R_2 \cdot i_{R2}(t)$$

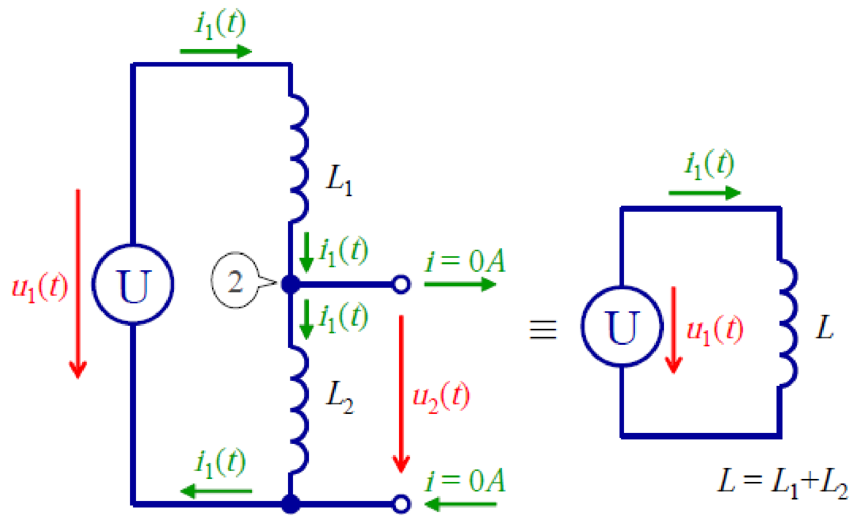
$$i_L(t = t0) = i_{L0};\ u_C(t = t0) = u_{C0}$$

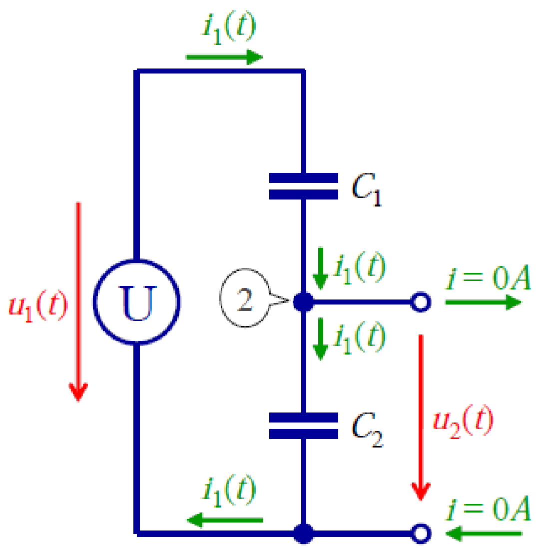


Circuit Elements Connection, Dividers

Connection in series







$$R = R_1 + R_2$$

$$L = L_1 + L_2$$

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

- Voltage divider (resistance divider)

$$u_2(t) = u_1(t) \cdot \frac{R_2}{R_1 + R_2}$$ (unloaded, non-reversible)

Connection in parallel

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

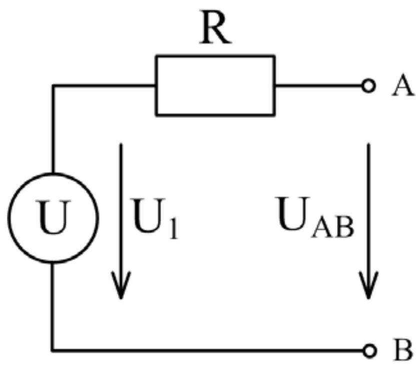
$$L = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}} = \frac{L_1 \cdot L_2}{L_1 + L_2}$$

