## Inductors and capacitors in ES

a) Series inductors

- reactors are used to limit short-circuit currents $\rightarrow$ current limiting reactors
- used in grids up to 35 kV , single-phase ( $\mathrm{I}_{\mathrm{n}}>200 \mathrm{~A}$ ) or three-phase ( $\mathrm{I}_{\mathrm{n}}<200 \mathrm{~A}$ )
- usually air-cooled (small L, no mag. saturation x leakage, mag. field induced current nearby metal objects)
- L optimization (small - lower voltage drop, higher - SC reduction)
- In fault-free state the inductor can be bypassed by a fuse to reduce a voltage drop.
- the same design in LC filters for harmonics suppression, SVC (TCR)



## SVC <br> (Static VAr Compensation)

TCR
(Thyristor Controlled Reactor)



$$
\mathrm{R}_{\mathrm{L}} \ll \mathrm{X}_{\mathrm{L}}
$$



Input: $\mathrm{X}_{\mathrm{L} \%}, \mathrm{~S}_{\mathrm{L}}, \mathrm{U}_{\mathrm{n}}, \mathrm{I}_{\mathrm{n}}$
Calculation: $\quad \mathrm{S}_{\mathrm{L}}=\sqrt{3} \cdot \mathrm{U}_{\mathrm{n}} \cdot \mathrm{I}_{\mathrm{n}}$

$$
\begin{aligned}
& X_{L}=\frac{X_{L \%} \cdot U_{n}}{100 \cdot \sqrt{3} \cdot I_{n}}=\frac{X_{L \%} \cdot U_{n}^{2}}{100 \cdot S_{L}} \\
& \Delta \hat{U}_{\mathrm{f}}=\hat{U}_{\mathrm{f} 1}-\hat{U}_{\mathrm{f} 2}=\left(R_{\mathrm{L}}+j \mathrm{X}_{\mathrm{L}}\right) \hat{\mathrm{I}}=\hat{\mathrm{Z}}_{\mathrm{L}} \hat{\mathrm{I}} \\
& \quad\left\langle\hat{\mathrm{Z}}_{\mathrm{Labc}}\right|=\hat{\mathrm{Z}}_{\mathrm{L}} \cdot[\mathrm{E}]-3 \text { ph inductor }
\end{aligned}
$$

$\rightarrow$ self-impedance $\hat{Z}_{\mathrm{L}}$, mutual impedances 0

## b) Shunt (parallel) inductors

- in the transmission systems (usually $\mathrm{U}_{\mathrm{N}}>220 \mathrm{kV}$ )
- oil cooling, Fe core
- used to compensate capacitive (charging) currents of long OHL for no-load or small loads $\rightarrow \underline{\mathrm{U} \text { control }}$ :

$$
\begin{aligned}
& \mathrm{X}_{\mathrm{L}}=\frac{\mathrm{U}_{\mathrm{Ln}}}{\sqrt{3} \cdot \mathrm{I}_{\mathrm{Ln}}}=\frac{\mathrm{U}_{\mathrm{Ln}}^{2}}{\mathrm{Q}_{\mathrm{Ln}}} \\
& \left\lfloor\hat{\mathrm{Z}}_{\mathrm{Labc}} \mid=\hat{\mathrm{Z}}_{\mathrm{L}} \cdot[\mathrm{E}]\right.
\end{aligned}
$$

- Q: 15, 30, 55 MVA

Connection in the system:

a) galvanic connection to the line

- Y winding
b) inductor connection to transformer tertiary winding
- lower voltage $\mathrm{U}_{\mathrm{n}} \approx 10 \div 35 \mathrm{kV}$


Kočín 400 kV


## c) Neutral point inductors

- used in networks with indirectly earthed neutral point to compensate currents during ground fault (capacitive currents)
- resonance compensation
- for distribution systems ( 6 to 35 kV )
- reactor is single-phased!, oil cooling
- capacitive current change (network reconfiguration) $\rightarrow$ change in inductance (air gap correction in the magnetic circuit)
 $=$ arc-suppression coil (Peterson coil)
_ $X_{L}=\frac{U_{\text {ph } n}}{I_{\text {Lset }}}$


## 4 MVAr, 13 kV, Ostrava - Kunčice



## d) Series capacitors

- capacitors in ES = capacitor banks
- in series $\rightarrow$ reduce TS line series inductance
- power flow control, voltage drop reduction, dynamic oscillation mitigation
- TCSR (Thyristor Controlled Series Capacitor)


$$
\hat{\mathrm{U}}_{\mathrm{C}}=-\mathrm{j} \frac{1}{\omega \mathrm{C}} \hat{\mathrm{I}}
$$

- device installed on insulated platforms - C under voltage
- during short-circuits and overcurrents there appears overvoltage on the capacitor (very fast protections are used)

