

Overhead Line (OHL) Electrical Parameters

4 basic (primary) el. parameters (for each phase)

- Resistance R_1 (Ω/km)
- Operational inductance L_1 (H/km)
- Conductance G_1 (S/km)
- Operational capacity C_1 (F/km)

Secondary parameters

- inductive reactance
 $X_1 = \omega L_1 = 2\pi f L_1$ (Ω / km)
- susceptance
 $B_1 = \omega C_1 = 2\pi f C_1$ (S / km)
- longitudinal (series) impedance
 $\hat{Z}_{11} = R_1 + jX_1$ (Ω / km)

- cross admittance

$$\hat{Y}_{q1} = G_1 + jB_1 \quad (\text{S/km})$$

- wave impedance

$$\hat{Z}_v = \sqrt{\frac{\hat{Z}_{11}}{\hat{Y}_{q1}}} \quad (\Omega)$$

- propagation constant

$$\hat{\gamma} = \sqrt{\hat{Z}_{11} \hat{Y}_{q1}} = \alpha + j\beta \quad (\text{km}^{-1})$$

α – specific damping

β – specific phase shift

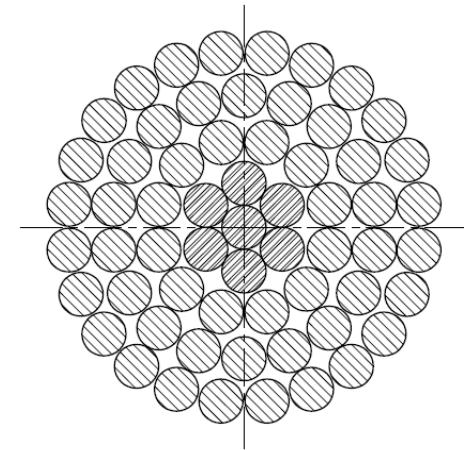
Note:

- we consider symmetrical supply and load
- networks

LV – mainly R		400 V
MV – R, L (in failures C)		6, 10, 22, 35 kV
HV – R, L, G, C (distributed)		110, 220, 400 kV

OHL conductors

- full cross section or twisted (1 or more materials)
- ropes Cu, Al, alloys, composites, optical wires, high-temperature materials
- ACSR (Aluminum Conductor Steel Reinforced) = carrying Fe core
+ conductive Al coat, $S \in (25 ; 680) \text{ mm}^2$
e.g. 25, 35, 42, 50, 70, 100, 120, 150, 180, 210, 240, 350, 450, 680
- example of labeling
 - 382-AL1/49-ST1A
 - 350AlFe4
 - AlFe450/52



Rope	Construction	Fe				Al			Rope		
		Nb. of wires	Diameter of wire	Diameter of inner tube	Cross-section	Nb. Of wires	Diameter of wire	Cross-section	Diameter	Cross-section	R _{DC+20}
		ks	mm	mm	mm ²	ks	mm	mm ²	mm	mm ²	Ω·km ⁻¹
350 AlFe 4	1+6+12/12+18	19	2,36	11,80	83,11	30	3,75	331,34	26,80	414,45	0,087
450 AlFe 8	3+9/18+14+20	12	2,36	9,90	52,49	18+34	1,90+3,75	426,55	28,70	479,05	0,0674
AlFe 450/52	3+9/12+18+24	12	2,36	9,81	52,49	54	3,25	447,97	29,31	500,46	0,0646
382-AL1/49-ST1A	1+6/12+18+24	7	3,00	3,00	49,48	54	3,00	381,70	27,00	431,18	0,0758
476-AL1/62-ST1A	1+6/12+18+24	7	3,35	10,05	61,70	54	3,35	475,96	30,15	537,66	0,0608

Resistance

Value influenced by:

conductor material, temperature, skin effect, elongation due to twisted wires, current density distribution along stripes, sag, unequal cross section, connections

With DC current (at 20°C)

$$R_{1dc0} = \frac{\rho_0}{S} \quad (\Omega / \text{km})$$

$$\text{Cu: } \rho_0 = 1,78 \cdot 10^{-8} \quad (\Omega\text{m})$$

$$\text{Al: } \rho_0 = 2,81 \cdot 10^{-8} \quad (\Omega\text{m})$$

$$\text{Fe: } \rho_0 = 12,8 \cdot 10^{-8} \quad (\Omega\text{m})$$

$$\rho_{\text{AlFeDC}} = \frac{\rho_{\text{Al}} \cdot S_{\text{Al}} + \rho_{\text{Fe}} \cdot S_{\text{Fe}}}{S_{\text{Al}} + S_{\text{Fe}}}$$

Temperature effect

$$k_T = 1 + \alpha(T_1 - T_0) + \beta(T_1 - T_0)^2 \quad (-)$$

$$\alpha \approx 4 \cdot 10^{-3} \quad (\text{K}^{-1})$$

$$\beta \approx 10^{-6} \text{ K}^{-2} \rightarrow \text{under normal } \Delta T \text{ neglected}$$

Influence of AC current, e.g.

$$k_{ac} = 1,004 \div 1,3 \quad (-)$$

In catalogue usually R_{1dc0}

$$\Rightarrow R_1 = R_{1dc0} \cdot k_T \cdot k_{ac} \quad (\Omega / \text{km})$$

$$\text{cca } R_{1dc0} \in (0,05 ; 2) \Omega / \text{km}$$

AlFe25 $R_{1dc0} \sim 1,2 \Omega/\text{km}$

AlFe42 $R_{1dc0} \sim 0,7 \Omega/\text{km}$

AlFe70 $R_{1dc0} \sim 0,4 \Omega/\text{km}$

AlFe95 $R_{1dc0} \sim 0,3 \Omega/\text{km}$

AlFe120 $R_{1dc0} \sim 0,2 \Omega/\text{km}$

AlFe185 $R_{1dc0} \sim 0,16 \Omega/\text{km}$

AlFe210 $R_{1dc0} \sim 0,14 \Omega/\text{km}$

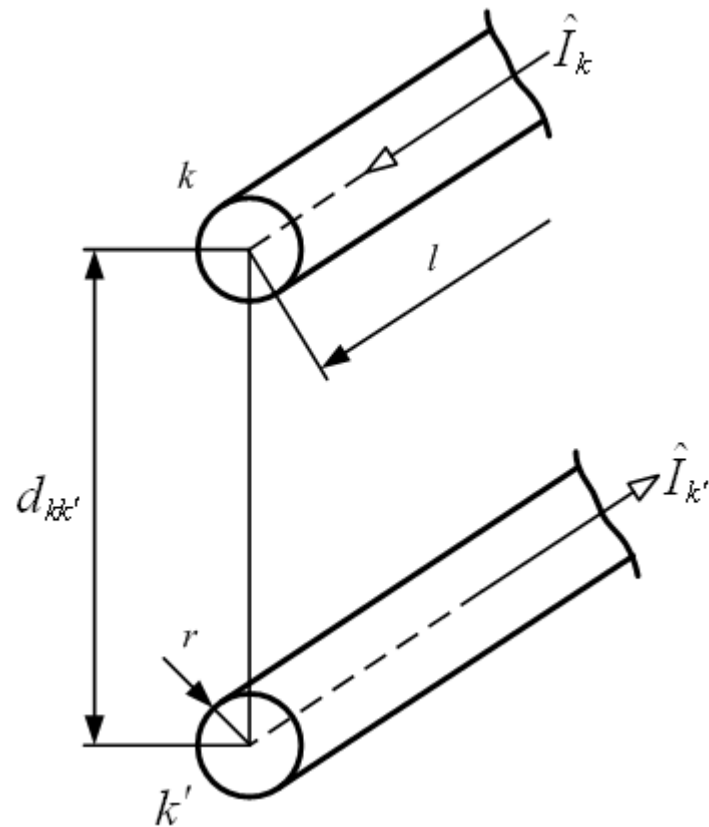
AlFe350 $R_{1dc0} \sim 0,09 \Omega/\text{km}$

AlFe450 $R_{1dc0} \sim 0,07 \Omega/\text{km}$

AlFe680 $R_{1dc0} \sim 0,04 \Omega/\text{km}$

Inductance and longitudinal impedance

Inductance and impedance in a loop



$$r \ll d \ll l \quad d_{kk'} = d \quad \hat{I}_k = -\hat{I}_{k'}$$

Internal inductance of a conductor (magnetic flux inside the conductor)

$$L_{ik} = \frac{\mu_0 \mu_{rv}}{8\pi} \alpha \quad (\text{H/m; H/m, -, -})$$

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ H} \cdot \text{m}^{-1}$$

μ_{rv}relative permeability of conductor

αinequality of current distribution through cross-section

External inductance of a conductor in a loop (magnetic flux outside the conductor)

$$L_{ek} = \frac{\mu_0}{2\pi} \ln \frac{d}{r} \quad (\text{H/m; H/m, m, m})$$

Self-inductance

$$L_v = L_{ik} + L_{ek} = \frac{\mu_0 \mu_{rv}}{8\pi} \alpha + \frac{\mu_0}{2\pi} \ln \frac{d}{r}$$

$$L_v = 0,05 \mu_{rv} \alpha + 0,46 \log \frac{d}{r} = 0,46 \log \frac{d}{\xi r} \quad (\text{mH} \cdot \text{km}^{-1}; \text{m, m})$$

ξ ... coefficient of current density inequality in cross section and permeability

$$\xi = 10^{\frac{0,05\mu_r\alpha}{0,46}}$$

$\xi \in (0,809 ; 0,826)$ for common ACSR ropes

Impedance of one conductor in a loop of two conductors

$$\hat{Z}_{kv} = R_{1k} + j\omega \cdot 0,46 \cdot 10^{-6} \cdot \log \frac{d}{\xi r} \quad (\Omega \cdot m^{-1})$$

Self-impedance of a loop conductor-ground

- Rüdénberg's conception – ground transformed in a loop conductor

3 components:

- a) R_{1k} – resistance respecting power losses in the conductor
- b) X_{1k} – reactance respecting part of magnetic flux coupled with the conductor and closed in the conductor and in the air
- c) Z_{1g} – impedance respecting part of magnetic flux in the ground coupled with conductor

$$\hat{Z}_{kk} = R_{kk} + jX_{kk} = R_{1k} + jX_{1k} + R_{1g} + jX_{1g}$$

$$R_{1g} = \pi^2 f \cdot 10^{-7} \quad (\Omega \cdot \text{m}^{-1}; \text{Hz})$$

$$\text{for } f = 50 \text{ Hz je } R_{1g} = 0,0495 \Omega \cdot \text{km}^{-1}$$

$$\hat{Z}_{kk} = R_{1k} + \pi^2 f \cdot 10^{-4} + j\omega \cdot 10^{-3} \cdot 0,46 \log \frac{D_g}{\xi \cdot r} \quad (\Omega \cdot \text{km}^{-1})$$