$$C = C_1 + C_2$$

Real Sources

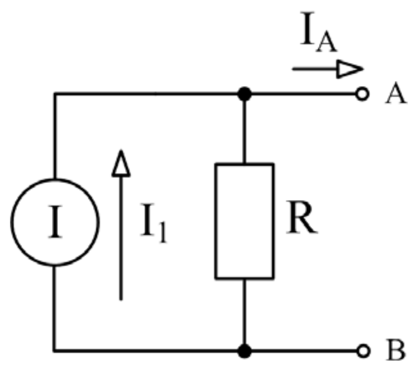
voltage source

$$U_{AB} = U_1 - R \cdot I$$



current source

$$I_A = I_1 - U_{AB} / R$$

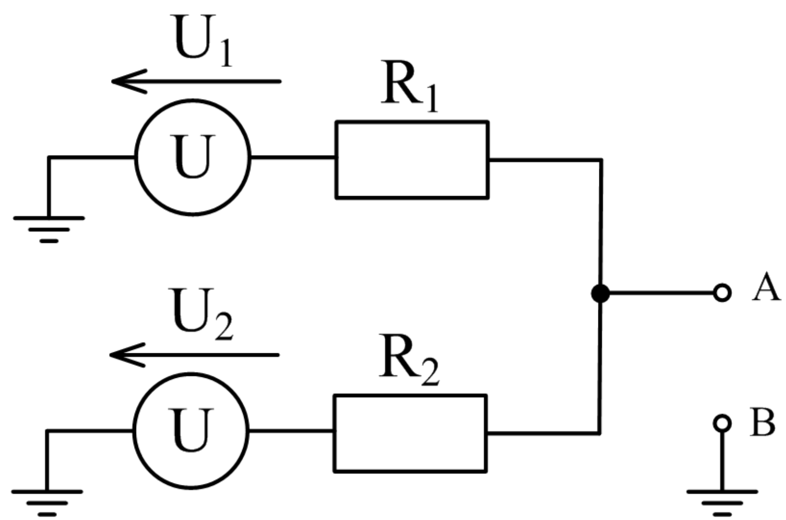


- real voltage sources in parallel

$$R_\Sigma = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

$$\frac{U_\Sigma}{R_\Sigma} = U_\Sigma \cdot \frac{R_1 + R_2}{R_1 \cdot R_2} = \frac{U_1}{R_1} + \frac{U_2}{R_2}$$

$$U_\Sigma = \frac{U_1 \cdot R_2 + U_2 \cdot R_1}{R_1 + R_2}$$



DC Circuits

Sources with constant output → transient phenomena → steady state.

$$u_R(t) = R \cdot i_R(t) = R \cdot i_R$$

$$u_L(t) = L \cdot \frac{di_L(t)}{dt} = 0$$

$$i_C(t) = C \cdot \frac{du_C(t)}{dt} = 0$$

- transients – differential equations
- steady – algebraic equations (disconnect C, short-circuit L)

Electric Power and Energy

Generally

$$p(t) = \frac{dW(t)}{dt} \quad (W; J, s)$$

$$p(t) = u(t) \cdot i(t) \quad (W; V, A)$$ - instantaneous power

Inductance

$$p(t) = L \cdot \frac{di(t)}{dt} \cdot i(t) = L \cdot \frac{d}{dt}\left(\frac{i(t)^2}{2}\right)$$

$$W = \int p(t)dt = L\int_{t_0}^{t_\infty} \frac{d}{dt}\left(\frac{i(t)^2}{2}\right)dt = L\int_{i_0}^{i_\infty} d\left(\frac{i(t)^2}{2}\right) = \frac{1}{2}L\left(i_\infty^2 - i_0^2\right)$$

- zero energy balance in overall time

Capacity

$$p(t) = u(t) \cdot C \cdot \frac{du(t)}{dt} = C \cdot \frac{d}{dt}\left(\frac{u(t)^2}{2}\right)$$

$$W = \int p(t)dt = C\int_{t_0}^{t_\infty} \frac{d}{dt}\left(\frac{u(t)^2}{2}\right)dt = C\int_{u_0}^{u_\infty} d\left(\frac{u(t)^2}{2}\right) = \frac{1}{2}C\left(u_\infty^2 - u_0^2\right)$$

- zero energy balance in overall time

Resistance

$$p(t) = R \cdot i(t) \cdot i(t) = R \cdot i(t)^2$$

$$W = \int p(t)dt = R\int_{t_0}^{t_\infty} i(t)^2 dt \geq 0$$

- electric energy conversion to heat energy

Harmonic Steady State

All quantities are harmonic functions or their linear combinations.

$$u(t) = U_M \cdot \sin(\omega \cdot t + \varphi) \qquad U_M \geq 0, \varphi \in \left(-\pi, \pi\right\rangle$$

$$\omega = 2\pi \cdot f = \frac{2\pi}{T}$$

$$U_M \cdot \sin(\omega \cdot t + \varphi) = U_M \cdot \cos\varphi \cdot \sin(\omega \cdot t) + U_M \cdot \sin\varphi \cdot \cos(\omega \cdot t)$$

Standard in AC systems.

Derivative of harmonic function is harmonic function with the same frequency.

$$u'(t) = \omega \cdot U_M \cdot \cos(\omega \cdot t + \varphi)$$

Using complex numbers

$$z = a + j \cdot b \qquad a, b \in R, a = Re(z), b = Im(z) \qquad j^2 = -1$$

$$Abs(z) = \sqrt{Re(z)^2 + Im(z)^2} = \sqrt{a^2 + b^2}$$

$$e^{j \cdot \varphi} = \cos\varphi + j \cdot \sin\varphi, \forall \varphi \in R$$ Euler's relation (φ in rad)

$$\sin(\varphi) = Im\{e^{j \cdot \varphi}\} \rightarrow$$

$$u(t) = U_M \cdot \sin(\omega \cdot t + \varphi) = Im\{U_M \cdot e^{j(\omega \cdot t + \varphi)}\}$$

$$u(t) = Im\{U_M \cdot e^{j \cdot \varphi} \cdot e^{j \cdot \omega \cdot t}\} = Im\{\hat{U}_M \cdot e^{j \cdot \omega \cdot t}\}$$

$\hat{U}_M = U_M \cdot e^{j \cdot \varphi}$ - phasor (in maximal values measure)

Phasors and Impedances

Time behaviour

$$u(t) = U_M \cdot \sin(\omega \cdot t + \varphi) = \mathrm{Im}\left\{\hat{U}_M \cdot e^{j \cdot \omega \cdot t}\right\}$$

$$U_M = \mathrm{Abs}(\hat{U}_M);\ \varphi = \mathrm{Arg}(\hat{U}_M)$$

Why phasors?: In harmonic states differential equations can be transformed to linear algebraic equations using only R, L, C values (time eliminated).