Canada 750 kV


## e) Shunt capacitors

- used in LV industrial networks (up to 1 kV )
- connection:
a) wye - Y
b) delta - $\Delta$ (D)


$$
\begin{array}{ll}
Q_{f}=U_{f} \cdot I_{C}=U_{f}^{2} \omega C_{Y} & Q_{f}=U \cdot I_{C}=U^{2} \omega C_{\Delta} \\
Q=3 U_{f}^{2} \omega C_{Y}=U^{2} \omega C_{Y} & Q=3 U^{2} \omega C_{\Delta}
\end{array}
$$

- with the same reactive power

$$
\mathrm{U}^{2} \omega \mathrm{C}_{\mathrm{Y}}=3 \mathrm{U}^{2} \omega \mathrm{C}_{\Delta} \rightarrow \mathrm{C}_{\mathrm{Y}}=3 \mathrm{C}_{\Delta} \rightarrow \text { rather delta }
$$


$\rightarrow$ power factor improvement, lower power losses, voltage drops

- individual or group compensation could be used
- shunt - also in harmonic filter (mainly MV, or SVC to HV via transformer)


## Transformer Concept in CR



- voltages, neutral point grounding, winding connection +D winding


## Transformers in ES

## DTS 22/0,4 kV (35/0,4 kV)




Industrial ( $22 / 6 \mathrm{kV}$ )




## Construction Issues

- winding material (Al, Cu)
- winding connection (D, Y, Z)
- clock hour number (phasor group) (1-11)
- core material (standard, amorphous) $\rightarrow$ no load losses
- tank (oil, dry)
- cooling (oil, air) - e.g. ONAN, OFAF
- noise
- weight
- voltage levels, ratio
- power

○DTS: 50, 63, 100, 160, 250, 400, 500, 630, 800, 1000, 1250, 1600, 2000, 2500, 4000 kVA

- $110 \mathrm{kV} / \mathrm{MV}: 10,16,25,40,50,63 \mathrm{MVA}$
o HV/MV: 66, 200, 250, 350
- parameters ...


## a) Two-winding transformers

- winding connection Y, Yn, D, Z, Zn, V

Yzn - distribution TRF MV/LV up to 250 kVA , for unbalanced load Dyn - distribution TRF MV/LV from 400 kVA
Yd - block TRF in power plants, the $3{ }^{\text {rd }}$ harmonic suppression
Yna-d, YNynd - power grid transformer (400, 220, 110 kV )
YNyd - power grid transformer (e.g. 110/23/6,3 kV)

- clock hour number (phasor group)

| Sr. <br> No. | Symbol | Windings and terminals | EMF vector diagrams | Equivalent clock method representation |
| :---: | :---: | :---: | :---: | :---: |
| 5. | $\begin{aligned} & \text { D y } 1 \\ & -30^{\circ} \end{aligned}$ |  |  |  |
| 6. | $\begin{aligned} & Y d 1 \\ & -30^{\circ} \end{aligned}$ |  |  |  |
| 7. | $\begin{gathered} \text { D y } 11 \\ +30^{\circ} \end{gathered}$ |  |  |  |
| 8. | $\begin{gathered} \text { Y d } 11 \\ +30^{\circ} \end{gathered}$ |  |  |  |

- equivalent circuit: T - network

$$
\hat{Z}_{\text {op }}=R_{p}+j X_{\text {op }} \quad \hat{Z}_{\text {os }}=R_{s}+j X_{\text {os }} \quad \hat{Y}_{q}=G_{q}-j B_{q}
$$



- each phase can be considered separately (unbalance is neglected)
- further operational impedance discussed
- values of the parameters are calculated, then verified by two tests
o no-load test - secondary winding open, primary winding supplied by rated voltage, no-load current is flowing (lower than rated current)
o short-circuit test - secondary winding short-circuited, primary winding supplied by short-circuit voltage (lower than rated voltage), so that rated current is flowing

$$
\begin{aligned}
& \Delta \mathrm{P}_{0}(\mathrm{~W}), \mathrm{i}_{0}(\%), \Delta \mathrm{P}_{\mathrm{k}}(\mathrm{~W}), \mathrm{z}_{\mathrm{k}}=\mathrm{u}_{\mathrm{k}}(\%), \mathrm{S}_{\mathrm{n}}(\mathrm{VA}), \mathrm{U}_{\mathrm{n}}(\mathrm{~V}) \\
& \mathrm{u}_{\mathrm{k}} \approx 4 \div 17 \% \text { (increases with TRF power) } \\
& \mathrm{p}_{\mathrm{k}} \approx 0,1 \div 1 \% \text { (decreases with TRF power) } \\
& \mathrm{p}_{0} \approx 0,01 \div 0,1 \% \text { (decreases with TRF power) }
\end{aligned}
$$