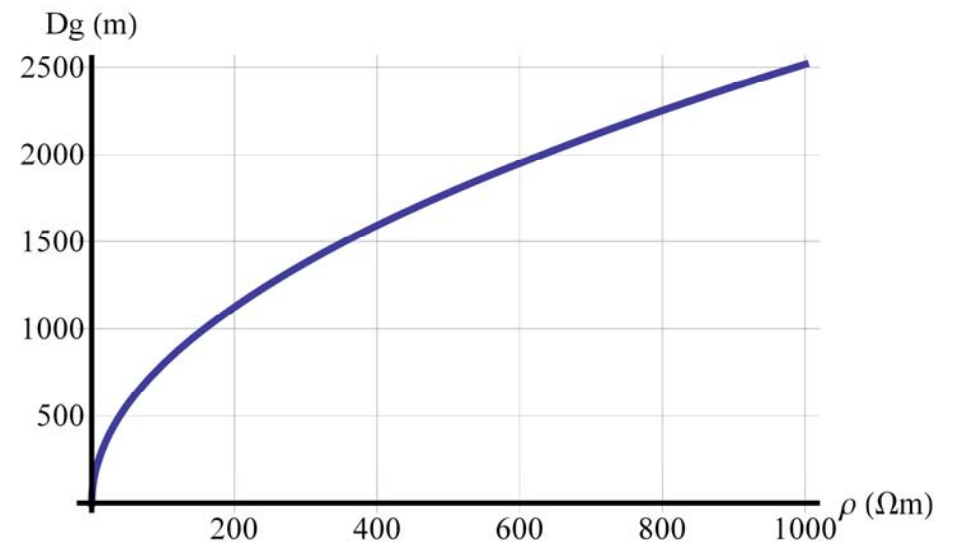
Fictitious conductor deep in the ground which has the same impacts as the real current in the ground

$$D_g = \frac{0,178\sqrt{\rho \cdot 10^7}}{\sqrt{f}} \quad (\text{m; } \Omega\text{m, Hz})$$

$D_g \sim 100\text{x m}$, i.e. $h \ll D_g$

ρ ...ground resistivity

Type of soil	ρ ($\Omega \cdot \text{m}$)
peat	30
topsoil and clay	100
moist sand	200 - 300
dry gravel and sand	1000 - 3000
stony soil	3000 - 10000



Mutual impedance of two loops conductor-ground

$D_g \gg d_{km} \rightarrow$ resulting electromagnetic impacts of return currents in the conductors k' , m' on the real conductors k , m is almost zero

Impedance of one conductor in a loop

$$\Delta \hat{U}_{kv} = \hat{Z}_{kv} \cdot \hat{I}_k = \hat{Z}_{kk} \cdot \hat{I}_k + \hat{Z}_{km} \cdot \hat{I}_m$$

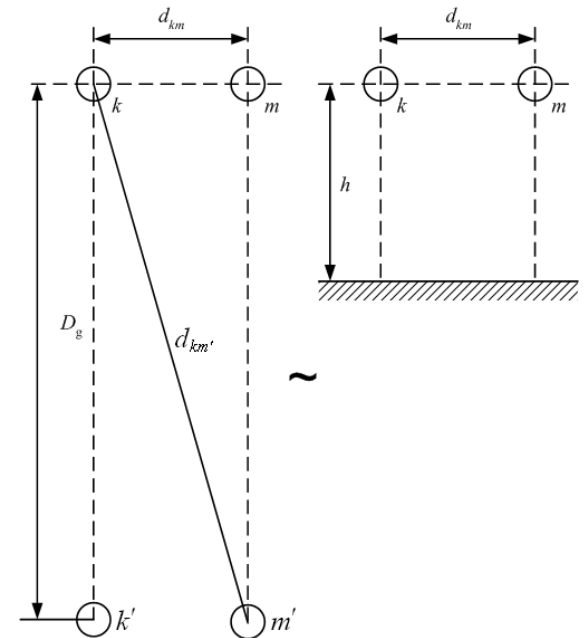
$$\hat{I}_k = -\hat{I}_m \Rightarrow \hat{Z}_{kv} = \hat{Z}_{kk} - \hat{Z}_{km}$$

Hence after substitutions

$$\hat{Z}_{kk} = R_{1k} + R_{1g} + j\omega \cdot 10^{-3} \cdot 0,46 \log \frac{D_g}{\xi_r} \quad (\Omega \cdot \text{km}^{-1})$$

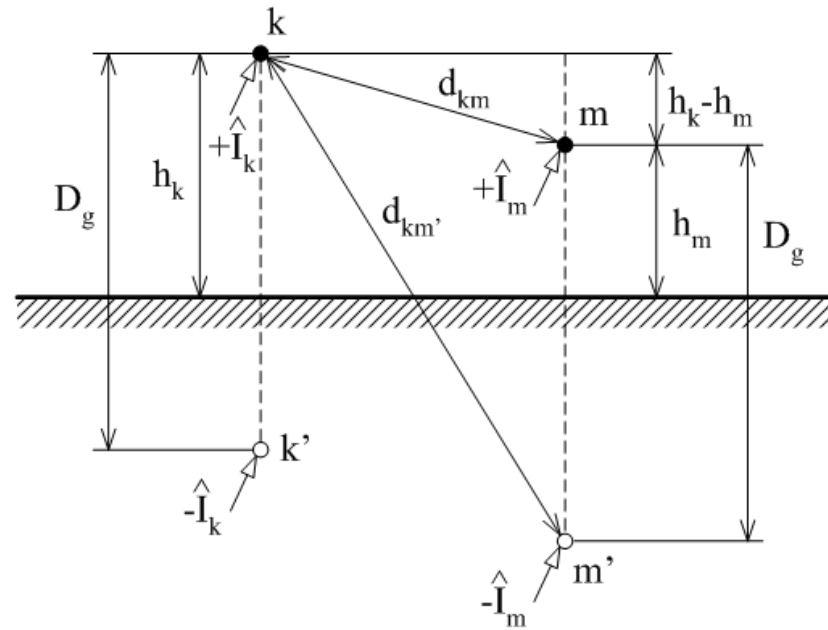
$$\hat{Z}_{kv} = R_{1k} + j\omega \cdot 0,46 \cdot 10^{-3} \cdot \log \frac{d_{km}}{\xi_r} \quad (\Omega \cdot \text{km}^{-1})$$

$$\hat{Z}_{km} = \hat{Z}_{kk} - \hat{Z}_{kv} = R_{1g} + j\omega \cdot 10^{-3} \cdot 0,46 \log \frac{D_g}{d_{km}} \quad (\Omega \cdot \text{km}^{-1})$$



Configuration of n real conductors

Configuration of loops n real conductors and the ground is substituted by n real and n fictitious conductors in mutual distance D_g .



n -conductor system

$$[\Delta \hat{U}] = [\hat{Z}_{km}] [\hat{I}]$$

voltage drop in k^{th} conductor

$$\Delta \hat{U}_k = \sum_{m=1}^n \hat{Z}_{km} \hat{I}_m \quad (\text{V / km})$$

Self-impedance (loop k-k')

$$\hat{Z}_{kk} = R_{kk} + j\omega L_{kk} = R_{1k} + R_{1g} + j0,1445 \log \frac{D_g}{\xi \cdot r_k} \left(\frac{\Omega}{\text{km}} \right)$$

Mutual impedance (loop k-k', m-m')

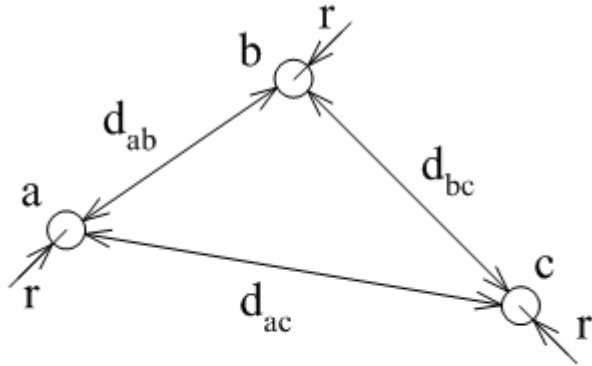
$$\hat{Z}_{km} = \hat{Z}_{mk} = R_{km} + j\omega L_{km} = R_{1g} + j0,1445 \log \frac{D_g}{d_{km}} \left(\frac{\Omega}{\text{km}} \right)$$

Operational impedance (inductance) – for 1 single conductor, it causes the same voltage drop as in the system of n conductors (it can be a complex number, done by operating condition)

$$\Delta \hat{U}_k = \sum_{m=1}^n \hat{Z}_{km} \hat{I}_m = \hat{Z}_k \hat{I}_k \Rightarrow \hat{Z}_k = \frac{\sum_{m=1}^n \hat{Z}_{km} \hat{I}_m}{\hat{I}_k} \quad \hat{L}_k = \frac{\sum_{m=1}^n M_{km} \hat{I}_m}{\hat{I}_k}$$

Simple (unbalanced) three-phase power line

Symmetrical loading



$$\hat{I}_a = \hat{I}_a$$

$$\hat{I}_b = \hat{a}^2 \hat{I}_a$$

$$\hat{I}_c = \hat{a} \hat{I}_a$$

$$\hat{a} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = e^{+j\frac{2\pi}{3}}$$

$$\hat{a}^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = e^{-j\frac{2\pi}{3}}$$

$$1 + \hat{a}^2 + \hat{a} = 0$$



Operational inductances

$$\hat{L}_a = \frac{M_{aa}\hat{I}_a + M_{ab}\hat{I}_b + M_{ac}\hat{I}_c}{\hat{I}_a} = M_{aa} + \hat{a}^2 M_{ab} + \hat{a} M_{ac}$$

$$\hat{L}_b = \frac{M_{ab}\hat{I}_a + M_{bb}\hat{I}_b + M_{bc}\hat{I}_c}{\hat{I}_b} = \hat{a} M_{ab} + M_{bb} + \hat{a}^2 M_{bc}$$

$$\hat{L}_c = \frac{M_{ac}\hat{I}_a + M_{bc}\hat{I}_b + M_{cc}\hat{I}_c}{\hat{I}_c} = \hat{a}^2 M_{ac} + \hat{a} M_{bc} + M_{cc}$$

