

Prvky a provoz elektroenergetických soustav (B1M15PPE)

ZS 2017/2018

B1M15PPE - Prvky a provoz elektroenergetických soustav (2+2s) - sudý a lichý týden podle Časového plánu																
hodina	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
čas	7:30 - 9:00		9:15 - 10:45		11:00 - 12:30		12:45 - 14:15		14:30 - 16:00		16:15 - 17:45		18:00 - 19:30		20:30	
Pondělí							T2:C4-364 - Pře J. Hlaváček 1(15 stud.)									
Úterý																
Středa					T2:C2-82 - Cvi O. Šefl 101(15 stud.)											
Čtvrtek																
Pátek																
	Přednášky				Cvičení				Laboratoře				Ostatní			

Prvky a provoz elektroenergetických soustav (B1M15PPE)



Výukové materiály

Přednášky / Lectures	Cvičení / Seminars
1. Vybraná témata teorie elektrických obvodů	Cvičení 1 - Výpočty z el. obvodů
2. Parametry vedení PS a DS	Cvičení 2 - Parametry vedení
3. Transformátory, tlumivky, kondenzátory v sítích	Cvičení 3 - Parametry transformátorů
4. Základní výpočty v sítích - ustálené stavy	Cvičení 4 - Ustálené stavy v DS
5. Základní výpočty v sítích - poruchové stavy	Cvičení 5 - Poruchy v DS
6. Řízení napětí a frekvence v ES	Cvičení 6 - Regulace f, U
7. Kvalita elektrické energie	Cvičení 7 - Ampacita vedení
8.	Cvičení 8 - Kvalita el. energie
9.	Cvičení 9
10.	Cvičení 10
11. Stejnoseměrné stroje	Cvičení 11 - Zkušební protokoly
12. Asynchronní stroje	Cvičení 12
13. Synchronní stroje	Cvičení 13

Témata

- Elektrické obvody, rovnice, zákony, výkon, energie
- Parametry distribučních a přenosových vedení
- Parametry a význam transformátorů, tlumivek a kondenzátorů v ES
- Základní výpočty v sítích – ustálené stavy soustav
- Základní výpočty v sítích – poruchové stavy
- Dimenzování prvků ES, rozvodny
- Chránění a jištění, ochrany proti přepětí
- Řízení napětí a frekvence v ES
- Kvalita elektrické energie, PPDS, vliv zdrojů na provoz DS
- Další vybrané aspekty: Stabilita přenosu, tepelná bilance vodiče, phase-shift TRF

Literatura

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- [8] Kasicki, Ismail. Analysis and design of low-voltage power systems: an engineer's field guide. Weinheim: Wiley, ©2002. xxii, 387 s. ISBN 9783527602339.
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- klasifikovaný zápočet – samostatná práce + test

Current and Voltage

Current

$$i, I, i(t), \hat{I} \quad (\text{A})$$

$$i = \frac{dq}{dt} \quad (\text{A; C, s}) \quad \text{charge flow in time}$$

Voltage

$$u, U, u(t), \hat{U} \quad (\text{V})$$

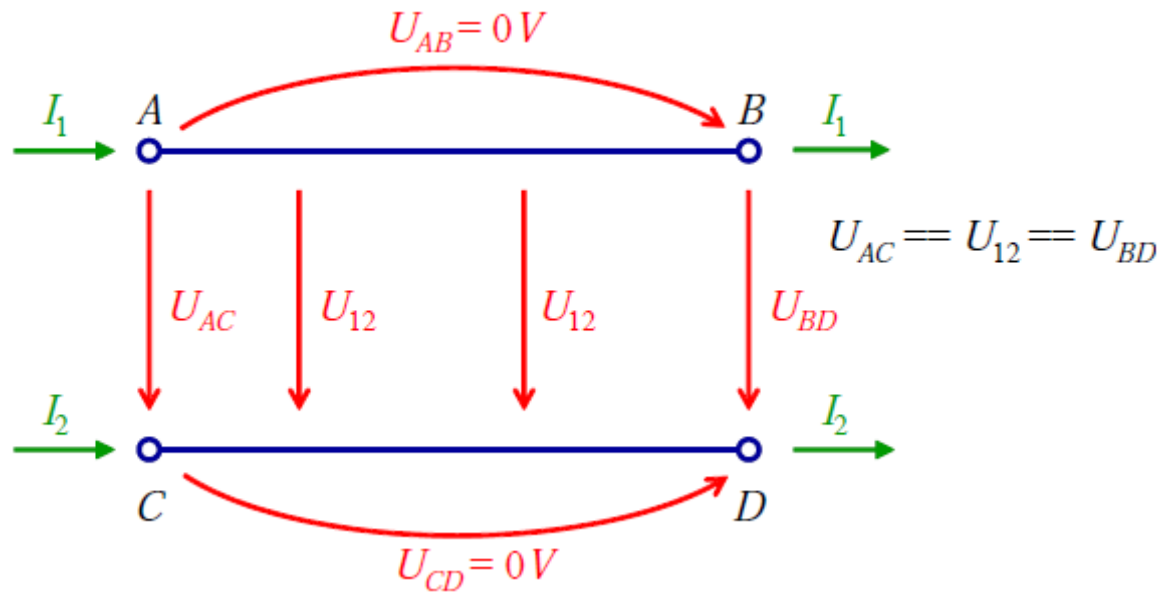
$$u_{AB} = \frac{A}{q} \quad (\text{V; J, C}) \quad \text{energy necessary to move unit charge}$$

Current doesn't appear or disappear in el. circuits, it "flows in a round".

Current flow from A to B = I_{AB} , then current flow from B to A = $-I_{AB}$.

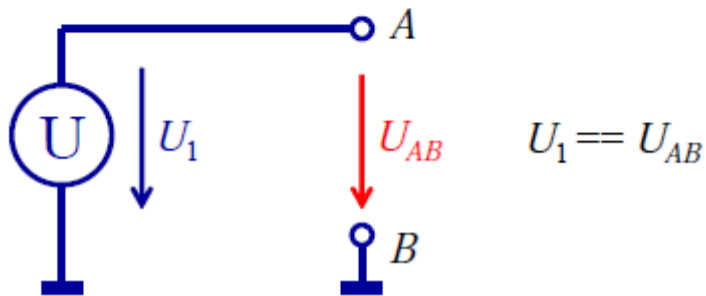
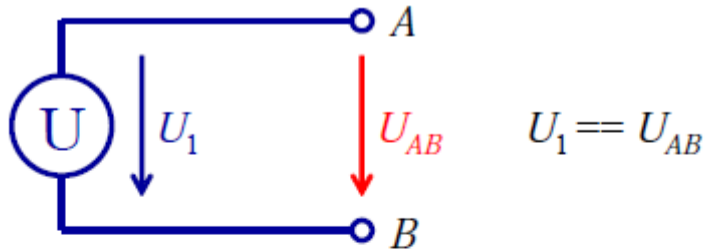
Voltage from A to B = U_{AB} , then voltage from B to A = $-U_{AB}$.