- shunt branch:

$$
\begin{aligned}
& g_{q}=\frac{\Delta P_{0}}{S_{n}} \quad y_{q}=\frac{i_{0 \%}}{100} \quad b_{q}=\sqrt{y_{q}^{2}-g_{q}^{2}} \\
& \hat{y}_{q}=\frac{\Delta P_{0}}{S_{n}}-j \sqrt{\left(\frac{i_{0 \%}}{100}\right)^{2}-\left(\frac{\Delta P_{0}}{S_{n}}\right)^{2}}=g_{q}-j \cdot b_{q} \\
& \hat{Y}_{q}=\hat{y}_{q} \frac{S_{n}}{U_{n}^{2}}=\frac{S_{n}}{U_{n}^{2}}\left[\frac{\Delta P_{0}}{S_{n}}-j \sqrt{\left(\frac{i_{0 \%}}{100}\right)^{2}-\left(\frac{\Delta P_{0}}{S_{n}}\right)^{2}}\right]=G_{q}-j \cdot B_{q}
\end{aligned}
$$

- series branch:

$$
\begin{aligned}
& \mathrm{r}_{\mathrm{k}}=\frac{\Delta \mathrm{P}_{\mathrm{k}}}{\mathrm{~S}_{\mathrm{n}}} \quad \mathrm{Z}_{\mathrm{k}}=\frac{\mathrm{u}_{\mathrm{k} \%}}{100} \quad \mathrm{x}_{\mathrm{k}}=\sqrt{\mathrm{z}_{\mathrm{k}}^{2}-\mathrm{r}_{\mathrm{k}}^{2}} \\
& \hat{\mathrm{Z}}_{\mathrm{k}}=\frac{\Delta \mathrm{P}_{\mathrm{k}}}{\mathrm{~S}_{\mathrm{n}}}+\mathrm{j} \sqrt{\left(\frac{\mathrm{u}_{\mathrm{k} \%}}{100}\right)^{2}-\left(\frac{\Delta \mathrm{P}_{\mathrm{k}}}{\mathrm{~S}_{\mathrm{n}}}\right)^{2}}=\mathrm{r}_{\mathrm{k}}+\mathrm{j} \cdot \mathrm{x}_{\mathrm{k}} \\
& \hat{Z}_{\mathrm{k}}=\hat{\mathrm{Z}}_{\mathrm{k}} \frac{\mathrm{U}_{\mathrm{n}}^{2}}{\mathrm{~S}_{\mathrm{n}}}=\frac{\mathrm{U}_{\mathrm{n}}^{2}}{\mathrm{~S}_{\mathrm{n}}}\left[\frac{\Delta \mathrm{P}_{\mathrm{k}}}{\mathrm{~S}_{\mathrm{n}}}+\mathrm{j} \sqrt{\left(\frac{\mathrm{u}_{\mathrm{k} \%}}{100}\right)^{2}-\left(\frac{\Delta \mathrm{P}_{\mathrm{k}}}{\mathrm{~S}_{\mathrm{n}}}\right)^{2}}\right]=\mathrm{R}_{\mathrm{k}}+\mathrm{j} \cdot \mathrm{X}_{\mathrm{k}} \\
& \hat{Z}_{\sigma p s}=\hat{Z}_{\mathrm{k}}=\left(\mathrm{R}_{\mathrm{p}}+\mathrm{R}_{\mathrm{s}}\right)+\mathrm{j}\left(\mathrm{X}_{\sigma p}+\mathrm{X}_{\sigma \mathrm{s}}\right)
\end{aligned}
$$

- we choose $\hat{Z}_{\sigma p}=0,5 \hat{Z}_{\text {ops }}=\hat{Z}_{\sigma s}$
- this division is not physically correct (different leakage flows, different resistances)


## Transformer losses and efficiency

$\Delta \mathrm{P}_{0} \approx \mathrm{U}$ - constant during operation
$\Delta \mathrm{P}_{\mathrm{k}} \cong \mathrm{R} \cdot \mathrm{I}^{2} \approx \mathrm{I}^{2}$ - changing during operation

- efficiency $\eta=\frac{P_{\text {out }}}{P_{\text {in }}}=1-\frac{\Delta \mathrm{P}_{0}+\Delta \mathrm{P}_{\mathrm{k}}}{\mathrm{P}_{\text {in }}}$