Generally

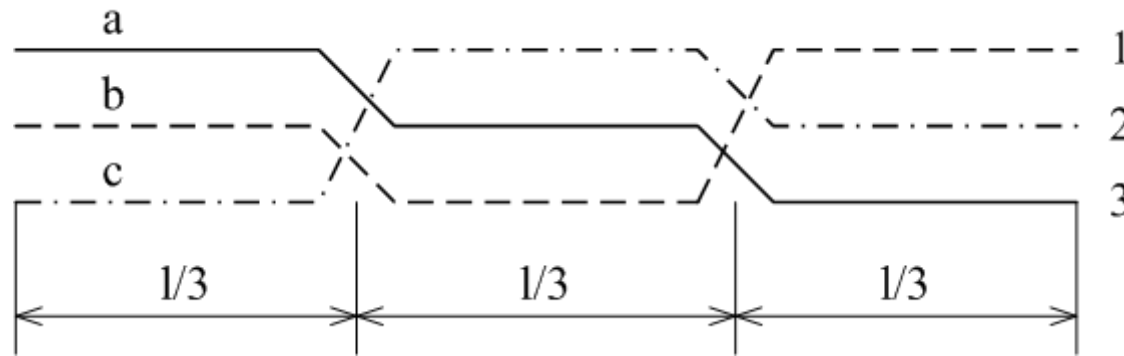
$$M_{aa} = M_{bb} = M_{cc} \quad M_{ab} \neq M_{bc} \neq M_{ac}$$

$$\hat{L}_a \neq \hat{L}_b \neq \hat{L}_c$$

→ unequal voltage drops (magnitude and phase) → voltage unbalance,
active power transfer between phases through electromagnetic coupling
without further sources loading → transposition

Transposition of three-phase power line

= conductors position exchange so that each one is in a definite position for 1/3 length



Voltage drops

$$\begin{pmatrix} \Delta \hat{U}_a \\ \Delta \hat{U}_b \\ \Delta \hat{U}_c \end{pmatrix} = \frac{1}{3} j\omega \left\{ \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{pmatrix} + \begin{pmatrix} M_{33} & M_{13} & M_{23} \\ M_{13} & M_{11} & M_{12} \\ M_{23} & M_{12} & M_{22} \end{pmatrix} + \begin{pmatrix} M_{22} & M_{23} & M_{12} \\ M_{23} & M_{33} & M_{13} \\ M_{12} & M_{13} & M_{11} \end{pmatrix} \right\} \begin{pmatrix} \hat{I}_a \\ \hat{I}_b \\ \hat{I}_c \end{pmatrix}$$



Let's mark

$$M = \frac{1}{3}(M_{11} + M_{22} + M_{33}) = 0,46 \log \frac{D_g}{\xi_r} \quad (\text{mH / km})$$

$$M' = \frac{1}{3}(M_{12} + M_{13} + M_{23}) = 0,46 \log \frac{D_g}{d} \quad (\text{mH / km})$$

mean geometrical distance

$$d = \sqrt[3]{d_{12}d_{13}d_{23}}$$

Then

$$\begin{pmatrix} \Delta \hat{U}_a \\ \Delta \hat{U}_b \\ \Delta \hat{U}_c \end{pmatrix} = j\omega \begin{pmatrix} M & M' & M' \\ M' & M & M' \\ M' & M' & M \end{pmatrix} \begin{pmatrix} \hat{I}_a \\ \hat{a}^2 \hat{I}_a \\ \hat{a} \hat{I}_a \end{pmatrix}$$

Phase operational inductances at transposed and symmetrically loaded power line are equal and real:

$$L_a = M + \hat{a}^2 M' + \hat{a} M'$$

$$L_a = L_b = L_c = M - M'$$

Finally

$$L_1 = L_a = L_b = L_c = 0,46 \log \frac{d}{\xi r} \quad (\text{mH / km})$$

$$\hat{Z}_1 = \hat{Z} - \hat{Z}' = R_1 + j0,1445 \log \frac{d}{\xi \cdot r} \quad \left(\frac{\Omega}{\text{km}} \right)$$

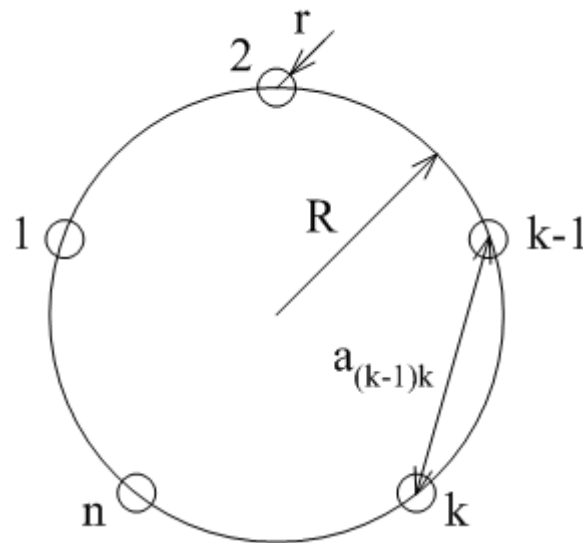
Power line with bundle conductors

Bundle conductor

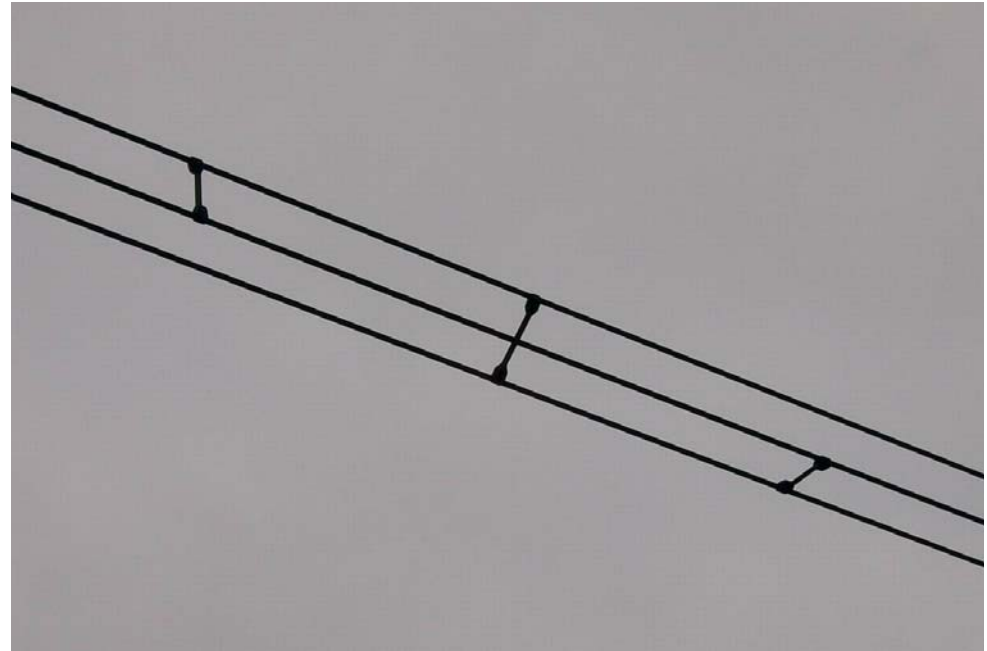
- each phase composed of n partial conductors connected in parallel
- arranged in regular n -polygon
- increases initial corona voltage
- from voltage 400 kV higher

U (kV)	400	750	1150	1800
n	3	4	8	16

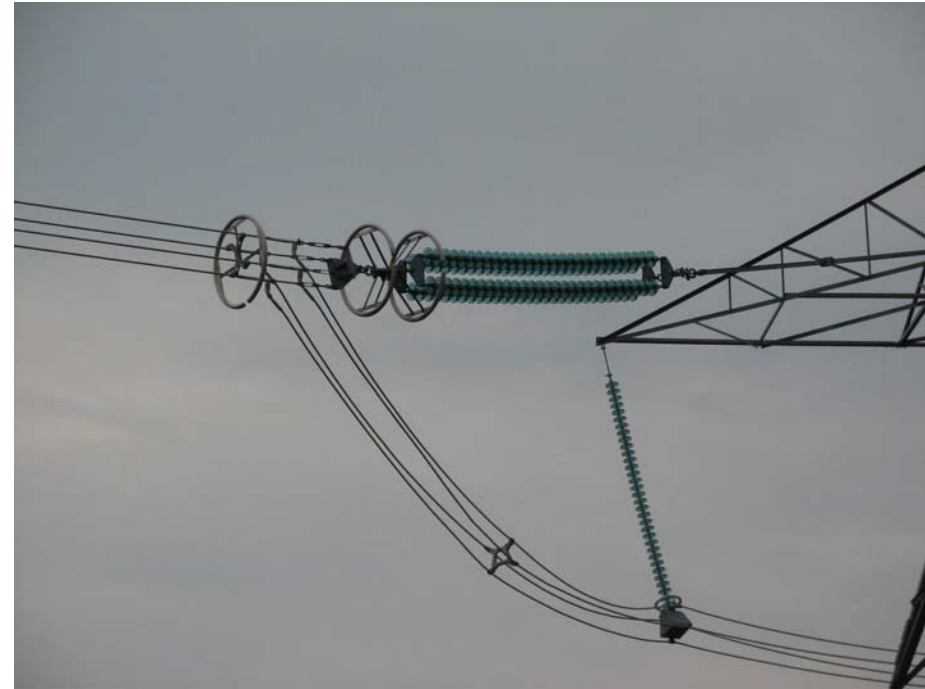
- $a_{400kV} = 40$ cm



Czech Republic: 400 kV – triple-bundle conductor



Kladno (CR) 110 kV (2), Canada 750 kV (4), China 1000 kV (8)



UHV conductor

Operational inductance

$$L_1 = 0,46 \log \frac{d}{\xi_e r_e} \quad (\text{mH / km})$$

equivalent bundle radius

$$r_e = R \sqrt[n]{r \frac{n}{R}}$$

equivalent coefficient

$$\xi_e = \sqrt[n]{\xi}$$

→ bundle conductor decreases L, R (conductors in parallel), increases C

22 kV **X ~ 0,35 Ω/km**

110 kV **X ~ 0,35÷0,4 Ω/km**

220 kV **X ~ 0,4 Ω/km**

400 kV **X ~ 0,3 Ω/km**

750 kV **X ~ 0,25 Ω/km**

Conductance

It causes active power losses by the conductance to the ground (through insulators, corona – dominant at overhead power lines). It depends on voltage, climatic conditions (p, T, humidity), conductors. Less dependant on loading.