Ideal conductors



Electrical Sources

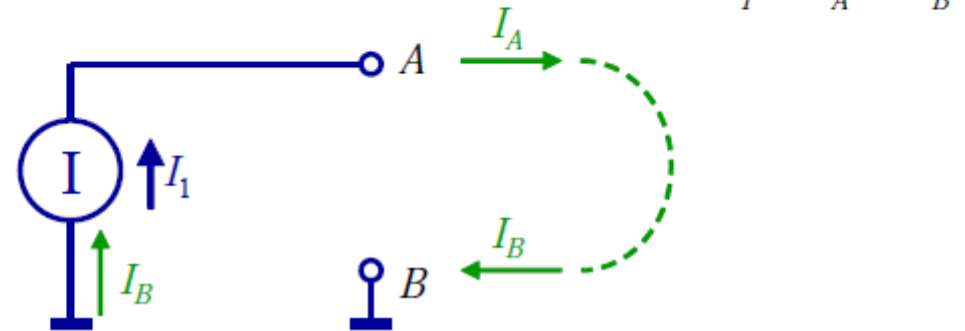
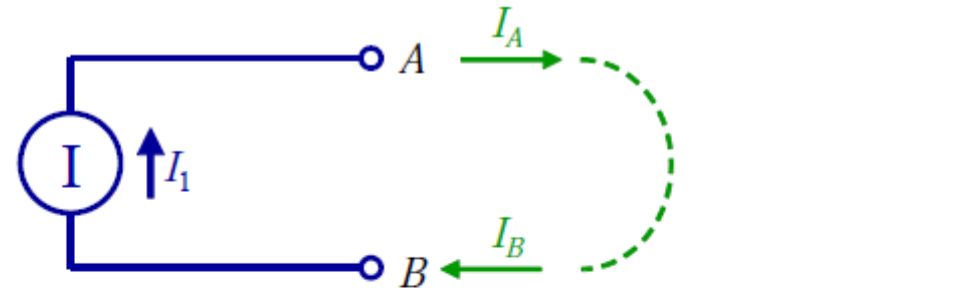
Ideal voltage source



$U = \text{const.}$

$U \neq f(I)$

Ideal current source

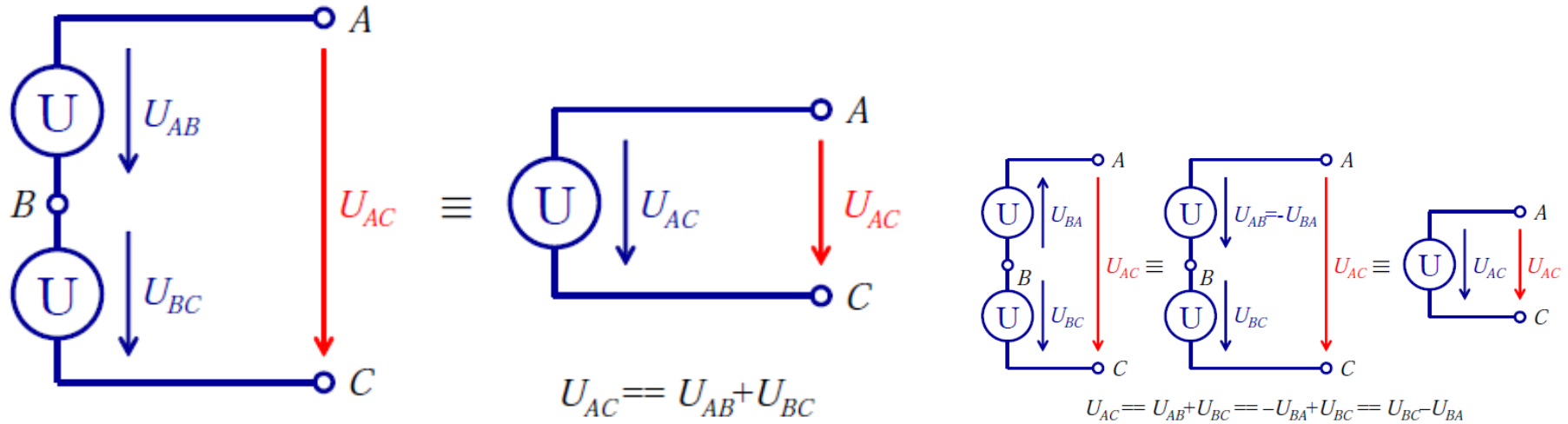


$I = \text{const.}$

$I \neq f(U)$

Voltage sources connection

- in series

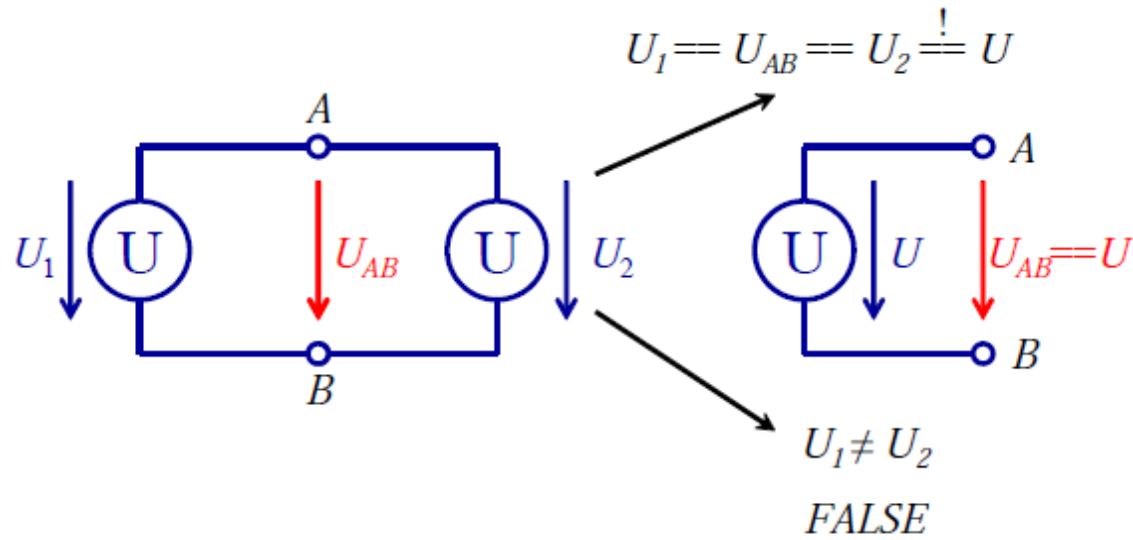


2nd Kirchhoff's law

$$\sum_{k=1}^n u_k = 0$$

Voltage sum in the circuit closed loop equals zero.