$$
\begin{aligned}
& \eta= 1-\frac{\Delta \mathrm{P}_{0}+\mathrm{R} \cdot \mathrm{I}^{2}}{\mathrm{U}_{\mathrm{n}} \cdot \mathrm{I} \cdot \cos \varphi}=1-\frac{\Delta \mathrm{P}_{0}}{\mathrm{U}_{\mathrm{n}} \cdot \mathrm{I} \cdot \cos \varphi}-\frac{\mathrm{R} \cdot \mathrm{I}}{\mathrm{U}_{n} \cdot \cos \varphi} \\
& \frac{\mathrm{~d} \mathrm{\eta}}{\mathrm{dI}}=0+\frac{\Delta \mathrm{P}_{0}}{\mathrm{U}_{\mathrm{n}} \cdot \mathrm{I}^{2} \cdot \cos \varphi}-\frac{\mathrm{R}}{\mathrm{U}_{\mathrm{n}} \cdot \cos \varphi}=0 \\
& \frac{\Delta \mathrm{P}_{0}}{\vdots} \frac{\vdots}{\mathrm{U}_{\mathrm{n}} \cdot \mathrm{I}^{2} \cdot \cos \varphi}=\frac{\mathrm{R}}{\mathrm{U}_{\mathrm{n}} \cdot \cos \varphi} \\
& \Delta \mathrm{P}_{0}=\mathrm{RI}^{2}=\Delta \mathrm{P}_{\mathrm{k}}
\end{aligned}
$$

## b) Three-winding transformers

- parameters are calculated, then verified by noload and short-circuit measurements (3 shortcircuit tests: 1 winding no-load, 1 short-circuit and 1 supplied):

$$
\begin{aligned}
& \Delta \mathrm{P}_{0}(\mathrm{~W}), \mathrm{i}_{0}(\%), \Delta \mathrm{P}_{\mathrm{k}}(\mathrm{~W}), \mathrm{z}_{\mathrm{K}}=\mathrm{u}_{\mathrm{K}}(\%), \\
& \mathrm{S}_{\mathrm{n}}(\mathrm{VA}), \mathrm{U}_{\mathrm{n}}(\mathrm{~V})
\end{aligned}
$$



- powers needn't be the same: $\mathrm{S}_{\mathrm{Sn}}=\mathrm{S}_{\mathrm{Tn}}=0,5 \cdot \mathrm{~S}_{\mathrm{Pn}}$
- equivalent circuit:

- no-load measurement: related to the primary rated power and rated voltage $S_{\text {Pn }}$ a $U_{\text {PN }}$ (supplied)

$$
\hat{y}_{q}=g_{q}-j \cdot b_{q}=\frac{\Delta P_{0}}{S_{P n}}-j \sqrt{\left(\frac{i_{0 \%}}{100}\right)^{2}-\left(\frac{\Delta P_{0}}{S_{P n}}\right)^{2}}
$$

denominated value ( S ) - related to UPN

$$
\hat{\mathrm{Y}}_{\mathrm{q}}=\hat{\mathrm{y}}_{\mathrm{q}} \frac{\mathrm{~S}_{\mathrm{P}_{\mathrm{n}}}}{U_{\mathrm{P}_{\mathrm{n}}}^{2}}=\mathrm{G}_{\mathrm{q}}-\mathrm{j} \cdot \mathrm{~B}_{\mathrm{q}}=\frac{\mathrm{S}_{\mathrm{P}_{\mathrm{n}}}}{\mathrm{U}_{\mathrm{P}_{\mathrm{n}}}^{2}}\left[\frac{\Delta \mathrm{P}_{0}}{\mathrm{~S}_{\mathrm{Pn}}}-\mathrm{j} \sqrt{\left(\frac{\mathrm{i}_{0 \%}}{100}\right)^{2}-\left(\frac{\Delta \mathrm{P}_{0}}{\mathrm{~S}_{\mathrm{P}_{\mathrm{n}}}}\right)^{2}}\right]
$$

- short-circuit measurement: (3x, supply - short-circuit - no-load) provided: $\mathrm{S}_{\mathrm{Pn}} \neq \mathrm{S}_{\mathrm{Sn}} \neq \mathrm{S}_{\mathrm{Tn}}$

| measurement between | $\mathrm{P}-\mathrm{S}$ | $\mathrm{P}-\mathrm{T}$ | $\mathrm{S}-\mathrm{T}$ |
| :--- | :---: | :---: | :---: |
| short-circuit losses (W) | $\Delta \mathrm{P}_{\mathrm{kPS}}$ | $\Delta \mathrm{P}_{\mathrm{kPT}}$ | $\Delta \mathrm{P}_{\mathrm{kST}}$ |
| short-circuit voltage (\%) | $\mathrm{u}_{\mathrm{kPS}}$ | $\mathrm{u}_{\mathrm{kPT}}$ | $\mathrm{u}_{\mathrm{kST}}$ |
| measurement corresponds to power (VA) | $\mathrm{S}_{\mathrm{Sn}}$ | $\mathrm{S}_{\mathrm{Tn}}$ | $\mathrm{S}_{\mathrm{Tn}}$ |

short-circuit tests S - T:
parameter to be found:

$$
\begin{aligned}
& \hat{Z}_{\mathrm{ST}}=\hat{Z}_{\sigma S}+\hat{Z}_{\sigma T}\left(\hat{Z}_{\sigma S}=R_{\mathrm{S}}+j \cdot X_{\sigma S}\right) \text { - recalculated to } U_{\mathrm{PN}} \\
& \hat{\mathrm{Z}}_{\mathrm{ST}}=\hat{\mathrm{Z}}_{\sigma \mathrm{S}}+\hat{Z}_{\sigma T}-\text { recalculated to } U_{\mathrm{PN}}, S_{\mathrm{PN}}
\end{aligned}
$$