Calculation from corona losses

$$P_S = 3U_f I_S = 3G_1 U_f^2 = G_1 U^2 \quad (\text{W} \cdot \text{km}^{-1})$$

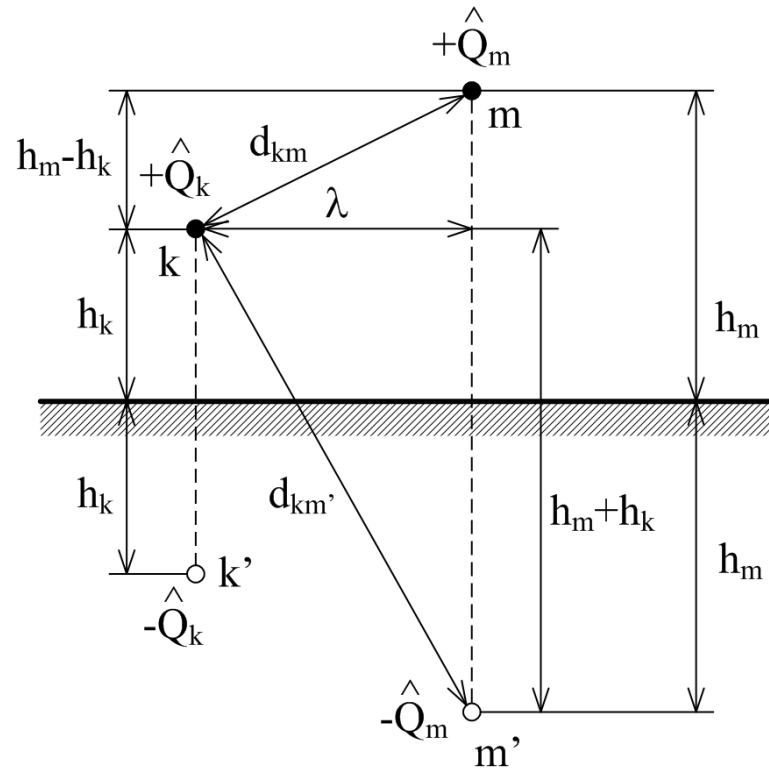
$$G_1 = \frac{P_S}{U^2} \quad (\text{S} / \text{km}; \text{W} / \text{km}, \text{V})$$

$$G_1 \approx 10^{-8} \text{ S} \cdot \text{km}^{-1} \quad \times \quad B_1 \approx 10^{-6} \text{ S} \cdot \text{km}^{-1}$$

U (kV)	G ₁ (S/km)	U (kV)	G ₁ (S/km)
110	(3,6 ÷ 5) · 10 ⁻⁸	750	(1,3 ÷ 2,5) · 10 ⁻⁸
220	(2,5 ÷ 3,6) · 10 ⁻⁸	1150	(1,0 ÷ 2) · 10 ⁻⁸
400	(1,4 ÷ 2) · 10 ⁻⁸		

OHL Capacities

El. potential at point P in the system of n parallel conductors ($d_{kk'} \ll 1$) and ground with zero potential - *mirror method*



$$\hat{U}_P = \sum_{k=1}^n (\hat{U}_{Pk} + \hat{U}_{Pk'}) = \sum_{k=1}^n \frac{\hat{Q}_k}{2\pi\epsilon} \ln \frac{d_{Pk'}}{d_{Pk}} \quad (\text{V}; \text{C}/\text{m}, \text{m}, \text{m})$$

Point P on the surface of the real wire k ($r_k \ll d_{km}$):

$$\hat{U}_k = \sum_{m=1}^n \frac{\hat{Q}_m}{2\pi\epsilon} \ln \frac{d_{km'}}{d_{km}} = \sum_{m=1}^n \delta_{mk} \hat{Q}_m$$

$$(d_{kk} = r_k; d_{kk'} = 2h_k)$$

Point on the ground

$$d_{zk} = d_{zk'}$$

$$\hat{U}_z = \sum_{m=1}^n \frac{\hat{Q}_m}{2\pi\epsilon} \ln \frac{d_{zm'}}{d_{zm}} = \sum_{m=1}^n \frac{\hat{Q}_m}{2\pi\epsilon} \ln 1 = 0$$

Self potential coefficient of the wire k ($m=k$)

$$\delta_{kk} = \frac{1}{2\pi\epsilon} \ln \frac{2h_k}{r_k} \quad (m / F; F / m, m, m)$$

h_k ...calculation height

The mutual potential coefficient (m ≠ k)

$$\delta_{km} = \delta_{mk} = \frac{1}{2\pi\epsilon} \ln \frac{d_{km'}}{d_{km}} \quad (\text{m / F; F / m, m, m})$$

$$\delta_{km} = \delta_{mk} = \frac{1}{2\pi\epsilon} \ln \frac{\sqrt{4h_k h_m + d_{km}^2}}{d_{km}}$$

Modified

$$\epsilon_0 = 8,854 \cdot 10^{-12} \approx \frac{10^{-9}}{36\pi} \text{ F / m ; } \epsilon_r = 1 ; \ln x = 2,3 \log x$$

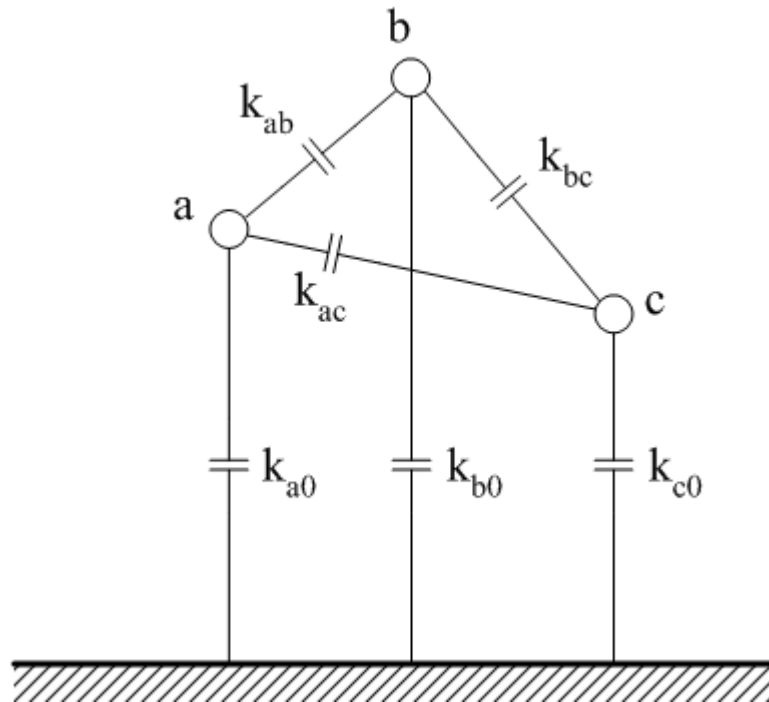
$$\Rightarrow \delta_{kk} = \frac{1}{0,0242} \log \frac{2h_k}{r_k} \quad (\text{km / } \mu\text{F})$$

$$\delta_{km} = \frac{1}{0,0242} \log \frac{\sqrt{4h_k h_m + d_{km}^2}}{d_{km}} \quad (\text{km / } \mu\text{F})$$

Matrix

$$(\hat{U}) = (\delta_{km})(\hat{Q})$$

A simple three-phase line without ground wires



Partial capacity to the ground: k_{x0}

Partial mutual capacity: k_{xy}

Symmetrical voltage

$$\hat{U}_a = \hat{U}_a \quad \hat{U}_b = \hat{a}^2 \hat{U}_a \quad \hat{U}_c = \hat{a} \hat{U}_a$$



Charges of the individual wires

$$\hat{Q}_a = k_{a0} \hat{U}_a + k_{ab} (\hat{U}_a - \hat{U}_b) + k_{ac} (\hat{U}_a - \hat{U}_c)$$

$$\hat{Q}_b = k_{b0} \hat{U}_b + k_{ab} (\hat{U}_b - \hat{U}_a) + k_{bc} (\hat{U}_b - \hat{U}_c)$$

$$\hat{Q}_c = k_{c0} \hat{U}_c + k_{ac} (\hat{U}_c - \hat{U}_a) + k_{bc} (\hat{U}_c - \hat{U}_b)$$

Modified

$$\hat{Q}_a = (k_{a0} + k_{ab} + k_{ac}) \hat{U}_a - k_{ab} \hat{U}_b - k_{ac} \hat{U}_c$$

$$\hat{Q}_b = -k_{ab} \hat{U}_a + (k_{b0} + k_{ab} + k_{bc}) \hat{U}_b - k_{bc} \hat{U}_c$$

$$\hat{Q}_c = -k_{ac} \hat{U}_a - k_{bc} \hat{U}_b + (k_{c0} + k_{ac} + k_{bc}) \hat{U}_c$$

The introduction of capacity coefficients

$$\hat{Q}_a = c_{aa} \hat{U}_a + c_{ab} \hat{U}_b + c_{ac} \hat{U}_c$$

$$\hat{Q}_b = c_{ab} \hat{U}_a + c_{bb} \hat{U}_b + c_{bc} \hat{U}_c$$

$$\hat{Q}_c = c_{ac} \hat{U}_a + c_{bc} \hat{U}_b + c_{cc} \hat{U}_c$$

Matrix

$$\begin{aligned}(\hat{Q}) &= (\mathbf{c}_{km})(\hat{U}) \\(\hat{Q}) &= (\delta_{km})^{-1}(\hat{U}) \quad \Rightarrow \quad (\mathbf{c}_{km}) = (\delta_{km})^{-1}\end{aligned}$$