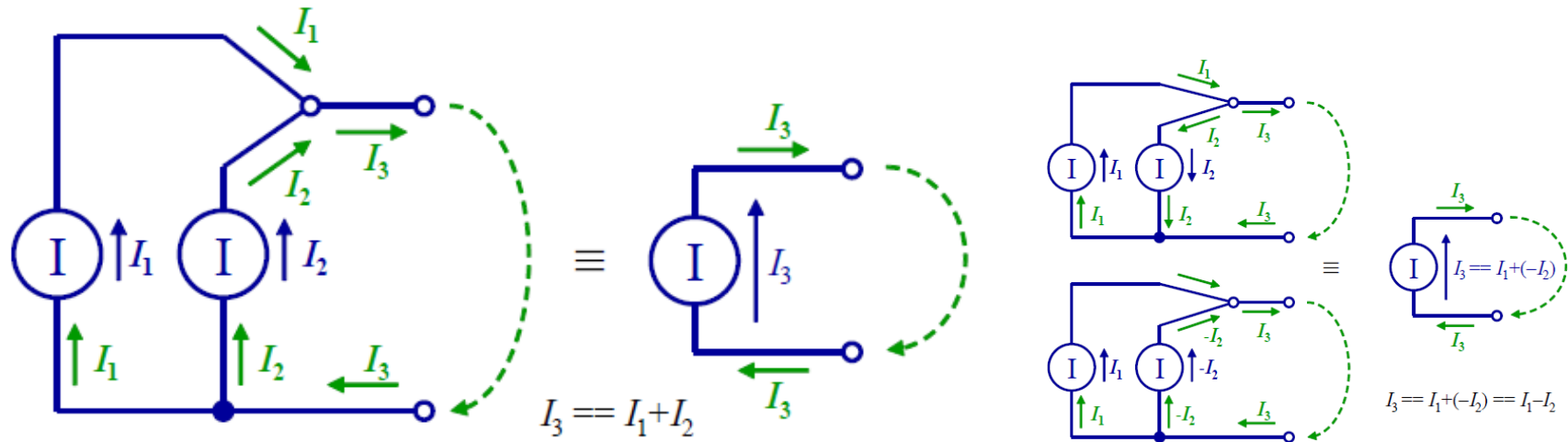
- in parallel



for real sources possible

Current sources connection

- in parallel

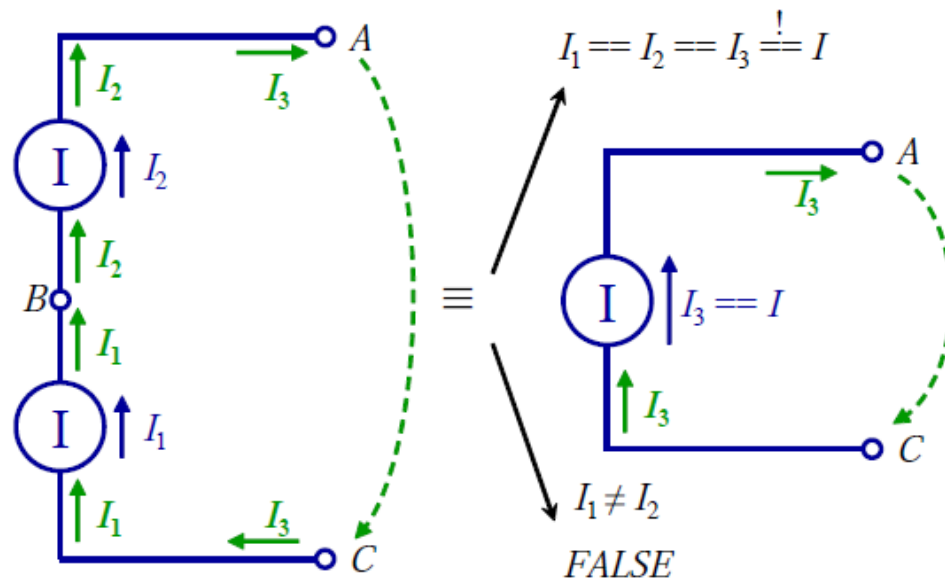


1st Kirchhoff's law

$$\sum_{k=1}^n i_k = 0$$

Current sum in the circuit bus (node) equals zero.

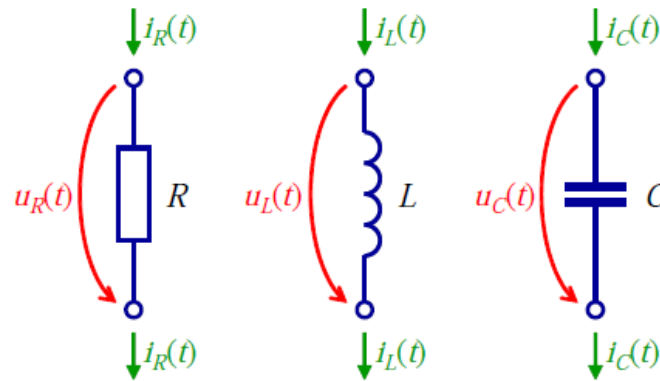
- in series



for real sources possible

Ideal voltage source mustn't be in short-circuit.
 Ideal current source mustn't be in open-circuit.

Basic Elements of Electric Circuits



- resistance R (Ω)

$$u_R(t) = R \cdot i_R(t) \quad (\text{Ohm's law})$$

- inductance L ($\text{H} = \Omega \cdot \text{s}$)

$$u_L(t) = L \cdot \frac{di_L(t)}{dt}$$

$$i_L(t) = \frac{1}{L} \int_0^t u_L(\tau) d\tau + i_L(0) \quad (\text{continuous current})$$

- capacity C (F = s/ Ω)

$$i_C(t) = C \cdot \frac{du_C(t)}{dt}$$

$$u_C(t) = \frac{1}{C} \int_0^t i_C(\tau) d\tau + u_C(0) \quad (\text{continuous voltage})$$

note: time constants

$$\tau = R \cdot C$$

$$\tau = \frac{L}{R}$$

$$\tau = \sqrt{L \cdot C}$$

Electric Circuit Description (Circuit Equations)

Bus voltage method: bus voltages, branch currents, 1st Kirchohoff's laws for buses, elements equations, initial conditions