- resistance R

$$u_R(t) = R \cdot i_R(t)$$

$$\mathrm{Im}\left\{\hat{U}_R \cdot e^{j \cdot \omega \cdot t}\right\} = R \cdot \mathrm{Im}\left\{\hat{I}_R \cdot e^{j \cdot \omega \cdot t}\right\} = \mathrm{Im}\left\{R \cdot \hat{I}_R \cdot e^{j \cdot \omega \cdot t}\right\}$$

$$\hat{U}_R = R \cdot \hat{I}_R$$

- inductance L

$$u_L(t) = L \cdot \frac{di_L(t)}{dt}$$

$$\mathrm{Im}\left\{\hat{U}_L \cdot e^{j\cdot\omega\cdot t}\right\} = L \cdot \frac{d}{dt}\mathrm{Im}\left\{\hat{I}_L \cdot e^{j\cdot\omega\cdot t}\right\} = \mathrm{Im}\left\{L \cdot \hat{I}_L \cdot j \cdot \omega \cdot e^{j\cdot\omega\cdot t}\right\}$$

$$\hat{U}_L = j \cdot \omega \cdot L \cdot \hat{I}_L$$

- capacity C

$$i_C(t) = C \cdot \frac{du_C(t)}{dt}$$

$$\hat{I}_C = j \cdot \omega \cdot C \cdot \hat{U}_C$$

$$\hat{U}_C = \frac{1}{j \cdot \omega \cdot C} \cdot \hat{I}_C$$

In fact Ohm's law for harmonic steady state.

<u>Impedances</u>

$$\hat{Z}_R = R,\ \hat{Z}_L = j \cdot \omega \cdot L,\ \hat{Z}_C = \frac{1}{j \cdot \omega \cdot C}$$

They are not phasors!

Phasors in el. circuits

$$i_3(t) = i_1(t) + i_2(t) \quad \Rightarrow \quad \hat{I}_3 = \hat{I}_1 + \hat{I}_2$$

All functions with the same ω!

1st Kirchhoff's law

$$\sum_{k=1}^{n} \hat{I}_k = 0$$

2nd Kirchhoff's law

$$\sum_{k=1}^{n} \hat{U}_k = 0$$

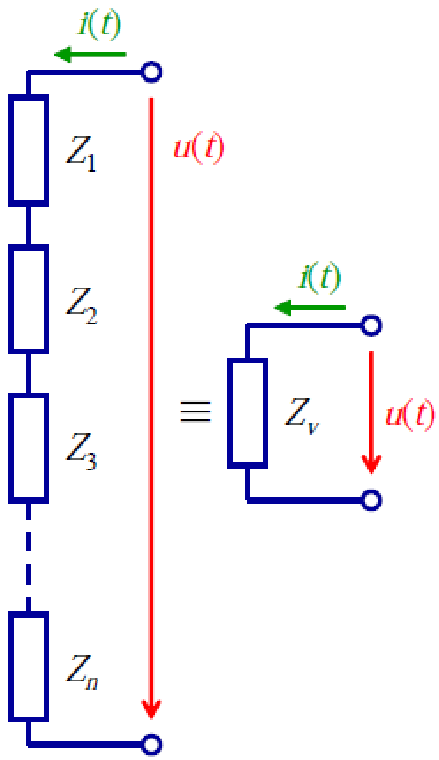
Impedances connection

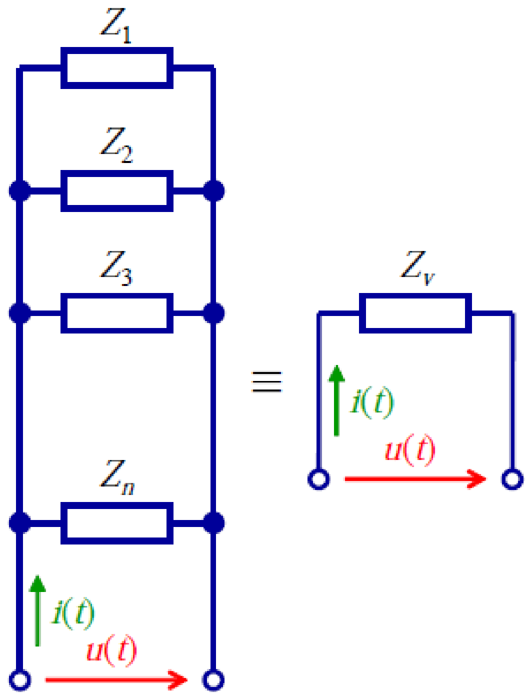
- in series

$$\hat{Z}_v = \sum_{i=1}^{n} \hat{Z}_i$$

- in parallel

$$\hat{Z}_v = \frac{1}{\sum_{i=1}^{n} \frac{1}{\hat{Z}_i}}$$





Other definitions

- admittance (S)