Calculation procedure:

geometry \rightarrow $(\delta_{km}) \rightarrow (\mathbf{c}_{km}) \rightarrow$ capacity

$$m = k : k_{k0} = \sum_{m=1}^n c_{km}$$

$$m \neq k : k_{km} = -c_{km}$$

Operational capacity - the k^{th} conductor alone has the same charge as in the system of n conductors

$$\hat{C}_k = \frac{\hat{Q}_k}{\hat{U}_k} = \frac{k_{k0} \hat{U}_k + \sum_{m=1, m \neq k}^n k_{km} (\hat{U}_k - \hat{U}_m)}{\hat{U}_k}$$

$$\hat{C}_a = \frac{(k_{a0} + k_{ab} + k_{ac})\hat{U}_a - k_{ab}\hat{U}_b - k_{ac}\hat{U}_c}{\hat{U}_a}$$

$$\hat{C}_b = \frac{-k_{ab}\hat{U}_a + (k_{b0} + k_{ab} + k_{bc})\hat{U}_b - k_{bc}\hat{U}_c}{\hat{U}_b}$$

$$\hat{C}_c = \frac{-k_{ac}\hat{U}_a - k_{bc}\hat{U}_b + (k_{c0} + k_{ac} + k_{bc})\hat{U}_c}{\hat{U}_c}$$

Generally

$$k_{a0} \neq k_{b0} \neq k_{c0}$$

$$k_{ab} \neq k_{bc} \neq k_{ac}$$

$$\hat{C}_a \neq \hat{C}_b \neq \hat{C}_c$$

→ current unbalance $(\hat{I}_{kc} = j\omega\hat{Q}_k) \rightarrow$ transposition

Transposed lines

The potential coefficients matrix

$$(\delta_{km}) = \frac{1}{3} \left\{ \begin{pmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{12} & \delta_{22} & \delta_{23} \\ \delta_{13} & \delta_{23} & \delta_{33} \end{pmatrix} + \begin{pmatrix} \delta_{33} & \delta_{13} & \delta_{23} \\ \delta_{13} & \delta_{11} & \delta_{12} \\ \delta_{23} & \delta_{12} & \delta_{22} \end{pmatrix} + \begin{pmatrix} \delta_{22} & \delta_{23} & \delta_{12} \\ \delta_{23} & \delta_{33} & \delta_{13} \\ \delta_{12} & \delta_{13} & \delta_{11} \end{pmatrix} \right\}$$

Let's introduce

$$\delta = \frac{1}{3} (\delta_{11} + \delta_{22} + \delta_{33})$$

$$\delta = \frac{1}{0,0242} \log \frac{2h}{r} \quad (\text{km} / \mu\text{F})$$

mean geometrical height

$$h = \sqrt[3]{h_1 h_2 h_3}$$

$$\delta' = \frac{1}{3} (\delta_{12} + \delta_{13} + \delta_{23})$$

$$\delta' = \frac{1}{0,0242} \log \frac{\sqrt{4h^2 + d^2}}{d} \quad (\text{km} / \mu\text{F})$$

$$d = \sqrt[3]{d_{12}d_{13}d_{23}}$$

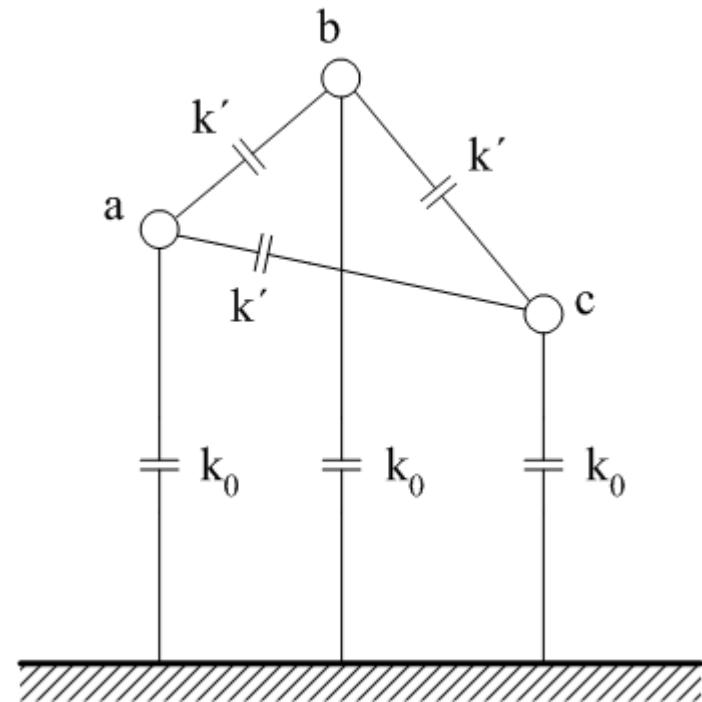
Then

$$\begin{pmatrix} \hat{U}_a \\ \hat{U}_b \\ \hat{U}_c \end{pmatrix} = \begin{pmatrix} \delta & \delta' & \delta' \\ \delta' & \delta & \delta' \\ \delta' & \delta' & \delta \end{pmatrix} \begin{pmatrix} \hat{Q}_a \\ \hat{Q}_b \\ \hat{Q}_c \end{pmatrix}$$

The diagrams include only 2 capacities

$$k_0 = k_{a0} = k_{b0} = k_{c0}$$

$$k' = k_{ab} = k_{bc} = k_{ac}$$



For the charges

$$\begin{pmatrix} \hat{Q}_a \\ \hat{Q}_b \\ \hat{Q}_c \end{pmatrix} = \begin{pmatrix} k_0 + 2k' & -k' & -k' \\ -k' & k_0 + 2k' & -k' \\ -k' & -k' & k_0 + 2k' \end{pmatrix} \begin{pmatrix} \hat{U}_a \\ \hat{U}_b \\ \hat{U}_c \end{pmatrix}$$

Solution:

Capacity to the ground

$$\underline{k_0 = \frac{1}{\delta + 2\delta'}}$$

Capacity between conductors

$$\underline{k' = \frac{\delta'}{(\delta + 2\delta') \cdot (\delta - \delta')}}}$$

Operational capacity (real number)

$$C = C_a = C_b = C_c = k_0 + 3k' \qquad \underline{C = \frac{1}{\delta - \delta'}}$$

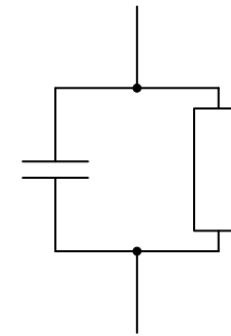
Values:

• 400 kV - $B_1 \approx (3,5 \div 4,5) \mu\text{S} \cdot \text{km}^{-1}$

• 110, 220 kV - $B_1 \approx (2,5 \div 3) \mu\text{S} \cdot \text{km}^{-1}$

• 22 kV - $B_1 \approx 1,4 \mu\text{S} \cdot \text{km}^{-1}$

$B \sim 10^{-6} \text{ S/km}$



$G \sim 10^{-8} \text{ S/km}$

Conductance is negligible in relation to capacities.

Electrical parameters of cables

Resistance

The same as for overhead lines.

Single-core – R increased due to eddy currents and hysteresis losses in the metal case (screen).

Often (up to) higher cross-sections than OHL.

e.g.	22 kV	CXEKVCE 240	$R_{1dc0} \sim 0,075 \Omega/\text{km}$
		AXEKVCEY 240	$R_{1dc0} \sim 0,125 \Omega/\text{km}$
		AXEKVCEY 400	$R_{1dc0} \sim 0,078 \Omega/\text{km}$
		AXEKVCEY 630	$R_{1dc0} \sim 0,047 \Omega/\text{km}$
	110 kV	A2XS(FL)2Y 800	$R_{1dc0} \sim 0,037 \Omega/\text{km}$

Inductance

Three-core – the same as for transposed power lines.

$$L_1 = 0,46 \log \frac{d}{\xi r} \quad (\text{mH / km})$$

Not valid $d \gg r \rightarrow L$ values less precise but technically applicable.

$$6 \text{ kV} \quad X_1 \sim 0,06 \Omega/\text{km}$$

$$22 \text{ kV} \quad X_1 \sim 0,10 \Omega/\text{km}$$

$$110 \text{ kV} \quad X_1 \sim 0,11 \Omega/\text{km}$$

Conductance

Determined by dielectric losses.

3 phase

$$P_d = 3U_f I_c = 3U_f I_j \operatorname{tg} \delta \quad (\text{W})$$

$$P_d = 3U_f \omega C U_f \operatorname{tg} \delta = \omega C U^2 \operatorname{tg} \delta = Q_c \operatorname{tg} \delta$$

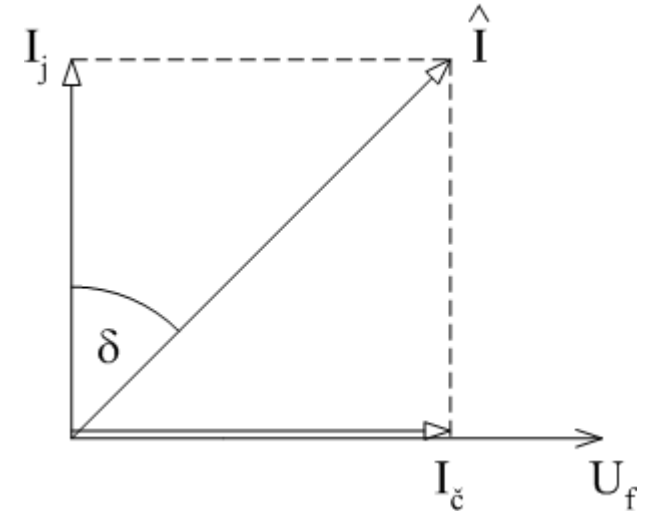
Q_c ...charging power

Conductance per length unit

$$G_1 = \frac{P_{d1}}{U^2} \quad (\text{S/km; W/km, V})$$

B/G ratio lower for cables than for OHL.

- OHL 20÷100
- cables 10÷30



Capacities

3 cable types:

- a) full plastic (without conducting screen)
- b) single-core with a metal screen or multi-core with a screen for each conductor
- c) three-core with a common metal screen

ad a)

C is changing with cable placement and environment. It is measured.

ad b) – the most often

Only capacity of the conductor to the screen = operational.