Example (2 buses, 4 elements) – 6 equations, 6 unknown variables

$$i_{R1}(t) + i_L(t) - i_C(t) = 0$$

$$i_L(t) + i_{R2}(t) = 0$$

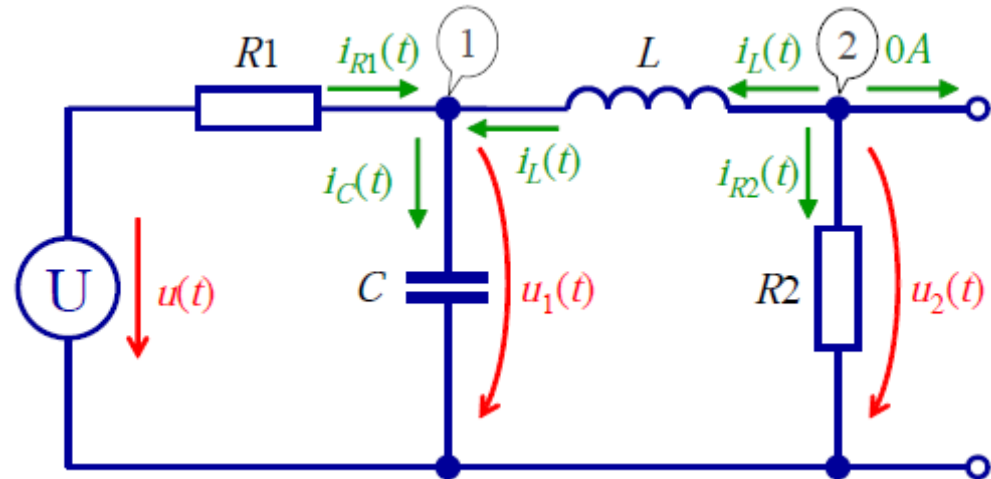
$$u(t) - u_1(t) = R_1 \cdot i_{R1}(t)$$

$$u_2(t) - u_1(t) = L \cdot \frac{di_L(t)}{dt}$$

$$i_C(t) = C \cdot \frac{du_1(t)}{dt}$$

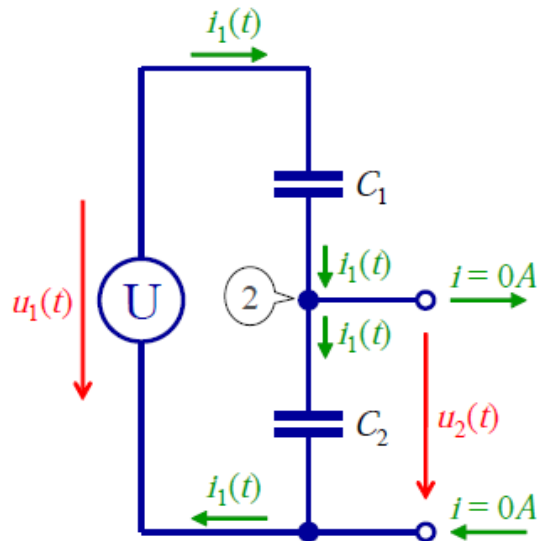
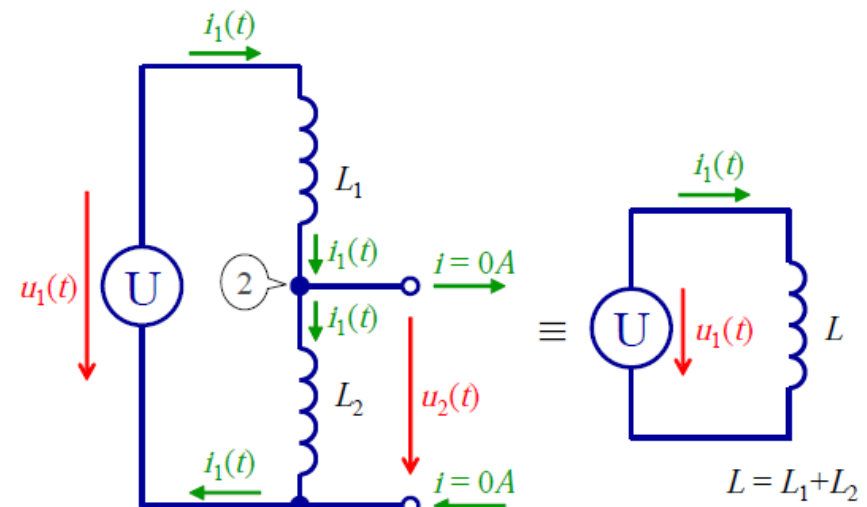
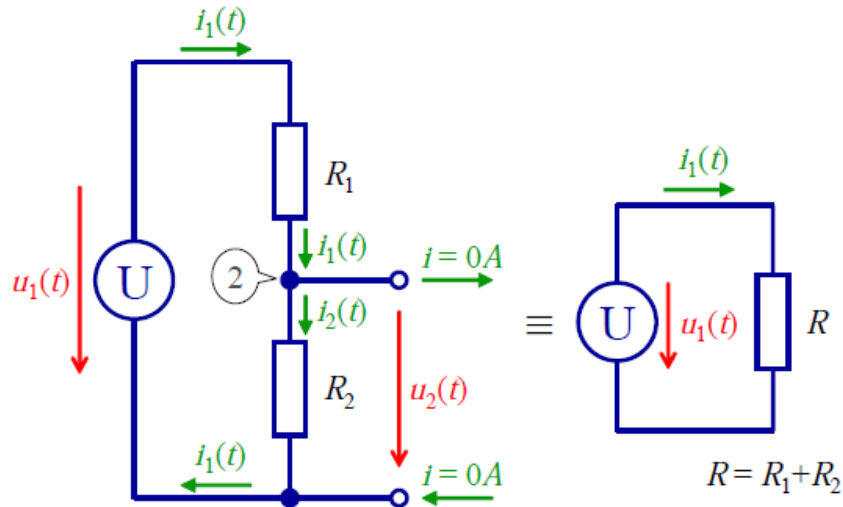
$$u_2(t) = R_2 \cdot i_{R2}(t)$$

$$i_L(t = t_0) = i_{L0}; u_C(t = t_0) = u_{C0}$$



Circuit Elements Connection, Dividers

Connection in series



$$R = R_1 + R_2$$

$$L = L_1 + L_2$$

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

- Voltage divider (resistance divider)

$$u_2(t) = u_1(t) \cdot \frac{R_2}{R_1 + R_2} \quad (\text{unloaded, non-reversible})$$

Connection in parallel

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

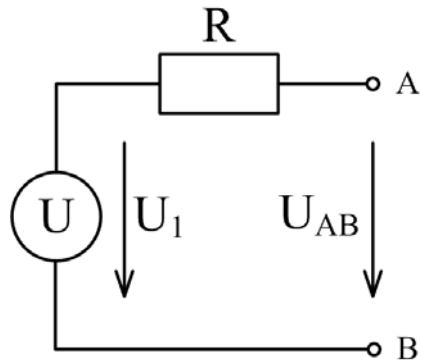
$$L = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}} = \frac{L_1 \cdot L_2}{L_1 + L_2}$$

$$C = C_1 + C_2$$

Real Sources

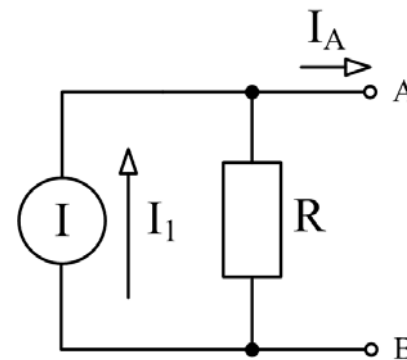
voltage source

$$U_{AB} = U_1 - R \cdot I$$



current source

$$I_A = I_1 - U_{AB} / R$$

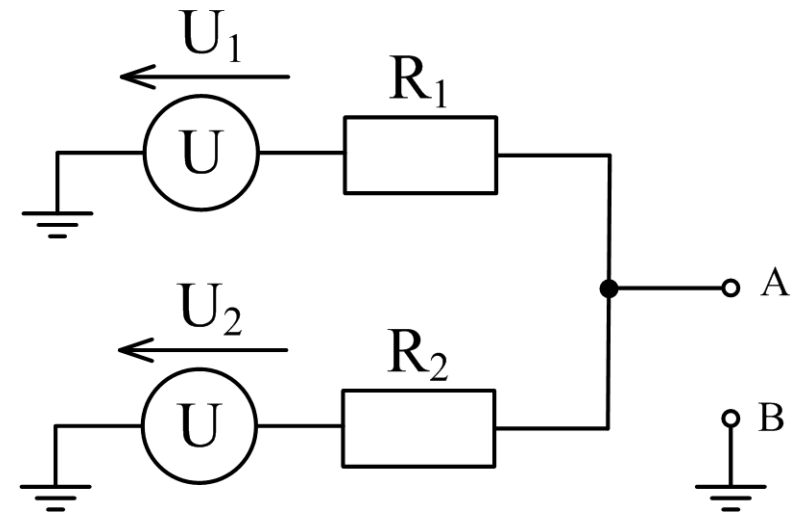


- real voltage sources in parallel

$$R_{\Sigma} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

$$\frac{U_{\Sigma}}{R_{\Sigma}} = U_{\Sigma} \cdot \frac{R_1 + R_2}{R_1 \cdot R_2} = \frac{U_1}{R_1} + \frac{U_2}{R_2}$$

$$U_{\Sigma} = \frac{U_1 \cdot R_2 + U_2 \cdot R_1}{R_1 + R_2}$$



DC Circuits

Sources with constant output → transient phenomena → steady state.

$$u_R(t) = R \cdot i_R(t) = R \cdot i_R$$

$$u_L(t) = L \cdot \frac{di_L(t)}{dt} = 0$$

$$i_C(t) = C \cdot \frac{du_C(t)}{dt} = 0$$

- transients – differential equations
- steady – algebraic equations (disconnect C, short-circuit L)

Electric Power and Energy

Generally

$$p(t) = \frac{dW(t)}{dt} \quad (W; J, s)$$

$$p(t) = u(t) \cdot i(t) \quad (W; V, A) \quad - \text{instantaneous power}$$

Inductance

$$p(t) = L \cdot \frac{di(t)}{dt} \cdot i(t) = L \cdot \frac{d}{dt} \left(\frac{i(t)^2}{2} \right)$$

$$W = \int p(t) dt = L \int_{t_0}^{t_\infty} \frac{d}{dt} \left(\frac{i(t)^2}{2} \right) dt = L \int_{i_0}^{i_\infty} d \left(\frac{i(t)^2}{2} \right) = \frac{1}{2} L (i_\infty^2 - i_0^2)$$

- zero energy balance in overall time

Capacity

$$p(t) = u(t) \cdot C \cdot \frac{du(t)}{dt} = C \cdot \frac{d}{dt} \left(\frac{u(t)^2}{2} \right)$$

$$W = \int p(t) dt = C \int_{t_0}^{t_\infty} \frac{d}{dt} \left(\frac{u(t)^2}{2} \right) dt = C \int_{u_0}^{u_\infty} d \left(\frac{u(t)^2}{2} \right) = \frac{1}{2} C (u_\infty^2 - u_0^2)$$

- zero energy balance in overall time

Resistance

$$p(t) = R \cdot i(t) \cdot i(t) = R \cdot i(t)^2$$

$$W = \int p(t) dt = R \int_{t_0}^{t_\infty} i(t)^2 dt \geq 0$$

- electric energy conversion to heat energy

Harmonic Steady State

All quantities are harmonic functions or their linear combinations.

$$u(t) = U_M \cdot \sin(\omega \cdot t + \varphi) \quad U_M \geq 0, \varphi \in (-\pi, \pi)$$

$$\omega = 2\pi \cdot f = \frac{2\pi}{T}$$

$$U_M \cdot \sin(\omega \cdot t + \varphi) = U_M \cdot \cos \varphi \cdot \sin(\omega \cdot t) + U_M \cdot \sin \varphi \cdot \cos(\omega \cdot t)$$

Standard in AC systems.