$$\hat{Y} = \frac{1}{\hat{Z}}$$

- conductance (S)

$$G = \frac{1}{R}$$

- reactance (Ω)

$$X_L = \omega \cdot L$$

$$X_C = \frac{1}{\omega \cdot C}$$

- susceptance (S)

$$B = \frac{1}{X}$$

Electric Power in Harmonic Steady State

Instantaneous power

$$p(t) = u(t) \cdot i(t)$$

Mean power for periodic course

$$P = \frac{1}{T} \int_{t=t_1}^{t_1+T} p(t) \cdot dt$$

for DC $\quad P = p(t) = P_{DC}$

$$u(t) = U_M \cdot \sin(\frac{2\pi}{T} \cdot t + \varphi_U) \qquad i(t) = I_M \cdot \sin(\frac{2\pi}{T} \cdot t + \varphi_I)$$

$$P = \frac{1}{T} \int_{t=0}^{T} u(t) \cdot i(t) \cdot dt = \frac{U_M \cdot I_M}{2} \cos(\varphi_U - \varphi_I)$$

$$U = \sqrt{\frac{1}{T} \int_{t=0}^{T} u^2(t) \cdot dt}$$

$$P = U \cdot I \cdot \cos\varphi$$

$$U = U_{RMS} = \frac{U_M}{\sqrt{2}}$$

"root mean square" value

$$\varphi = \varphi_U - \varphi_I$$

- resistance $\varphi = 0 \;\rightarrow\; P = U \cdot I$
- inductance, capacity $\varphi = \pm\pi/2 \;\rightarrow\; P = 0$

Complex Power in AC Grids

Instantaneous power

$$p(t) = u(t) \cdot i(t) = U_M \cdot \sin(\frac{2\pi}{T} \cdot t + \varphi_U) \cdot I_M \cdot \sin(\frac{2\pi}{T} \cdot t + \varphi_I)$$

$$p(t) = \frac{1}{2} U_M \cdot I_M \cdot \left[\cos(\varphi_I - \varphi_U) - \cos(2\omega \cdot t + \varphi_U + \varphi_I)\right]$$

e.g. for $\varphi_U = 0$

$$\varphi = \varphi_U - \varphi_I = -\varphi_I$$

$$p(t) = U \cdot I \cdot \left[\cos\varphi - \cos(2\omega \cdot t - \varphi)\right]$$

- resistance $\varphi = 0$

$$p(t) = U \cdot I \cdot [1 - \cos(2\omega \cdot t)]$$

$$P = \frac{1}{T} \int_{t=0}^{T} p(t) \cdot dt = U \cdot I$$

"active power"

- inductance $\varphi = \pi / 2$

$$p(t) = U \cdot I \cdot [0 - \cos(2\omega \cdot t - \pi / 2)]$$

$$p(t) = U \cdot I \cdot \sin(2\omega \cdot t)$$

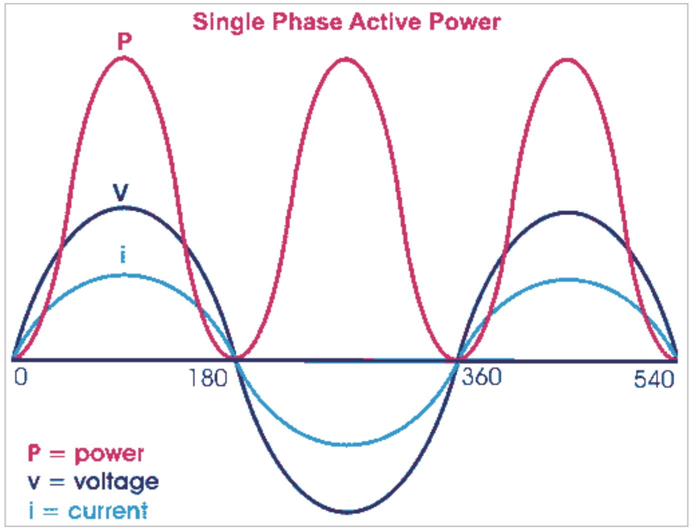
$$P = \frac{1}{T} \int_{t=0}^{T} p(t) \cdot dt = 0$$