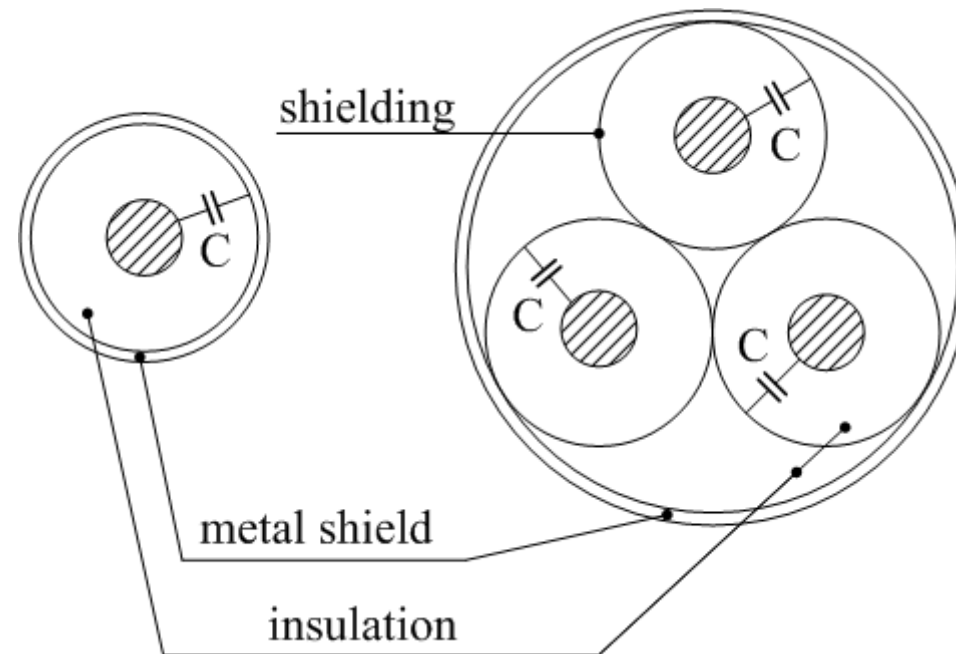
For coaxial cylinder

$$C = \frac{0,0242\epsilon_r}{\log \frac{r_2}{r_1}} \quad (\mu\text{F} / \text{km})$$

ϵ_r ...insulation relative permittivity

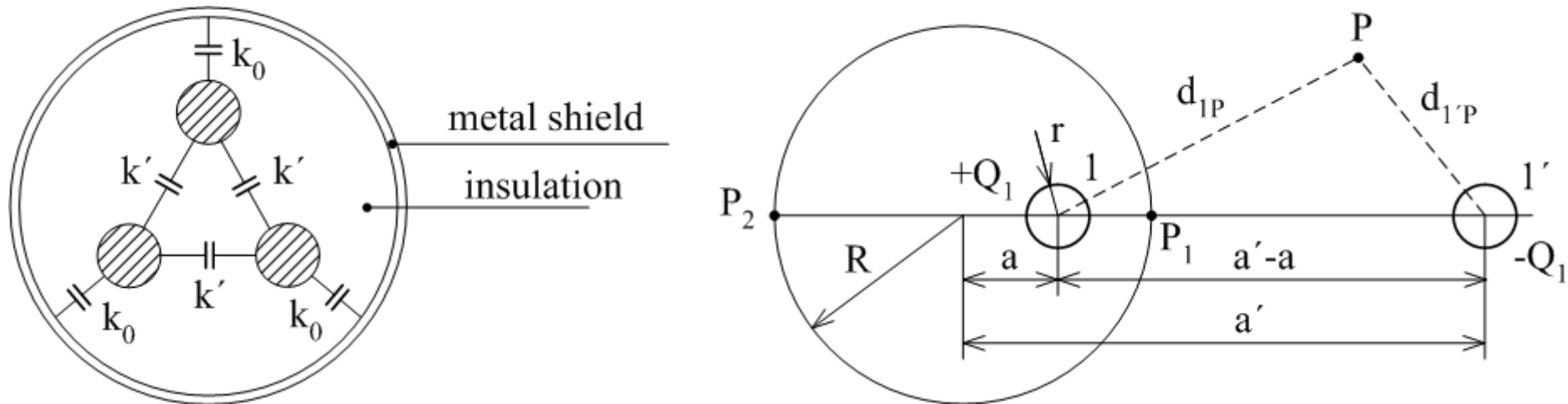
r_1 ...conductor radius

r_2 ...screen mean radius



ad c)

As three phase symmetrical power lines.
Mirror method x along the metal screen.



Cable capacities are much higher than for overhead lines (c. 30÷50 times)
→ limited lengths of cable networks because of charging currents
(10x km).

- 22kV - $B_1 \approx (50 \div 150) \mu\text{S} \cdot \text{km}^{-1}$ $S \approx (50 \div 630) \text{mm}^2$
- 110 kV - $B_1 \approx (50 \div 90) \mu\text{S} \cdot \text{km}^{-1}$ $S \approx (400 \div 2000) \text{mm}^2$

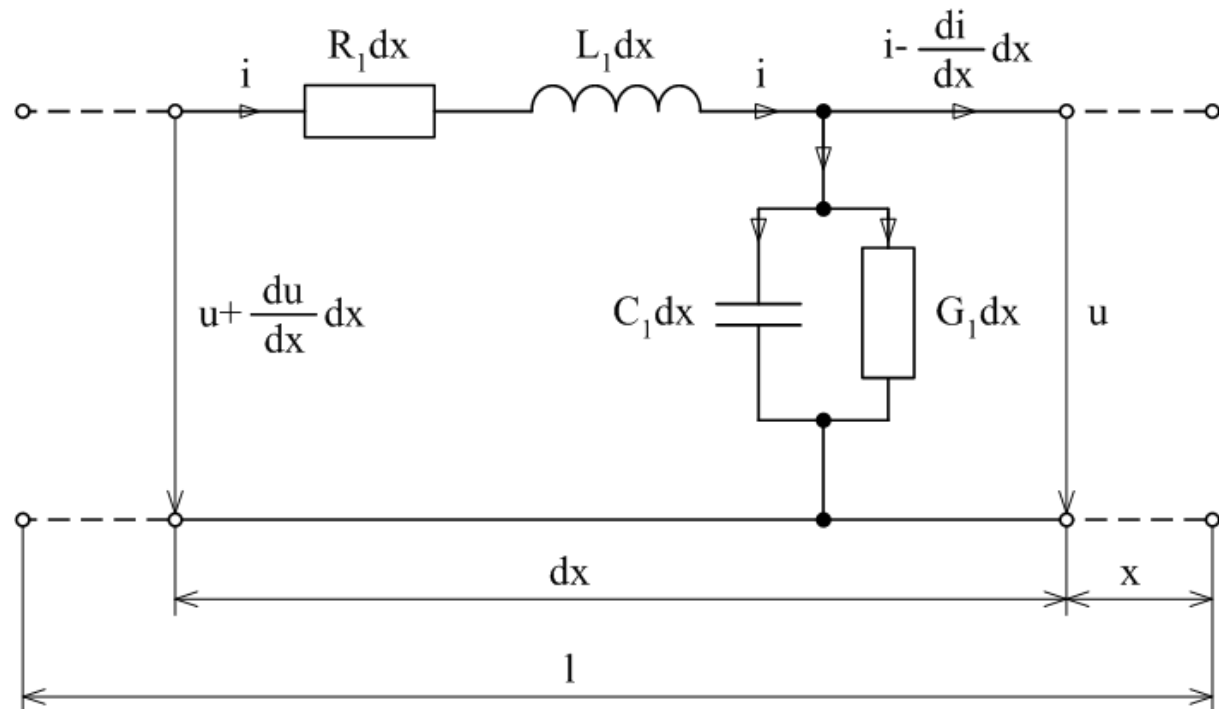
Three-phase HV Power Lines

Transmission systems, international connections.

Aim: relations between sending and receiving ends conditions, losses, efficiency. RLGC

Line with equally distributed parameters

Homogeneous line - parameters R_1 , L_1 , G_1 , C_1 are equally distributed along the total line length.



2nd Kirchhoff's law

$$u + \frac{\partial u}{\partial x} dx - u - R_1 dx \cdot i - L_1 dx \frac{\partial i}{\partial t} = 0 \rightarrow \frac{\partial u}{\partial x} = R_1 i + L_1 \frac{\partial i}{\partial t}$$

1st Kirchhoff's law

$$i - \frac{\partial i}{\partial x} dx - i + G_1 dx \cdot u + C_1 dx \frac{\partial u}{\partial t} = 0 \rightarrow \frac{\partial i}{\partial x} = G_1 u + C_1 \frac{\partial u}{\partial t}$$

In phasors

$$\frac{d\hat{U}_f}{dx} = (R_1 + j\omega L_1)\hat{I} = \hat{Z}_{l_1} \hat{I} \qquad \frac{d\hat{I}}{dx} = (G_1 + j\omega C_1)\hat{U}_f = \hat{Y}_{q_1} \hat{U}_f$$

After derivation and substitution we get wave equations

$$\frac{d^2 \hat{U}_f}{dx^2} = \hat{Z}_{l_1} \frac{d\hat{I}}{dx} = \hat{Z}_{l_1} \hat{Y}_{q_1} \hat{U}_f = \hat{\gamma}^2 \hat{U}_f$$

$$\frac{d^2 \hat{I}}{dx^2} = \hat{Y}_{q_1} \frac{d\hat{U}_f}{dx} = \hat{Z}_{l_1} \hat{Y}_{q_1} \hat{I} = \hat{\gamma}^2 \hat{I}$$

General wave equations solution, i.e. second-order linear ordinary differential equation (char. equation $\hat{\lambda}^2 - \hat{\gamma}^2 = 0$) – progressive and reflected wave

$$\hat{U}_f = \hat{K}_1 e^{\hat{\gamma}x} + \hat{K}_2 e^{-\hat{\gamma}x}$$

$$\hat{I} = \frac{d\hat{U}_f}{dx} \frac{1}{\hat{Z}_{l_1}} = \frac{\hat{\gamma}}{\hat{Z}_{l_1}} (\hat{K}_1 e^{\hat{\gamma}x} - \hat{K}_2 e^{-\hat{\gamma}x}) = \sqrt{\frac{\hat{Y}_{q_1}}{\hat{Z}_{l_1}}} (\hat{K}_1 e^{\hat{\gamma}x} - \hat{K}_2 e^{-\hat{\gamma}x})$$

$$\hat{I} = \frac{1}{\hat{Z}_v} (\hat{K}_1 e^{\hat{\gamma}x} - \hat{K}_2 e^{-\hat{\gamma}x})$$

After solution using boundary conditions

$$\hat{U}_{f1} = \hat{U}_{f2} \cosh \hat{\gamma}l + \hat{Z}_v \hat{I}_2 \sinh \hat{\gamma}l$$

$$\hat{I}_1 = \frac{\hat{U}_{f2}}{\hat{Z}_v} \sinh \hat{\gamma}l + \hat{I}_2 \cosh \hat{\gamma}l$$