$\Delta \mathrm{P}_{\mathrm{k}}$ for $\mathrm{I}_{\mathrm{Tn}} \rightarrow \Delta \mathrm{P}_{\mathrm{kST}}=3 \cdot \mathrm{R}^{+}{ }_{\mathrm{ST}} \cdot \mathrm{I}^{2} \mathrm{Tn}, \quad \mathrm{I}_{\mathrm{Tn}}=\frac{\mathrm{S}_{\mathrm{Tn}}}{\sqrt{3} \cdot \mathrm{U}_{\mathrm{Tn}}}$
$\mathrm{R}^{+}$st....resistance of secondary and tertiary windings (related to $\mathrm{U}_{\mathrm{Tn}}$ )

$$
\begin{aligned}
& \mathrm{R}^{+}{ }_{\mathrm{ST}}=\frac{\Delta \mathrm{P}_{\mathrm{kST}}}{\mathrm{~S}_{\mathrm{Tn}}^{2}} \cdot \mathrm{U}_{\mathrm{Tn}}^{2} \\
& \mathrm{R}_{\mathrm{ST}}=\mathrm{R}^{+}{ }_{\mathrm{ST}} \cdot \frac{\mathrm{U}_{\mathrm{Pn}}^{2}}{\mathrm{U}_{\mathrm{Tn}}^{2}} \rightarrow \mathrm{R}_{\mathrm{ST}}=\mathrm{R}_{\mathrm{S}}+\mathrm{R}_{\mathrm{T}}=\frac{\Delta \mathrm{P}_{\mathrm{kST}}}{\mathrm{~S}_{\mathrm{Tn}}^{2}} \cdot \mathrm{U}_{\mathrm{Pn}}^{2}
\end{aligned}
$$

$\mathrm{R}_{\mathrm{S}}\left(\mathrm{R}_{\mathrm{T}}\right)$...resistance of sec. and ter. windings recalculated to primary

$$
\mathrm{r}_{\mathrm{ST}}=\mathrm{R}_{\mathrm{ST}} \cdot \frac{\mathrm{~S}_{\mathrm{PN}}}{\mathrm{U}_{\mathrm{Pn}}^{2}}=\frac{\Delta \mathrm{P}_{\mathrm{kST}}}{\mathrm{~S}_{\mathrm{Tn}}^{2}} \cdot \mathrm{~S}_{\mathrm{Pn}}
$$

- impedance:

$$
\begin{aligned}
& \mathrm{z}_{\mathrm{ST}}=\frac{\mathrm{u}_{\mathrm{kST} \%}}{100} \cdot \frac{\mathrm{~S}_{\mathrm{Pn}}}{\mathrm{~S}_{\mathrm{Tn}}}, \mathrm{Z}_{\mathrm{ST}}=\mathrm{z}_{\mathrm{ST}} \cdot \frac{\mathrm{U}_{\mathrm{Pn}}^{2}}{\mathrm{~S}_{\mathrm{Pn}}}=\frac{\mathrm{u}_{\mathrm{kST} \%}}{100} \cdot \frac{\mathrm{U}_{\mathrm{Pn}}^{2}}{\mathrm{~S}_{\mathrm{Tn}}} \\
& \hat{\mathrm{z}}_{\mathrm{ST}}=\mathrm{r}_{\mathrm{ST}}+\mathrm{j} \cdot \mathrm{x}_{\mathrm{ST}}, \mathrm{x}_{\mathrm{ST}}=\sqrt{\mathrm{z}_{\mathrm{ST}}^{2}-\mathrm{r}_{\mathrm{ST}}^{2}}, \quad \mathrm{x}_{\mathrm{ST}}=\mathrm{x}_{\sigma \mathrm{S}}+\mathrm{x}_{\sigma \mathrm{T}}
\end{aligned}
$$

- based on the derived relations we can write:

P-S:

$$
\begin{aligned}
& \hat{Z}_{\mathrm{PS}}=\mathrm{r}_{\mathrm{PS}}+\mathrm{j} \cdot \mathrm{x}_{\mathrm{PS}}=\frac{\Delta \mathrm{P}_{\mathrm{kPS}}}{\mathrm{~S}_{\mathrm{Sn}}^{2}} \cdot \mathrm{~S}_{\mathrm{Pn}}+\mathrm{j} \cdot \sqrt{\left(\frac{\mathrm{u}_{\mathrm{kPS} \%}}{100} \cdot \frac{\mathrm{~S}_{\mathrm{Pn}}}{\mathrm{~S}_{\mathrm{Sn}}}\right)^{2}-\left(\frac{\Delta \mathrm{P}_{\mathrm{kPS}}}{\mathrm{~S}_{\mathrm{Sn}}^{2}} \cdot \mathrm{~S}_{\mathrm{Pn}}\right)^{2}} \\
& \hat{Z}_{\mathrm{PS}}=R_{\mathrm{PS}}+j \cdot X_{\mathrm{PS}}=\frac{\Delta \mathrm{P}_{\mathrm{kPS}}}{\mathrm{~S}_{\mathrm{Sn}}^{2}} \cdot \mathrm{U}_{\mathrm{Pn}}^{2}+\mathrm{j} \cdot \sqrt{\left(\frac{\mathrm{u}_{\mathrm{kPS} \%}}{100} \cdot \frac{\mathrm{U}_{\mathrm{Pn}}^{2}}{\mathrm{~S}_{\mathrm{Sn}}}\right)^{2}-\left(\frac{\Delta \mathrm{P}_{\mathrm{kPS}}}{\mathrm{~S}_{\mathrm{Sn}}^{2}} \cdot \mathrm{U}_{\mathrm{Pn}}^{2}\right)^{2}}
\end{aligned}
$$

- analogous for $\mathrm{P}-\mathrm{T}$ and $\mathrm{S}-\mathrm{T}$
- leakage reactances for $\mathrm{P}, \mathrm{S}, \mathrm{T}$ :

$$
\begin{aligned}
& \hat{Z}_{\sigma \mathrm{P}}=\mathrm{R}_{\mathrm{P}}+\mathrm{j} \cdot \mathrm{X}_{\sigma \mathrm{CP}}=0,5 \cdot\left(\hat{\mathrm{Z}}_{\mathrm{PS}}+\hat{\mathrm{Z}}_{\mathrm{PT}}-\hat{\mathrm{Z}}_{\mathrm{ST}}\right) \\
& \hat{Z}_{\sigma \mathrm{S}}=\mathrm{R}_{\mathrm{S}}+\mathrm{j} \cdot \mathrm{X}_{\sigma \mathrm{S}}=0,5 \cdot\left(\hat{\mathrm{Z}}_{\mathrm{PS}}+\hat{\mathrm{Z}}_{\mathrm{ST}}-\hat{Z}_{\mathrm{PT}}\right) \\
& \hat{Z}_{\sigma \mathrm{T}}=\mathrm{R}_{\mathrm{T}}+\mathrm{j} \cdot \mathrm{X}_{\sigma \mathrm{T}}=0,5 \cdot\left(\hat{\mathrm{Z}}_{\mathrm{PT}}+\hat{Z}_{\mathrm{ST}}-\hat{Z}_{\mathrm{PS}}\right)
\end{aligned}
$$

- knowledge of the series impedances and shunt admittances allows to study voltage and power conditions of 3-winding transformers


## Symmetrical system components

Decomposition of unsymmetrical (unbalanced) voltage:

$$
\begin{aligned}
& \hat{\mathrm{U}}_{\mathrm{A}}=\hat{\mathrm{U}}_{\mathrm{A} 1}+\hat{\mathrm{U}}_{\mathrm{A} 2}+\hat{\mathrm{U}}_{\mathrm{A} 0} \\
& \hat{\mathrm{U}}_{\mathrm{B}}=\hat{\mathrm{U}}_{\mathrm{B} 1}+\hat{\mathrm{U}}_{\mathrm{B} 2}+\hat{\mathrm{U}}_{\mathrm{B} 0} \\
& \hat{\mathrm{U}}_{\mathrm{C}}=\hat{\mathrm{U}}_{\mathrm{C} 1}+\hat{\mathrm{U}}_{\mathrm{C} 2}+\hat{\mathrm{U}}_{\mathrm{C} 0}
\end{aligned}
$$




Positive sequence (1), negative (2) and zero (0) sequence.
Hence (reference phase A)

$$
\begin{array}{cl}
\hat{U}_{A}=\hat{U}_{1}+\hat{U}_{2}+\hat{U}_{0} & \hat{\mathrm{I}}_{A}=\hat{\mathrm{I}}_{1}+\hat{\mathrm{I}}_{2}+\hat{\mathrm{I}}_{0} \\
\hat{U}_{B}=\hat{a}^{2} \hat{U}_{1}+\hat{a} \hat{\mathrm{U}}_{2}+\hat{U}_{0} & \hat{\mathrm{I}}_{B}=\hat{a}^{2} \hat{\mathrm{I}}_{1}+\hat{\mathrm{a}} \hat{\mathrm{I}}_{2}+\hat{\mathrm{I}}_{0} \\
\hat{U}_{C}=\hat{a} \hat{\mathrm{U}}_{1}+\hat{a}^{2} \hat{U}_{2}+\hat{U}_{0} & \hat{\mathrm{I}}_{C}=\hat{a}_{1}+\hat{\mathrm{a}}^{2} \hat{\mathrm{I}}_{2}+\hat{\mathrm{I}}_{0} \\
\text { where } \hat{\mathrm{a}}=-\frac{1}{2}+\mathrm{j} \frac{\sqrt{3}}{2}=\mathrm{e}^{\mathrm{j} \frac{2 \pi}{3}} & \hat{a}^{2}=-\frac{1}{2}-\mathrm{j} \frac{\sqrt{3}}{2}=e^{\mathrm{j} \frac{4 \pi}{3}}
\end{array}
$$