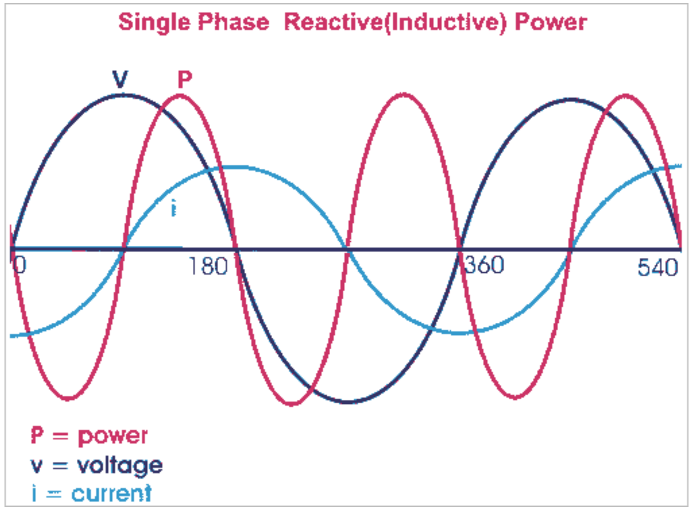
- capacity $\varphi = -\pi / 2$

$$p(t) = U \cdot I \cdot [0 - \cos(2\omega \cdot t + \pi / 2)]$$

$$p(t) = -U \cdot I \cdot \sin(2\omega \cdot t)$$

$$P = 0$$





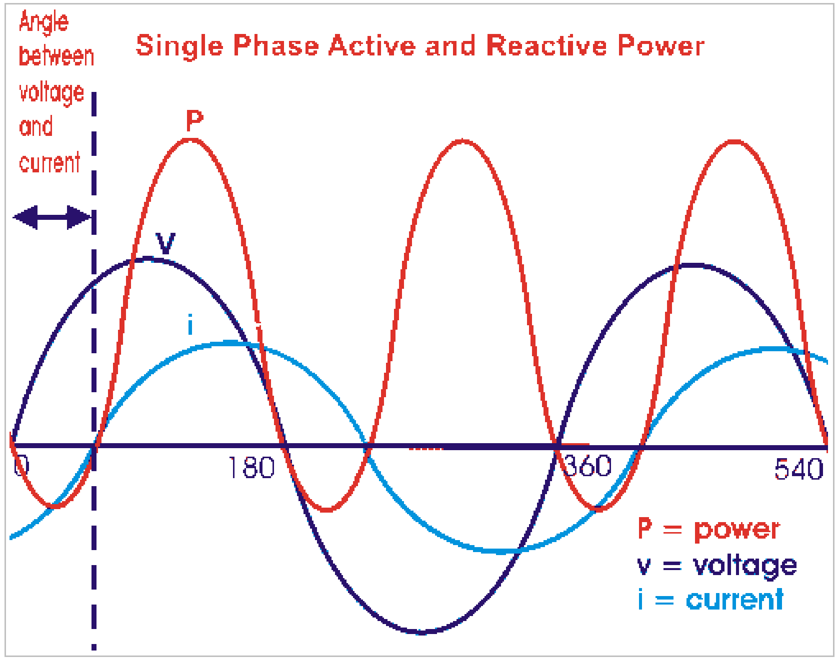
- general load

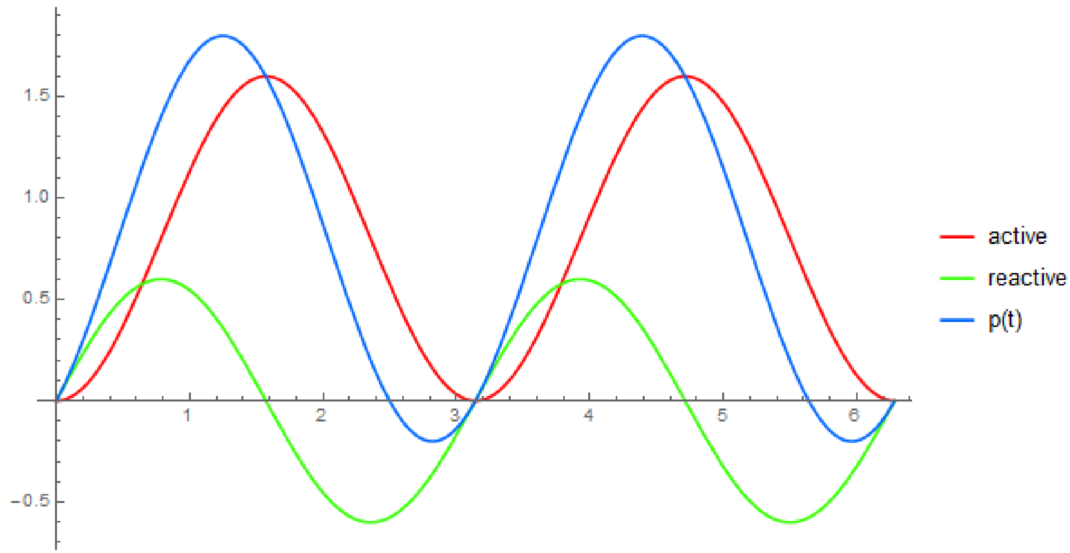
$$p(t) = U \cdot I \cdot [\cos\varphi - \cos(2\omega \cdot t - \varphi)]$$