Matrix form

$$\begin{pmatrix} \hat{U}_{f1} \\ \hat{I}_1 \end{pmatrix} = \begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix} \begin{pmatrix} \hat{U}_{f2} \\ \hat{I}_2 \end{pmatrix}$$

where $\hat{A}(-)$, $\hat{B}(\Omega)$, $\hat{C}(S)$, $\hat{D}(-)$ are Blondel's constants

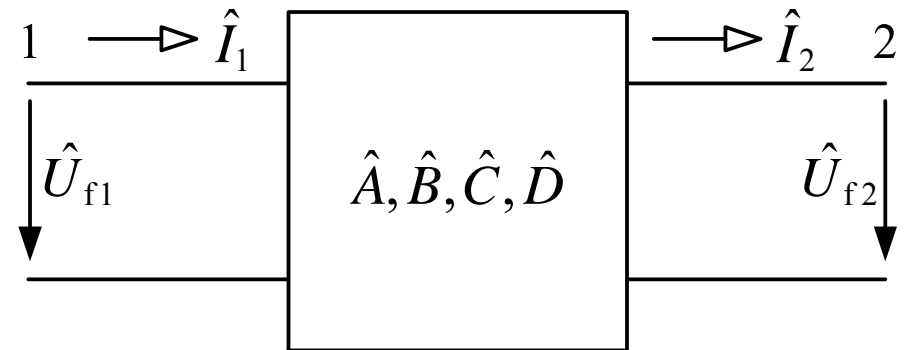
it is valid $\hat{A} = \hat{D}$, $\hat{A}\hat{D} - \hat{B}\hat{C} = 1$

$$(\cosh \hat{\gamma}l)^2 - (\sinh \hat{\gamma}l)^2 = 1$$

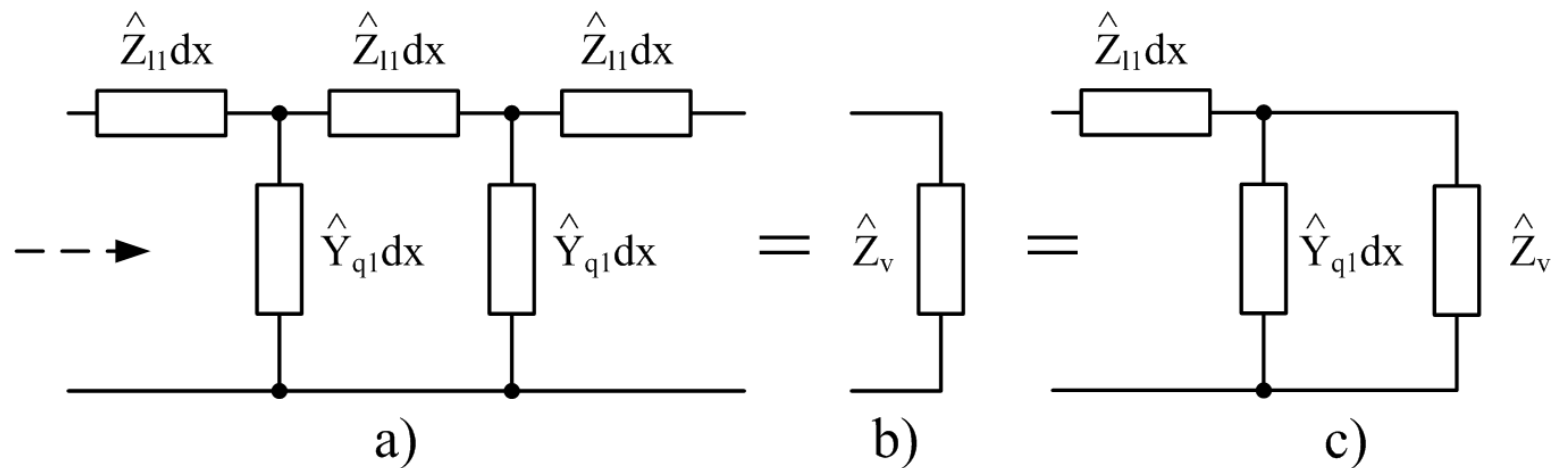
(symmetrical, passive two-port network)

Values at the line beginning are set

$$\begin{pmatrix} \hat{U}_{f2} \\ \hat{I}_2 \end{pmatrix} = \begin{pmatrix} \hat{D} & -\hat{B} \\ -\hat{C} & \hat{A} \end{pmatrix} \begin{pmatrix} \hat{U}_{f1} \\ \hat{I}_1 \end{pmatrix}$$



Surge impedance = impedance of the infinite long line



Input impedance

$$\hat{Z}_v = \hat{Z}_{11} dx + \frac{\hat{Z}_v \cdot (\hat{Y}_{q1} dx)^{-1}}{\hat{Z}_v + (\hat{Y}_{q1} dx)^{-1}}$$

$$\hat{Z}_v^2 - \hat{Z}_{11} dx \cdot \hat{Z}_v - \hat{Z}_{11} dx \cdot (\hat{Y}_{q1} dx)^{-1} = 0$$

$$\hat{Z}_v = \frac{\hat{Z}_{11} dx \pm \sqrt{(\hat{Z}_{11} dx)^2 + 4\hat{Z}_{11} dx \cdot (\hat{Y}_{q1} dx)^{-1}}}{2} \stackrel{dx \rightarrow 0}{=} \sqrt{\frac{\hat{Z}_{11}}{\hat{Y}_{q1}}}$$

Hyperbolic functions series (Taylor series at $x = 0$)

$$f(x)_{x_0} = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!} (x - x_0)^k = f(x_0) + f'(x_0) \cdot (x - x_0) + \frac{f''(x_0)}{2} \cdot (x - x_0)^2 + \dots$$

$$\cosh \hat{\gamma}l = 1 + \frac{(\hat{\gamma}l)^2}{2} + \frac{(\hat{\gamma}l)^4}{24} + \dots = 1 + \frac{\hat{Z}_{l_1} \hat{Y}_{q_1}}{2} l^2 + \dots$$

$$\sinh \hat{\gamma}l = \hat{\gamma}l + \frac{(\hat{\gamma}l)^3}{6} + \dots = \sqrt{\hat{Z}_{l_1} \hat{Y}_{q_1}} l + \frac{(\hat{Z}_{l_1} \hat{Y}_{q_1})^{3/2}}{6} l^3 + \dots$$

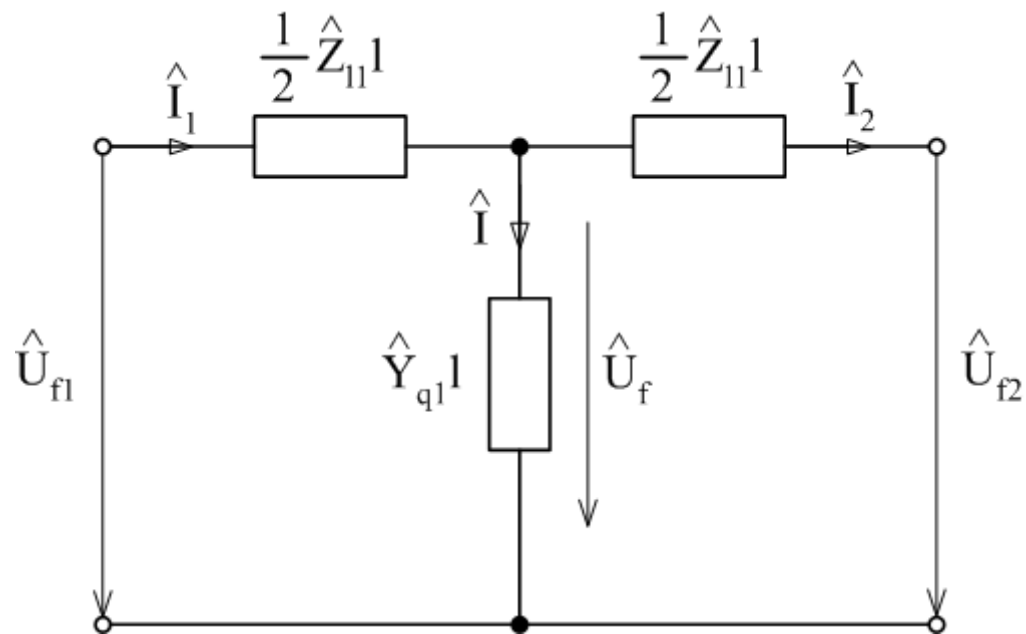
$$\hat{Z}_v \sinh \hat{\gamma}l = \sqrt{\frac{\hat{Z}_{l_1}}{\hat{Y}_{q_1}}} \sinh \hat{\gamma}l = \hat{Z}_{l_1} l + \hat{Z}_{l_1} l \frac{\hat{Z}_{l_1} \hat{Y}_{q_1}}{6} l^2 + \dots$$

$$\frac{1}{\hat{Z}_v} \sinh \hat{\gamma}l = \sqrt{\frac{\hat{Y}_{q_1}}{\hat{Z}_{l_1}}} \sinh \hat{\gamma}l = \hat{Y}_{q_1} l + \hat{Y}_{q_1} l \frac{\hat{Z}_{l_1} \hat{Y}_{q_1}}{6} l^2 + \dots$$

Line with lumped parameters

For ordinary calculations (meshed grids) it is possible to use substitution networks with a good accuracy (according to line length).