Derivative of harmonic function is harmonic function with the same frequency.

$$u'(t) = \omega \cdot U_M \cdot \cos(\omega \cdot t + \varphi)$$

Using complex numbers

$$z = a + j \cdot b \quad a, b \in \mathbb{R}, a = \operatorname{Re}(z), b = \operatorname{Im}(z) \quad j^2 = -1$$

$$\operatorname{Abs}(z) = \sqrt{\operatorname{Re}(z)^2 + \operatorname{Im}(z)^2} = \sqrt{a^2 + b^2}$$

$$e^{j\varphi} = \cos \varphi + j \cdot \sin \varphi, \quad \forall \varphi \in \mathbb{R} \quad \text{Euler's relation } (\varphi \text{ in rad})$$

$$\sin(\varphi) = \operatorname{Im}\{e^{j\varphi}\} \rightarrow$$

$$u(t) = U_M \cdot \sin(\omega \cdot t + \varphi) = \operatorname{Im}\{U_M \cdot e^{j(\omega t + \varphi)}\}$$

$$u(t) = \operatorname{Im}\{U_M \cdot e^{j\varphi} \cdot e^{j\omega t}\} = \operatorname{Im}\{\hat{U}_M \cdot e^{j\omega t}\}$$

$$\hat{U}_M = U_M \cdot e^{j\varphi} \quad \text{- phasor (in maximal values measure)}$$

Phasors and Impedances

Time behaviour

$$u(t) = U_M \cdot \sin(\omega \cdot t + \varphi) = \text{Im} \left\{ \hat{U}_M \cdot e^{j \cdot \omega \cdot t} \right\}$$

$$U_M = \text{Abs}(\hat{U}_M); \varphi = \text{Arg}(\hat{U}_M)$$

Why phasors?: In harmonic states differential equations can be transformed to linear algebraic equations using only R, L, C values (time eliminated).

- resistance R

$$u_R(t) = R \cdot i_R(t)$$

$$\text{Im} \left\{ \hat{U}_R \cdot e^{j \cdot \omega \cdot t} \right\} = R \cdot \text{Im} \left\{ \hat{I}_R \cdot e^{j \cdot \omega \cdot t} \right\} = \text{Im} \left\{ R \cdot \hat{I}_R \cdot e^{j \cdot \omega \cdot t} \right\}$$

$$\hat{U}_R = R \cdot \hat{I}_R$$

- inductance L

$$u_L(t) = L \cdot \frac{di_L(t)}{dt}$$

$$\text{Im}\{\hat{U}_L \cdot e^{j\omega t}\} = L \cdot \frac{d}{dt} \text{Im}\{\hat{I}_L \cdot e^{j\omega t}\} = \text{Im}\{L \cdot \hat{I}_L \cdot j \cdot \omega \cdot e^{j\omega t}\}$$

$$\hat{U}_L = j \cdot \omega \cdot L \cdot \hat{I}_L$$

- capacity C

$$i_C(t) = C \cdot \frac{du_C(t)}{dt}$$

$$\hat{I}_C = j \cdot \omega \cdot C \cdot \hat{U}_C$$

$$\hat{U}_C = \frac{1}{j \cdot \omega \cdot C} \cdot \hat{I}_C$$

In fact Ohm's law for harmonic steady state.

Impedances

$$\hat{Z}_R = R, \quad \hat{Z}_L = j \cdot \omega \cdot L, \quad \hat{Z}_C = \frac{1}{j \cdot \omega \cdot C}$$

They are not phasors!

Phasors in el. circuits

$$i_3(t) = i_1(t) + i_2(t) \Rightarrow \hat{I}_3 = \hat{I}_1 + \hat{I}_2 \quad \text{All functions with the same } \omega!$$

1st Kirchhoff's law

$$\sum_{k=1}^n \hat{I}_k = 0$$

2nd Kirchhoff's law

$$\sum_{k=1}^n \hat{U}_k = 0$$

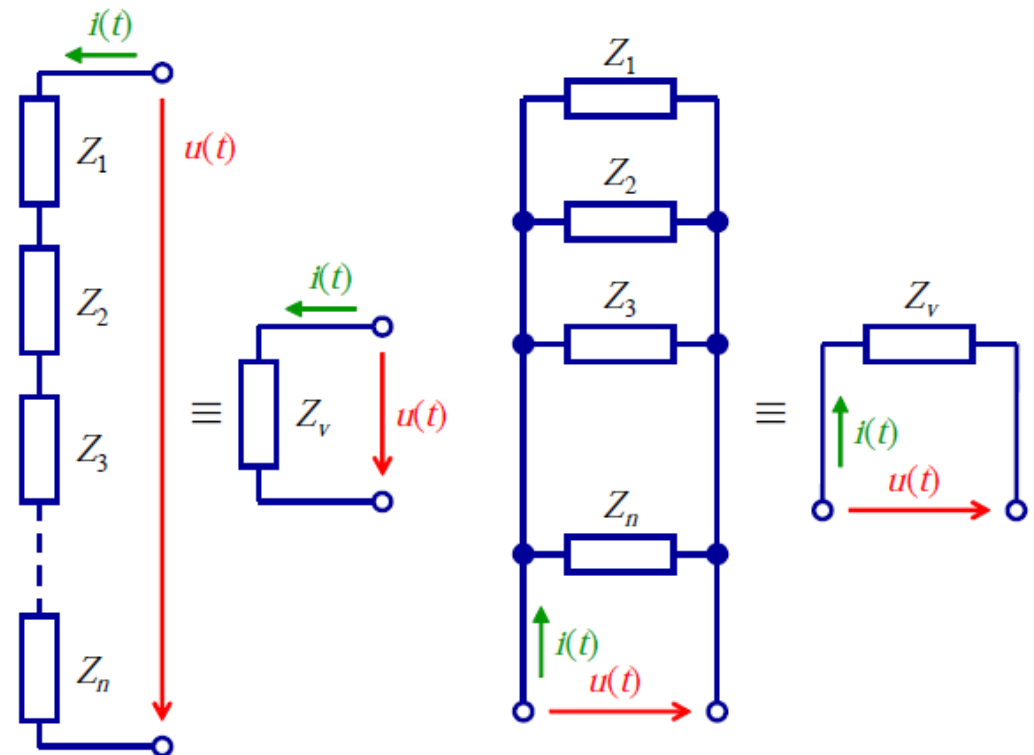
Impedances connection

- in series

$$\hat{Z}_v = \sum_{i=1}^n \hat{Z}_i$$

- in parallel

$$\hat{Z}_v = \frac{1}{\sum_{i=1}^n \frac{1}{\hat{Z}_i}}$$



Other definitions

- admittance (S)

$$\hat{Y} = \frac{1}{\hat{Z}}$$

- conductance (S)

$$G = \frac{1}{R}$$

- reactance (Ω)