$$p(t) = U \cdot I \cdot (\cos\varphi - \cos\varphi \cdot \cos 2\omega \cdot t - \sin\varphi \cdot \sin 2\omega \cdot t)$$

$$p(t) = U \cdot I \cdot [\cos\varphi \cdot (1 - \cos 2\omega \cdot t) - \sin\varphi \cdot \sin 2\omega \cdot t]$$

"active and reactive" component





Power and Phasors

Definitions

apparent power $S = U \cdot I \quad (VA)$

active power $P = \frac{1}{T} \int_{t=0}^{T} p(t) \cdot dt \quad (W)$

reactive power $S = \sqrt{P^2 + Q^2}$

$Q = \sqrt{S^2 - P^2} \quad (VAr / var)$

power factor $\cos \varphi = \frac{P}{S}$

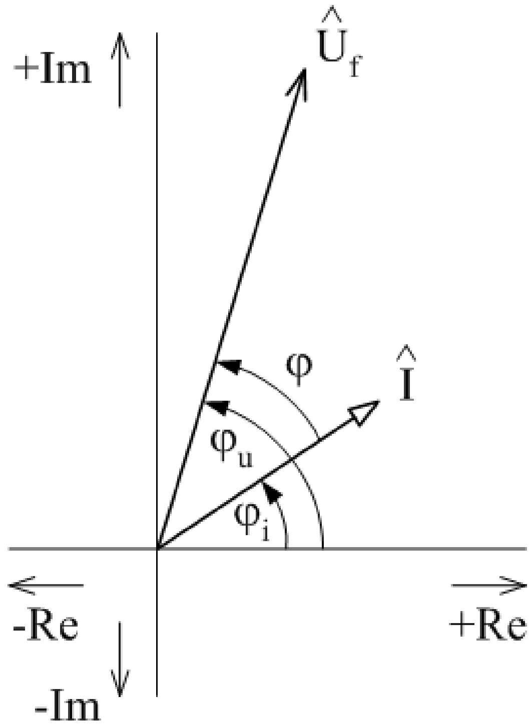
Complex conjugation (current)

$$\hat{S}_1 = \hat{U} \cdot \hat{I}^* = U \cdot I \cdot e^{j(\varphi_u - \varphi_i)} = U \cdot I \cdot e^{j\varphi}$$

$$\hat{S}_1 = \hat{U} \cdot \hat{I}^* = P_1 \pm jQ_1 \quad \begin{matrix} \text{IND} \\ \text{CAP} \end{matrix}$$

$$\hat{S}_1 = P_1 \pm jQ_1 = U \cdot I \cdot (\cos\varphi \pm j\sin\varphi) = S_1 e^{\pm j\varphi}$$

Sign according to convention.



Inductive load

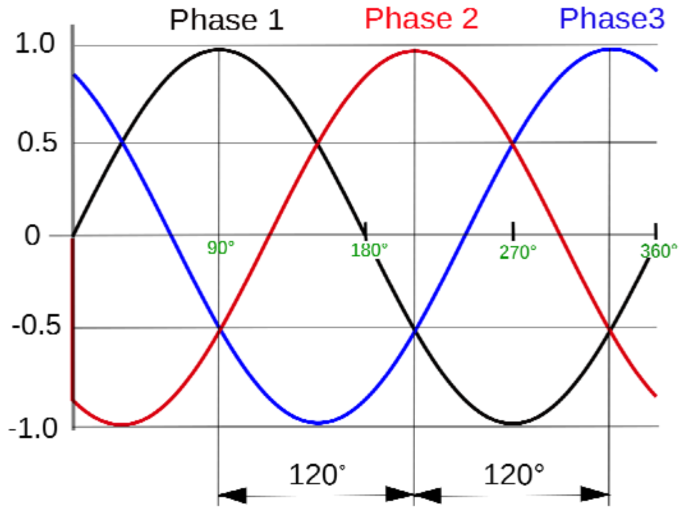
$$\hat{U}_f = U_f e^{j\varphi_u} \quad , \hat{I}_f = I e^{j\varphi_i}$$