T-network – short lines, transformers; it adds another node (substitution diagram for overhead lines up to 200 km, cable lines up to 80 km)



Voltage and current at the line beginning

$$\hat{U}_{f1} = \hat{U}_{f2} + \frac{1}{2} \hat{Z}_{l_1} \mathbf{1} \cdot \hat{I}_2 + \frac{1}{2} \hat{Z}_{l_1} \mathbf{1} \cdot \hat{I}_1 \quad \hat{I}_1 = \hat{I}_2 + \hat{I}$$

Voltage and current for the cross branch

$$\hat{U}_f = \hat{U}_{f2} + \frac{1}{2} \hat{Z}_{l_1} \mathbf{1} \cdot \hat{I}_2$$

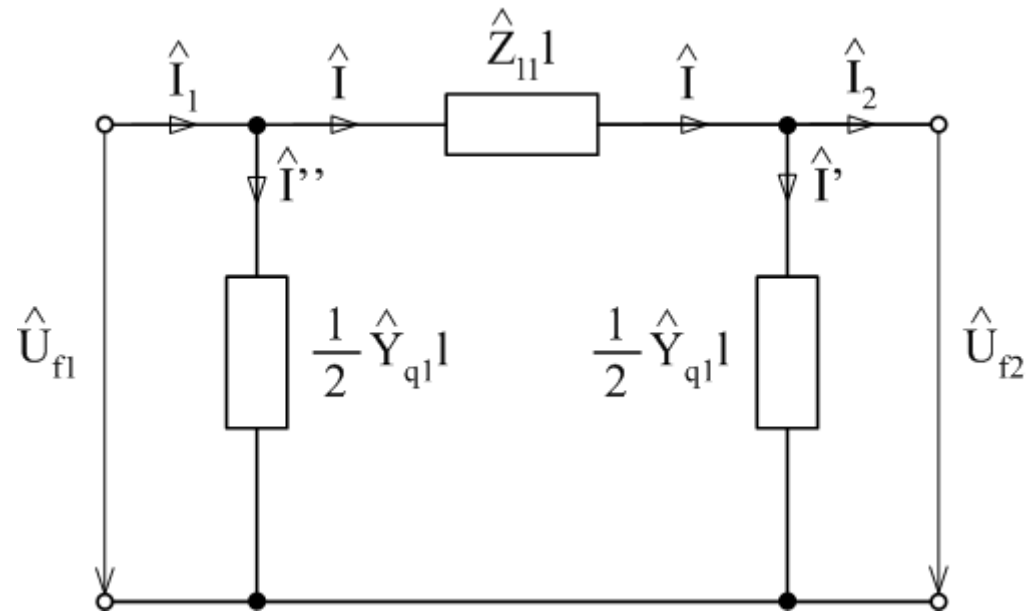
$$\hat{I} = \hat{Y}_{q_1} \mathbf{1} \cdot \hat{U}_f = \hat{U}_{f2} \hat{Y}_{q_1} \mathbf{1} + \hat{I}_2 \frac{\hat{Z}_{l_1} \hat{Y}_{q_1} \mathbf{1}^2}{2}$$

Hence (Blondel's constants relations)

$$\hat{U}_{f1} = \hat{U}_{f2} \left(1 + \frac{\hat{Z}_{l_1} \hat{Y}_{q_1} \mathbf{1}^2}{2} \right) + \hat{I}_2 \hat{Z}_{l_1} \mathbf{1} \left(1 + \frac{\hat{Z}_{l_1} \hat{Y}_{q_1} \mathbf{1}^2}{4} \right)$$

$$\hat{I}_1 = \hat{U}_{f2} \hat{Y}_{q_1} \mathbf{1} + \hat{I}_2 \left(1 + \frac{\hat{Z}_{l_1} \hat{Y}_{q_1} \mathbf{1}^2}{2} \right)$$

π -network – longer lines, more accurate (substitution diagram for overhead lines up to 250 km, cable lines up to 100 km)



Voltage and current at the line beginning

$$\hat{U}_{f1} = \hat{U}_{f2} + \hat{Z}_{l1} \cdot \hat{I} = \hat{U}_{f2} + \hat{Z}_{l1} \cdot (\hat{I}_2 + \hat{I}')$$

$$\hat{I}_1 = \hat{I}_2 + \hat{I}' + \hat{I}''$$

Currents for cross branches

$$\hat{I}' = \frac{1}{2} \hat{Y}_{q_1} \mathbf{1} \hat{U}_{f_2}$$

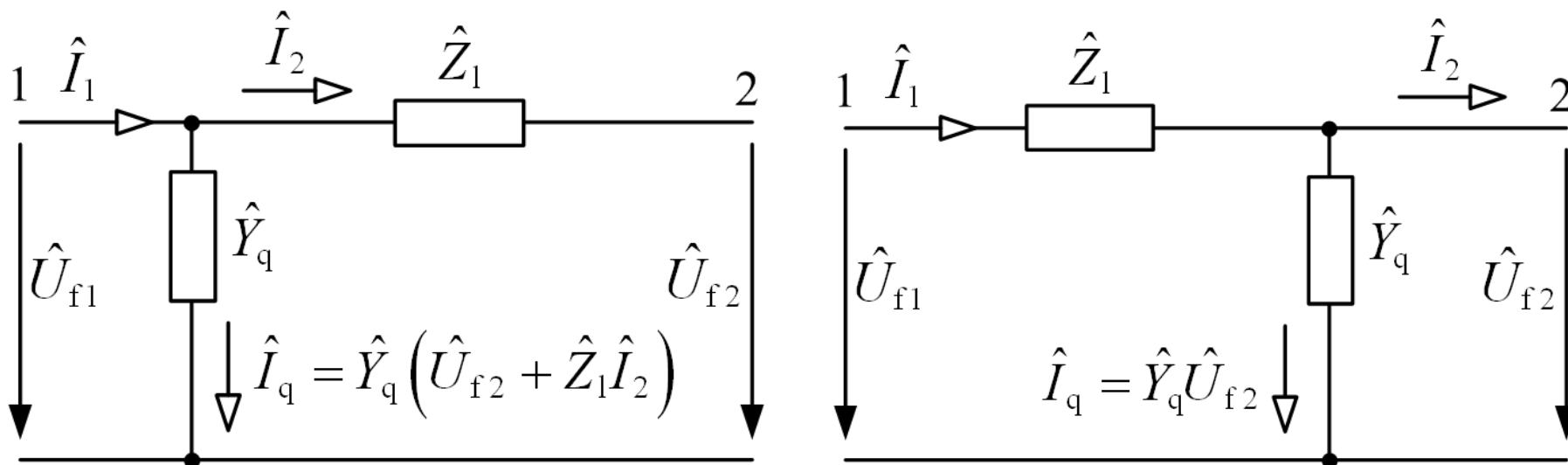
$$\hat{I}'' = \frac{1}{2} \hat{Y}_{q_1} \mathbf{1} \hat{U}_{f_1}$$

After modification (Blondel's constants relations)

$$\hat{U}_{f_1} = \hat{U}_{f_2} \left(1 + \frac{\hat{Z}_{l_1} \hat{Y}_{q_1}}{2} \mathbf{1}^2 \right) + \hat{I}_2 \hat{Z}_{l_1} \mathbf{1}$$

$$\hat{I}_1 = \hat{U}_{f_2} \hat{Y}_{q_1} \mathbf{1} \left(1 + \frac{\hat{Z}_{l_1} \hat{Y}_{q_1}}{4} \mathbf{1}^2 \right) + \hat{I}_2 \left(1 + \frac{\hat{Z}_{l_1} \hat{Y}_{q_1}}{2} \mathbf{1}^2 \right)$$

Γ -network (gamma) – relatively rare utilization, for shorter lines (overhead up to 80 km, cable up to 25 km), transformers



$$\hat{U}_{f1} = \hat{U}_{f2} + \hat{I}_2 \hat{Z}_{l_1} 1$$

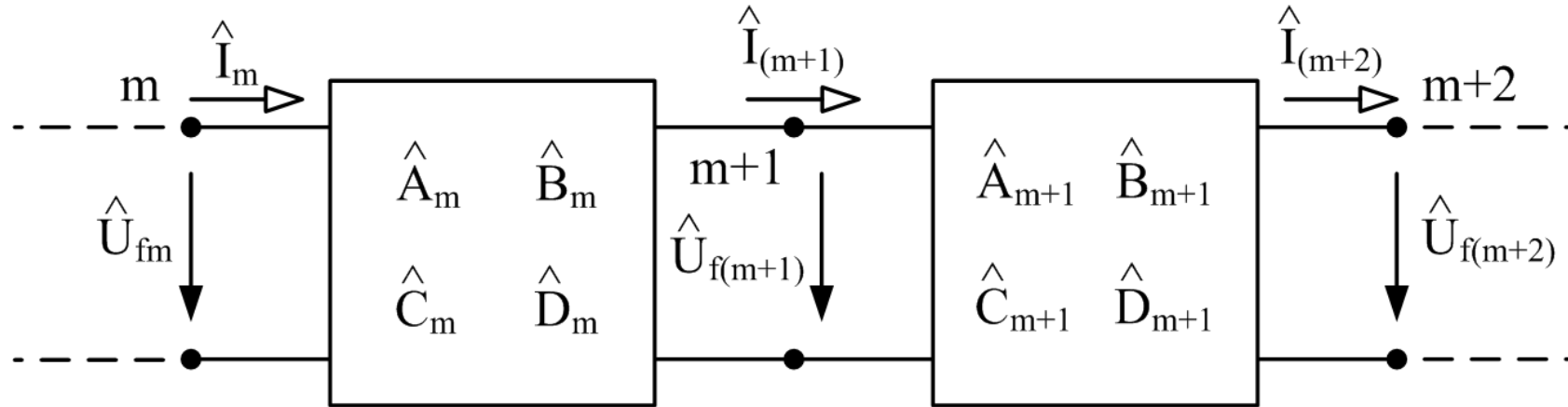
$$\hat{I}_1 = \hat{U}_{f2} \hat{Y}_{q_1} 1 + \hat{I}_2 (1 + \hat{Z}_{l_1} \hat{Y}_{q_1} 1^2)$$

$$\hat{U}_{f1} = \hat{U}_{f2} (1 + \hat{Z}_{l_1} \hat{Y}_{q_1} 1^2) + \hat{I}_2 \hat{Z}_{l_1} 1$$

$$\hat{I}_1 = \hat{U}_{f2} \hat{Y}_{q_1} 1 + \hat{I}_2$$

it is valid $\hat{A} \neq \hat{D}$, $\hat{A}\hat{D} - \hat{B}\hat{C} = 1$ (unbalanced, passive two-port network)

Longer lines → cascade connection of networks for shorter sections
(additional nodes)



$$\begin{pmatrix} \hat{U}_{fm} \\ \hat{I}_m \end{pmatrix} = \begin{pmatrix} \hat{A}_m & \hat{B}_m \\ \hat{C}_m & \hat{D}_m \end{pmatrix} \begin{pmatrix} \hat{U}_{f(m+1)} \\ \hat{I}_{(m+1)} \end{pmatrix}$$

$$\begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix} = \prod_{m=1}^n \begin{pmatrix} \hat{A}_m & \hat{B}_m \\ \hat{C}_m & \hat{D}_m \end{pmatrix}$$