Matrix

$$
\left(\mathrm{U}_{\mathrm{ABC}}\right)=\left(\begin{array}{l}
\hat{\mathrm{U}}_{\mathrm{A}} \\
\hat{\mathrm{U}}_{\mathrm{B}} \\
\hat{\mathrm{U}}_{\mathrm{C}}
\end{array}\right)=\left(\begin{array}{ccc}
1 & 1 & 1 \\
\hat{\mathrm{a}}^{2} & \hat{a} & 1 \\
\hat{\mathrm{a}} & \hat{a}^{2} & 1
\end{array}\right)\binom{\hat{\mathrm{U}}_{1}}{\hat{\mathrm{U}}_{2}}=(\mathrm{T})\left(\mathrm{U}_{120}\right)
$$

Inversely

$$
\left(\mathrm{U}_{120}\right)=\left(\begin{array}{l}
\hat{\mathrm{U}}_{1} \\
\hat{\mathrm{U}}_{2} \\
\hat{\mathrm{U}}_{0}
\end{array}\right)=\frac{1}{3}\left(\begin{array}{ccc}
1 & \hat{\mathrm{a}} & \hat{\mathrm{a}}^{2} \\
1 & \hat{\mathrm{a}}^{2} & \hat{\mathrm{a}} \\
1 & 1 & 1
\end{array}\right)\left(\begin{array}{c}
\hat{\mathrm{U}}_{\mathrm{A}} \\
\hat{\mathrm{U}}_{\mathrm{B}} \\
\hat{\mathrm{U}}_{\mathrm{C}}
\end{array}\right)=\left(\mathrm{T}^{-1}\right)\left(\mathrm{U}_{\mathrm{ABC}}\right)
$$

## Series symmetrical segments in ES

$$
\begin{aligned}
& \left(\begin{array}{l}
\Delta \hat{U}_{\mathrm{A}} \\
\Delta \hat{U}_{\mathrm{B}} \\
\Delta \hat{\mathrm{U}}_{\mathrm{C}}
\end{array}\right)=\left(\begin{array}{ccc}
\hat{\mathrm{Z}} & \hat{Z}^{\prime} & \hat{\mathrm{Z}}^{\prime} \\
\hat{Z}^{\prime} & \hat{\mathrm{Z}} & \hat{Z}^{\prime} \\
\hat{Z}^{\prime} & \hat{\mathrm{Z}}^{\prime} & \hat{\mathrm{Z}}
\end{array}\right)\left(\begin{array}{l}
\hat{\mathrm{I}}_{\mathrm{A}} \\
\hat{\mathrm{I}}_{\mathrm{B}} \\
\hat{\mathrm{I}}_{\mathrm{C}}
\end{array}\right) \\
& \left(\Delta \mathrm{U}_{\mathrm{ABC}}\right)=\left(\mathrm{Z}_{\mathrm{ABC}}\right)\left(\mathrm{I}_{\mathrm{ABC}}\right) \\
& (\mathrm{T})\left(\Delta \mathrm{U}_{120}\right)=\left(\mathrm{Z}_{\mathrm{ABC}}\right)(\mathrm{T})\left(\mathrm{I}_{120}\right) \\
& \left(\Delta \mathrm{U}_{120}\right)=(\mathrm{T})^{-1}\left(\mathrm{Z}_{\mathrm{ABC}}\right)(\mathrm{T})\left(\mathrm{I}_{120}\right)=\left(\mathrm{Z}_{120}\right)\left(\mathrm{I}_{120}\right) \\
& \left(\mathrm{Z}_{120}\right)=(\mathrm{T})^{-1}\left(\mathrm{Z}_{\mathrm{ABC}}\right)(\mathrm{T}) \\
& \left(\mathrm{Z}_{120}\right)=\left(\begin{array}{ccc}
\hat{\mathrm{Z}}_{1} & 0 & 0 \\
0 & \hat{Z}_{2} & 0 \\
0 & 0 & \hat{\mathrm{Z}}_{0}
\end{array}\right)=\left(\begin{array}{ccc}
\hat{\mathrm{Z}}-\hat{\mathrm{Z}}^{\prime} & 0 & 0 \\
0 & \hat{Z}-\hat{\mathrm{Z}}^{\prime} & 0 \\
0 & 0 & \hat{Z}+2 \hat{Z}^{\prime}
\end{array}\right)
\end{aligned}
$$