3 phase systems (symmetrical)

$$u_A(t) = U_M \cdot \sin(\omega \cdot t + \varphi_U)$$

$$u_B(t) = U_M \cdot \sin(\omega \cdot t + \varphi_U - \frac{2\pi}{3})$$

$$u_C(t) = U_M \cdot \sin(\omega \cdot t + \varphi_U + \frac{2\pi}{3})$$



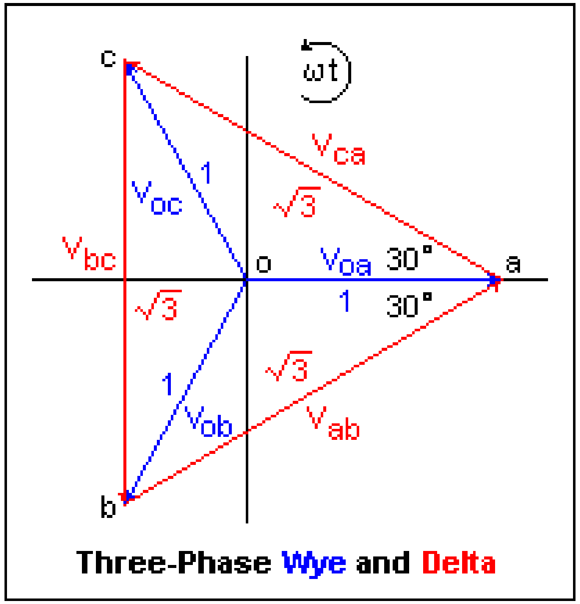
$$\hat{U}_A, \hat{U}_B = \hat{U}_A \cdot e^{-j\frac{2\pi}{3}}, \hat{U}_C = \hat{U}_A \cdot e^{j\frac{2\pi}{3}}$$

$$\hat{U}_A, \hat{U}_B = \hat{a}^2 \cdot \hat{U}_A, \hat{U}_C = \hat{a} \cdot \hat{U}_A$$

$$\hat{a} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = e^{+j\frac{2\pi}{3}}$$

$$\hat{a}^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = e^{-j\frac{2\pi}{3}}$$

$$1 + \hat{a}^2 + \hat{a} = 0 \quad \Rightarrow \quad \hat{U}_A + \hat{U}_B + \hat{U}_C = 0$$



$$U = \sqrt{3} \cdot U_{ph}$$

"phase-to-ground", "phase-to-phase" voltage

Power

- general

$$\hat{S} = \hat{U}_A \cdot \hat{I}_A^* + \hat{U}_B \cdot \hat{I}_B^* + \hat{U}_C \cdot \hat{I}_C^*$$

- symmetrical

$$\hat{S} = 3\hat{U}_{ph} \cdot \hat{I}^* = \sqrt{3}\hat{U} \cdot \hat{I}^*$$

$$S = 3U_{ph}I = \sqrt{3}UI = \sqrt{P^2 + Q^2} \quad (\mathrm{VA})$$

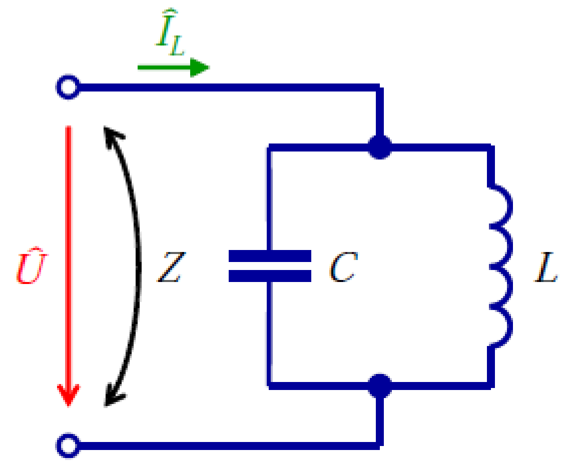
$$P = 3U_{ph}I\cos\varphi = \sqrt{3}UI\cos\varphi \quad (\mathrm{W})$$

$$Q = 3U_{ph}I\sin\varphi = \sqrt{3}UI\sin\varphi \quad (\mathrm{VAr})$$

Resonance

- parallel

$$\hat{Z} = \frac{1}{\frac{1}{j\cdot\omega\cdot L} + \frac{1}{\frac{1}{j\cdot\omega\cdot C}}} = \frac{j\cdot\omega\cdot L}{1-\omega^2\cdot L\cdot C}$$



$$\omega_0 = \frac{1}{\sqrt{L\cdot C}}$$ - resonance frequency

$$\lim_{\omega\to\omega_0} \mathrm{Abs}(\hat{Z}) = \lim_{\omega\to\omega_0} \mathrm{Abs}\left(\frac{\hat{U}}{\hat{I}}\right) = \infty \quad \Rightarrow \quad \mathrm{Abs}(\hat{U}) < \infty \rightarrow \lim_{\omega\to\omega_0} \mathrm{Abs}(\hat{I}) = 0$$