$$X_L = \omega \cdot L$$

$$X_C = \frac{1}{\omega \cdot C}$$

- susceptance (S)

$$B = \frac{1}{X}$$

Electric Power in Harmonic Steady State

Instantaneous power

$$p(t) = u(t) \cdot i(t)$$

Mean power for periodic course

$$P = \frac{1}{T} \int_{t=t_1}^{t_1+T} p(t) \cdot dt \quad \text{for DC} \quad P = p(t) = P_{DC}$$

$$u(t) = U_M \cdot \sin\left(\frac{2\pi}{T} \cdot t + \varphi_U\right) \quad i(t) = I_M \cdot \sin\left(\frac{2\pi}{T} \cdot t + \varphi_I\right)$$

$$P = \frac{1}{T} \int_{t=0}^T u(t) \cdot i(t) \cdot dt = \frac{U_M \cdot I_M}{2} \cos(\varphi_U - \varphi_I) \quad U = \sqrt{\frac{1}{T} \int_{t=0}^T u^2(t) \cdot dt}$$

$$P = U \cdot I \cdot \cos \varphi \quad U = U_{RMS} = \frac{U_M}{\sqrt{2}} \quad \text{“root mean square” value}$$

$$\varphi = \varphi_U - \varphi_I$$

- resistance $\varphi = 0 \rightarrow P = U \cdot I$
- inductance, capacity $\varphi = \pm\pi/2 \rightarrow P = 0$

Complex Power in AC Grids

Instantaneous power

$$p(t) = u(t) \cdot i(t) = U_M \cdot \sin\left(\frac{2\pi}{T} \cdot t + \varphi_U\right) \cdot I_M \cdot \sin\left(\frac{2\pi}{T} \cdot t + \varphi_I\right)$$

$$p(t) = \frac{1}{2} U_M \cdot I_M \cdot [\cos(\varphi_I - \varphi_U) - \cos(2\omega \cdot t + \varphi_U + \varphi_I)]$$

e.g. for $\varphi_U = 0$

$$\varphi = \varphi_U - \varphi_I = -\varphi_I$$

$$p(t) = U \cdot I \cdot [\cos \varphi - \cos(2\omega \cdot t - \varphi)]$$

- **resistance** $\varphi = 0$

$$p(t) = U \cdot I \cdot [1 - \cos(2\omega \cdot t)]$$

$$P = \frac{1}{T} \int_{t=0}^T p(t) \cdot dt = U \cdot I$$

“active power”

- **inductance** $\varphi = \pi/2$

$$p(t) = U \cdot I \cdot [0 - \cos(2\omega \cdot t - \pi/2)]$$

$$p(t) = U \cdot I \cdot \sin(2\omega \cdot t)$$

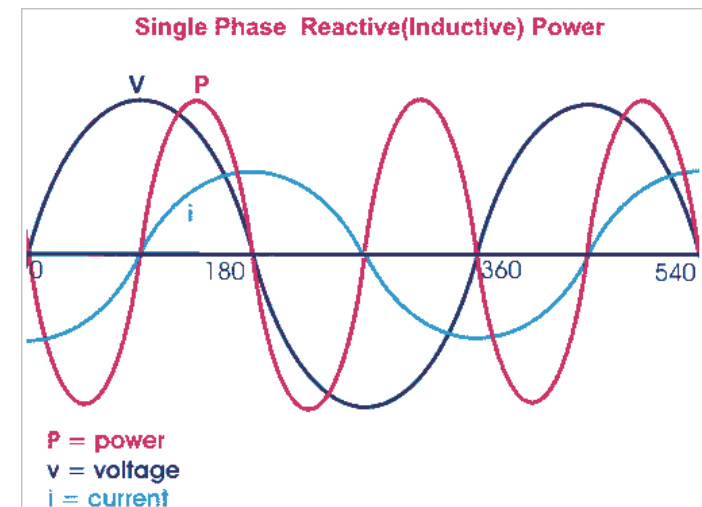
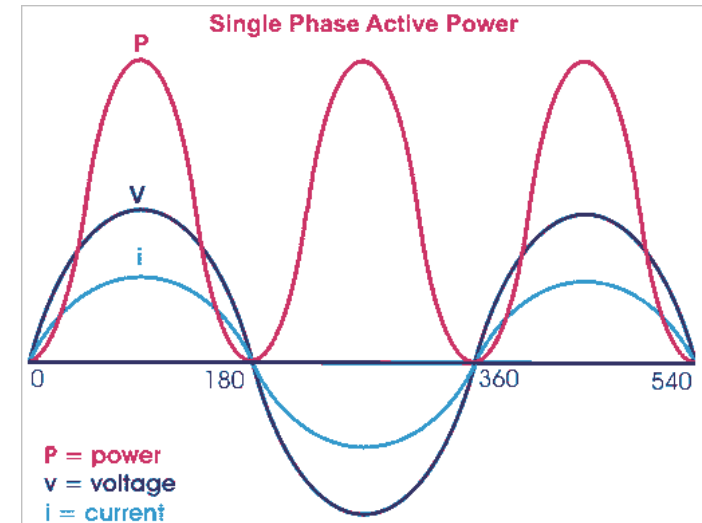
$$P = \frac{1}{T} \int_{t=0}^T p(t) \cdot dt = 0$$

- **capacity** $\varphi = -\pi/2$

$$p(t) = U \cdot I \cdot [0 - \cos(2\omega \cdot t + \pi/2)]$$

$$p(t) = -U \cdot I \cdot \sin(2\omega \cdot t)$$

$$P = 0$$



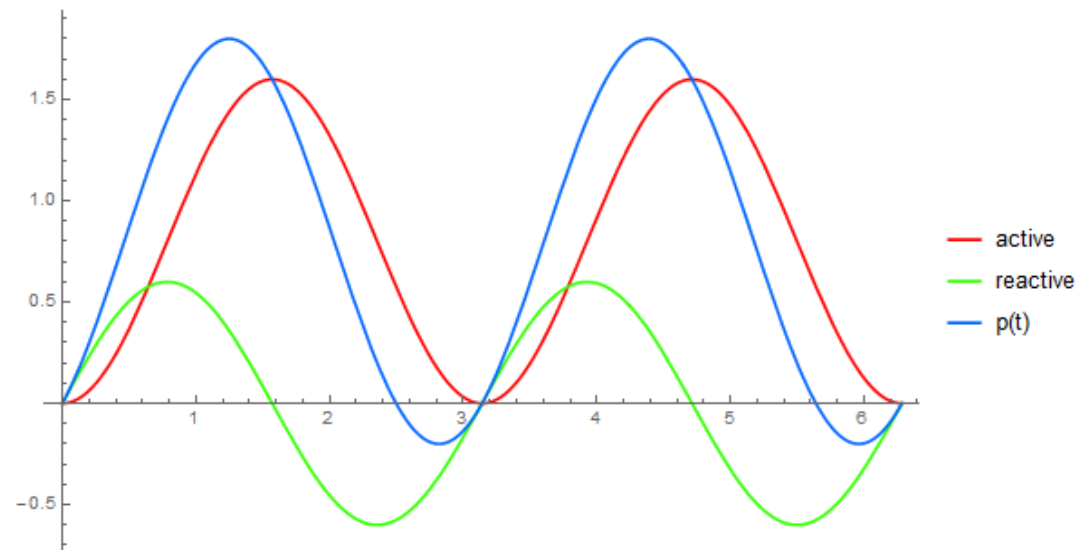
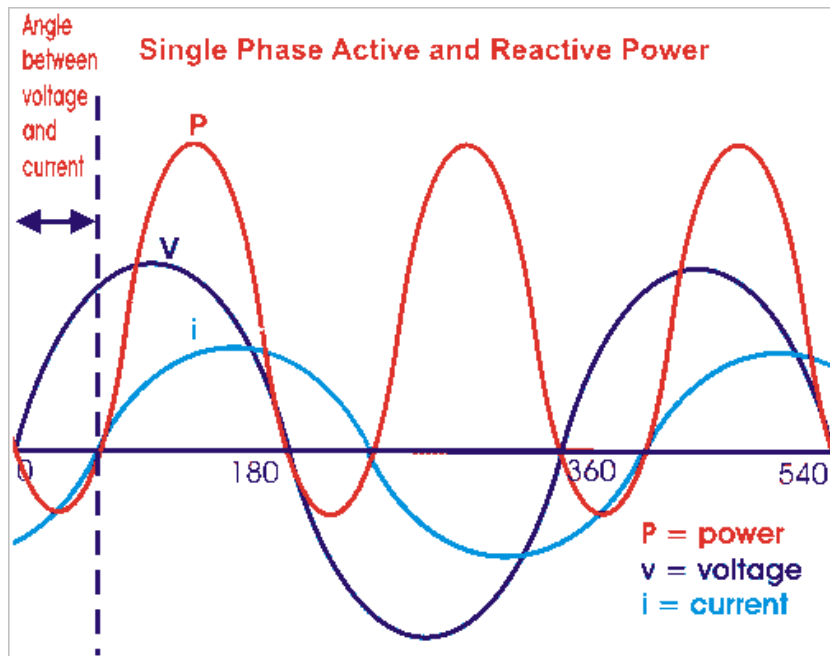
- general load

$$p(t) = U \cdot I \cdot [\cos \varphi - \cos(2\omega \cdot t - \varphi)]$$

$$p(t) = U \cdot I \cdot (\cos \varphi - \cos \varphi \cdot \cos 2\omega \cdot t - \sin \varphi \cdot \sin 2\omega \cdot t)$$

$$p(t) = U \cdot I \cdot [\cos \varphi \cdot (1 - \cos 2\omega \cdot t) - \sin \varphi \cdot \sin 2\omega \cdot t]$$

“active and reactive” component



Power and Phasors

Definitions

apparent power $S = U \cdot I$ (VA)

active power $P = \frac{1}{T} \int_{t=0}^T p(t) \cdot dt$ (W)

reactive power $S = \sqrt{P^2 + Q^2}$

$$Q = \sqrt{S^2 - P^2} \quad (\text{VAr / var})$$

power factor $\cos \varphi = \frac{P}{S}$

Complex conjugation (current)

$$\hat{S}_1 = \hat{U} \cdot \hat{I}^* = U \cdot I \cdot e^{j(\varphi_u - \varphi_i)} = U \cdot I \cdot e^{j\varphi}$$

$$\hat{S}_1 = \hat{U} \cdot \hat{I}^* = P_1 \pm jQ_1 \begin{array}{l} \text{IND} \\ \text{CAP} \end{array}$$

$$\hat{S}_1 = P_1 \pm jQ_1 = U \cdot I \cdot (\cos \varphi \pm j \sin \varphi) = S_1 e^{\pm j\varphi}$$

Sign according to convention.

Inductive load

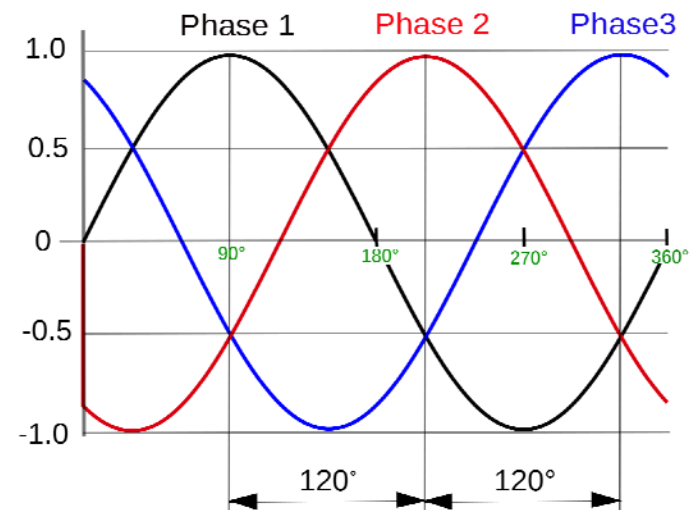
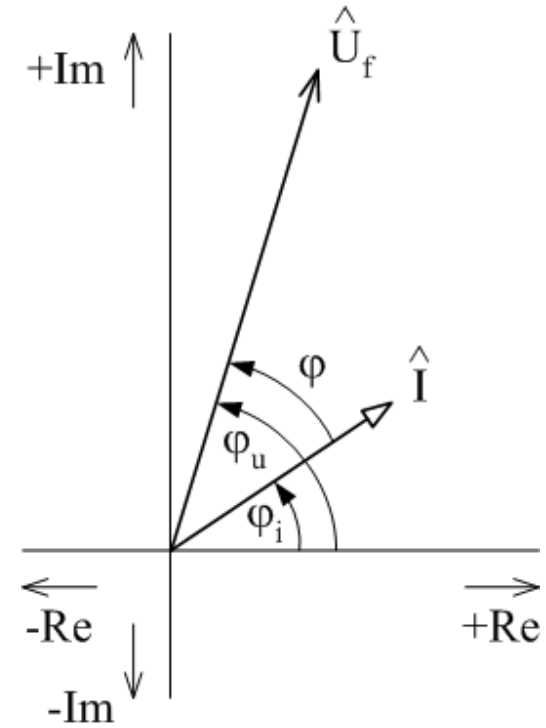
$$\hat{U}_f = U_f e^{j\varphi_u}, \hat{I}_f = I e^{j\varphi_i}$$

3 phase systems (symmetrical)

$$u_A(t) = U_M \cdot \sin(\omega \cdot t + \varphi_U)$$

$$u_B(t) = U_M \cdot \sin(\omega \cdot t + \varphi_U - \frac{2\pi}{3})$$

$$u_C(t) = U_M \cdot \sin(\omega \cdot t + \varphi_U + \frac{2\pi}{3})$$



$$\hat{U}_A, \hat{U}_B = \hat{U}_A \cdot e^{-j\frac{2\pi}{3}}, \hat{U}_C = \hat{U}_A \cdot e^{j\frac{2\pi}{3}}$$

$$\hat{U}_A, \hat{U}_B = \hat{a}^2 \cdot \hat{U}_A, \hat{U}_C = \hat{a} \cdot \hat{U}_A$$

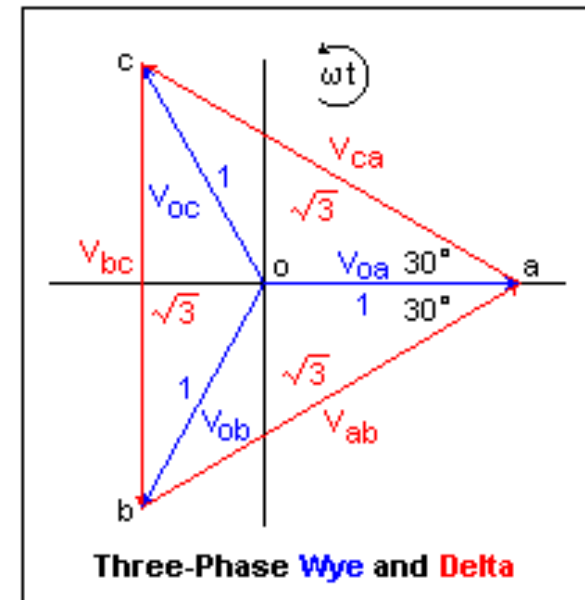
$$\hat{a} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = e^{+j\frac{2\pi}{3}}$$

$$\hat{a}^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = e^{-j\frac{2\pi}{3}}$$

$$1 + \hat{a}^2 + \hat{a} = 0 \Rightarrow \hat{U}_A + \hat{U}_B + \hat{U}_C = 0$$

$$U = \sqrt{3} \cdot U_{\text{ph}}$$

“phase-to-ground”, “phase-to-phase” voltage



Power

- general

$$\hat{S} = \hat{U}_A \cdot \hat{I}_A^* + \hat{U}_B \cdot \hat{I}_B^* + \hat{U}_C \cdot \hat{I}_C^*$$