$\omega < \omega_0$ - L mode

$\omega > \omega_0$ - C mode

e.g. Ripple control signal (HDO) support

- series (for ideal circuit)

$$\hat{Z} = j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C} = -j \cdot \frac{1 - \omega^2 \cdot L \cdot C}{\omega \cdot C}$$

$$\omega_0 = \frac{1}{\sqrt{L \cdot C}}$$ - resonance frequency

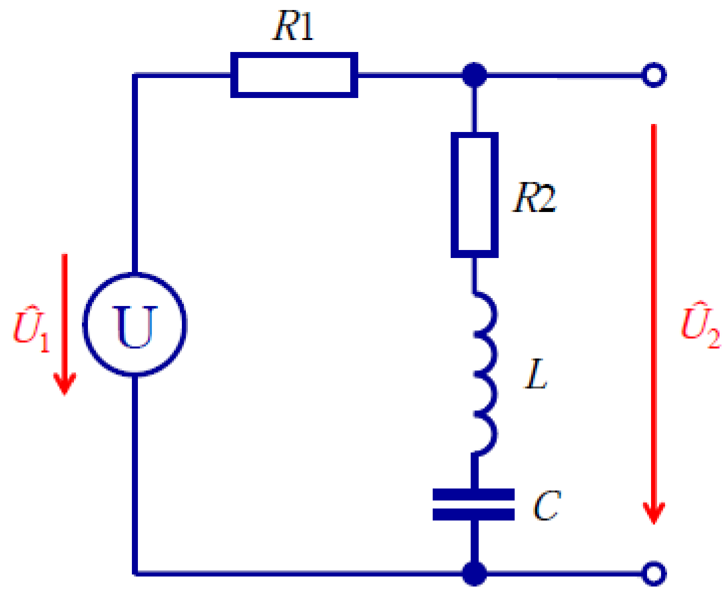
$$\lim_{\omega \to \omega_0} \mathrm{Abs}(\hat{Z}) = \lim_{\omega \to \omega_0} \mathrm{Abs}\left(\frac{\hat{U}}{\hat{I}}\right) = 0$$

$$\Rightarrow \quad \mathrm{Abs}(\hat{I}) < \infty \rightarrow \lim_{\omega \to \omega_0} \mathrm{Abs}(\hat{U}) = 0$$

$\omega < \omega_0$ - C mode

$\omega > \omega_0$ - L mode

e.g. higher harmonic filters (power quality)



Fourier Series

For periodical continuous functions

$$f(t) = a_0 + \sum_{i=1}^{n} \left(a_i \cdot \cos\left(i \cdot 2\pi \cdot \frac{t}{T} \right) + b_i \cdot \sin\left(i \cdot 2\pi \cdot \frac{t}{T} \right) \right)$$

ideally $n \to \infty$, technically $n < \infty$ is sufficient

$$a_0 = \frac{1}{T} \cdot \int_{t=0}^{T} f(t) \cdot dt$$

(mean value, DC value)

$$a_k = \frac{2}{T} \cdot \int_{t=0}^{T} f(t) \cdot \cos\left(k \cdot 2\pi \cdot \frac{t}{T} \right) \cdot dt$$

$$b_k = \frac{2}{T} \cdot \int_{t=0}^{T} f(t) \cdot \sin\left(k \cdot 2\pi \cdot \frac{t}{T} \right) \cdot dt$$

Fourier series and phasors – harmonic components

$$u_{b_k}(t) = b_k \cdot \sin\left(k \cdot 2\pi \cdot \frac{t}{T}\right) = b_k \cdot \sin(\omega_k \cdot t) = \mathrm{Im}\left\{b_k \cdot e^{j\omega_k t}\right\} = \mathrm{Im}\left\{\hat{U}_{b_k} \cdot e^{j\omega_k t}\right\}$$

$$\rightarrow \hat{U}_{b_k} = b_k$$

$$u_{a_k}(t) = a_k \cdot \cos\left(k \cdot 2\pi \cdot \frac{t}{T}\right) = a_k \cdot \cos(\omega_k \cdot t) = \mathrm{Im}\left\{a_k \cdot e^{j\pi/2} \cdot e^{j\omega_k t}\right\} =$$

$$= \mathrm{Im}\left\{\hat{U}_{a_k} \cdot e^{j\omega_k t}\right\}$$

$$\rightarrow \hat{U}_{a_k} = a_k \cdot e^{j\pi/2}$$

$$u_k(t) = u_{a_k}(t) + u_{b_k}(t) = \mathrm{Im}\left\{\left(b_k + a_k \cdot e^{j\pi/2}\right) \cdot e^{j\omega_k t}\right\} = \mathrm{Im}\left\{\hat{U}_k \cdot e^{j\omega_k t}\right\}$$

$\hat{U}_k = \hat{U}_{a_k} + \hat{U}_{b_k} = b_k + j \cdot a_k$ - analysis for each harmonic separately

$$\hat{U}_k = U_k \cdot e^{j \cdot \varphi_k} \qquad \left|\hat{U}_k\right| = \sqrt{a_k^2 + b_k^2} \qquad \varphi_k = \mathrm{arctg}\frac{a_k}{b_k}$$