- symmetrical

$$\hat{S} = 3\hat{U}_{\text{ph}} \cdot \hat{I}^* = \sqrt{3}\hat{U} \cdot \hat{I}^*$$

$$S = 3U_{\text{ph}}I = \sqrt{3}UI = \sqrt{P^2 + Q^2} \quad (\text{VA})$$

$$P = 3U_{\text{ph}}I \cos \varphi = \sqrt{3}UI \cos \varphi \quad (\text{W})$$

$$Q = 3U_{\text{ph}}I \sin \varphi = \sqrt{3}UI \sin \varphi \quad (\text{VAr})$$

Resonance

- parallel

$$\hat{Z} = \frac{1}{\frac{1}{j \cdot \omega \cdot L} + \frac{1}{j \cdot \omega \cdot C}} = \frac{j \cdot \omega \cdot L}{1 - \omega^2 \cdot L \cdot C}$$

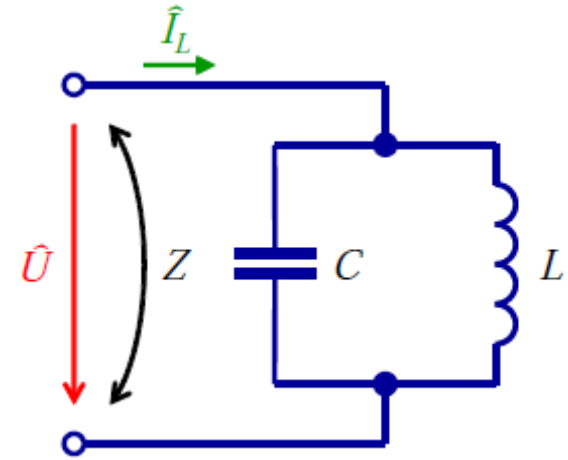
$$\omega_0 = \frac{1}{\sqrt{L \cdot C}} \quad \text{- resonance frequency}$$

$$\lim_{\omega \rightarrow \omega_0} \text{Abs}(\hat{Z}) = \lim_{\omega \rightarrow \omega_0} \text{Abs}\left(\frac{\hat{U}}{\hat{I}}\right) = \infty \Rightarrow \text{Abs}(\hat{U}) < \infty \rightarrow \lim_{\omega \rightarrow \omega_0} \text{Abs}(\hat{I}) = 0$$

$\omega < \omega_0$ - L mode

$\omega > \omega_0$ - C mode

e.g. Ripple control signal (HDO) support



- series (for ideal circuit)

$$\hat{Z} = j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C} = -j \cdot \frac{1 - \omega^2 \cdot L \cdot C}{\omega \cdot C}$$

$$\omega_0 = \frac{1}{\sqrt{L \cdot C}} \quad \text{- resonance frequency}$$

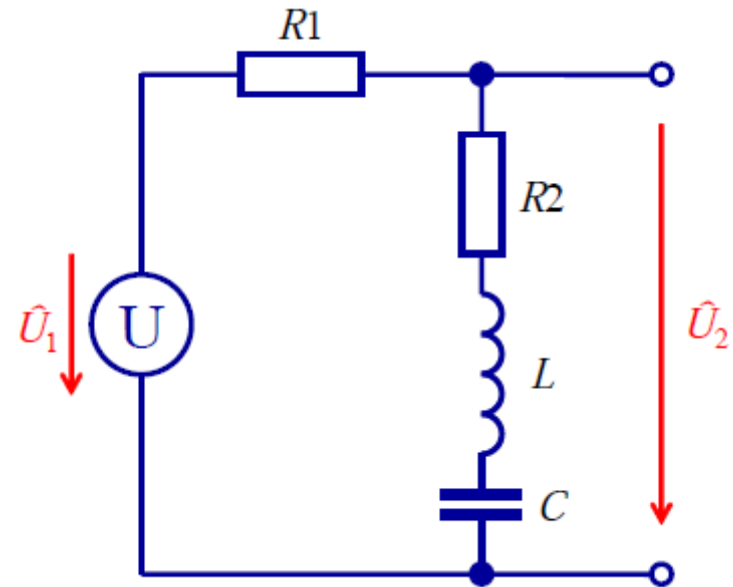
$$\lim_{\omega \rightarrow \omega_0} \text{Abs}(\hat{Z}) = \lim_{\omega \rightarrow \omega_0} \text{Abs}\left(\frac{\hat{U}}{\hat{I}}\right) = 0$$

$$\Rightarrow \text{Abs}(\hat{I}) < \infty \rightarrow \lim_{\omega \rightarrow \omega_0} \text{Abs}(\hat{U}) = 0$$

$\omega < \omega_0$ - C mode

$\omega > \omega_0$ - L mode

e.g. higher harmonic filters (power quality)



Fourier Series

For periodical continuous functions

$$f(t) = a_0 + \sum_{i=1}^n \left(a_i \cdot \cos\left(i \cdot 2\pi \cdot \frac{t}{T}\right) + b_i \cdot \sin\left(i \cdot 2\pi \cdot \frac{t}{T}\right) \right)$$

ideally $n \rightarrow \infty$, technically $n < \infty$ is sufficient

$$a_0 = \frac{1}{T} \cdot \int_{t=0}^T f(t) \cdot dt \quad (\text{mean value, DC value})$$

$$a_k = \frac{2}{T} \cdot \int_{t=0}^T f(t) \cdot \cos\left(k \cdot 2\pi \cdot \frac{t}{T}\right) \cdot dt$$

$$b_k = \frac{2}{T} \cdot \int_{t=0}^T f(t) \cdot \sin\left(k \cdot 2\pi \cdot \frac{t}{T}\right) \cdot dt$$

Fourier series and phasors – harmonic components

$$u_{b_k}(t) = b_k \cdot \sin\left(k \cdot 2\pi \cdot \frac{t}{T}\right) = b_k \cdot \sin(\omega_k \cdot t) = \text{Im}\{b_k \cdot e^{j\omega_k t}\} = \text{Im}\{\hat{U}_{b_k} \cdot e^{j\omega_k t}\}$$

$$\rightarrow \hat{U}_{b_k} = b_k$$

$$u_{a_k}(t) = a_k \cdot \cos\left(k \cdot 2\pi \cdot \frac{t}{T}\right) = a_k \cdot \cos(\omega_k \cdot t) = \text{Im}\{a_k \cdot e^{j\pi/2} \cdot e^{j\omega_k t}\} =$$

$$= \text{Im}\{\hat{U}_{a_k} \cdot e^{j\omega_k t}\}$$

$$\rightarrow \hat{U}_{a_k} = a_k \cdot e^{j\pi/2}$$

$$u_k(t) = u_{a_k}(t) + u_{b_k}(t) = \text{Im}\{(b_k + a_k \cdot e^{j\pi/2}) \cdot e^{j\omega_k t}\} = \text{Im}\{\hat{U}_k \cdot e^{j\omega_k t}\}$$

$$\hat{U}_k = \hat{U}_{a_k} + \hat{U}_{b_k} = b_k + j \cdot a_k \text{ - analysis for each harmonic separately}$$

$$\hat{U}_k = U_k \cdot e^{j\varphi_k} \quad \left| \hat{U}_k \right| = \sqrt{a_k^2 + b_k^2} \quad \varphi_k = \text{arctg} \frac{a_k}{b_k}$$