## Shunt symmetrical segments in ES

$$
\begin{aligned}
& \left(\mathrm{U}_{\mathrm{ABC}}\right)=\left(\mathrm{Z}_{\mathrm{ABC}}\right)\left(\mathrm{I}_{\mathrm{ABC}}\right)+\left(\mathrm{Z}_{\mathrm{N}}\right)\left(\mathrm{I}_{\text {ABC }}\right) \\
& \left(Z_{N}\right)=\left(\begin{array}{lll}
\hat{Z}_{N} & \hat{Z}_{N} & \hat{Z}_{N} \\
\hat{Z}_{N} & \hat{Z}_{N} & \hat{Z}_{N} \\
\hat{Z}_{N} & \hat{Z}_{\mathrm{N}} & \hat{Z}_{\mathrm{N}}
\end{array}\right) \\
& \left(\mathrm{U}_{120}\right)=(\mathrm{T})^{-1}\left(\mathrm{Z}_{\mathrm{ABC}}\right)(\mathrm{T})\left(\mathrm{I}_{120}\right)+(\mathrm{T})^{-1}\left(\mathrm{Z}_{\mathrm{N}}\right)(\mathrm{T})\left(\mathrm{I}_{120}\right) \\
& \left(\mathrm{Z}_{120}\right)=(\mathrm{T})^{-1}\left[\left(\mathrm{Z}_{\mathrm{ABC}}\right)+\left(\mathrm{Z}_{\mathrm{N}}\right)\right](\mathrm{T}) \\
& \left(Z_{120}\right)=\left(\begin{array}{ccc}
\hat{Z}-\hat{Z}^{\prime} & 0 & 0 \\
0 & \hat{Z}-\hat{Z}^{\prime} & 0 \\
0 & 0 & \hat{Z}+2 \hat{Z}^{\prime}+3 Z_{N}
\end{array}\right)
\end{aligned}
$$



Symmetrical components voltages in the symmetrical segments depend only on the corresponding component current and component impedance.

## Transformers zero sequence impedances



Series parameters are the same as for the positive sequence, the shunt always need to be determined.
Assumptions:

- Zero sequence voltage supplies the primary winding.
- The relative values are related to UPN and SpN.
- We distinguish free and tied magnetic flows (core x shell TRF).
$\mathrm{Z}_{0}$ depends on the winding connection.


## a) $\mathbf{Y} /$ any connection

$$
\begin{aligned}
& 3 \mathrm{i}_{0}=0 \\
& \mathrm{z}_{0}=\frac{\mathrm{u}_{0}}{\mathrm{i}_{0}} \rightarrow \infty \\
& \mathrm{z}_{0 \mathrm{p} 0} \rightarrow \infty \\
& \mathrm{z}_{0 \mathrm{ps}} \rightarrow \infty
\end{aligned}
$$



Secondary winding

b) D / any connection

Zero sequence voltage is attached to $\mathrm{D} \rightarrow$ voltage at each phase $\mathrm{u}_{0}-\mathrm{u}_{0}=0 \rightarrow \mathrm{i}_{\mathrm{a}}=\mathrm{i}_{\mathrm{b}}=\mathrm{i}_{\mathrm{c}}=0 \rightarrow \mathrm{i}_{0}=0$

$$
\begin{aligned}
& \mathrm{Z}_{0}=\frac{\mathrm{u}_{0}}{\mathrm{i}_{0}} \rightarrow \infty \\
& \mathrm{Z}_{0 \mathrm{p} 0} \rightarrow \infty \\
& \mathrm{Z}_{0 \mathrm{ps}} \rightarrow \infty
\end{aligned}
$$



## c) $\mathrm{YN} / \mathrm{D}$

Currents in the primary winding $i_{0}$ induce currents $i_{0}{ }^{\prime}$ in the secondary winding to achieve magnetic balance.
Currents $\mathrm{i}_{0}{ }^{\prime}$ in the secondary winding are short-closed and do not flow further into the grid.
$\hat{\mathrm{z}}_{\mathrm{op} 0}=\hat{\mathrm{z}}_{\mathrm{op}}+\hat{\mathrm{z}}_{\mathrm{q} 0}$
$\hat{\mathrm{z}}_{0}=\frac{\hat{\mathrm{u}}_{0}}{\hat{\mathrm{i}}_{0}}=\hat{\mathrm{z}}_{\mathrm{\sigma p}}+\frac{\hat{\mathrm{z}}_{\text {os }} \cdot \hat{\mathrm{z}}_{\mathrm{q} 0}}{\hat{\mathrm{z}}_{\text {os }}+\hat{\mathrm{z}}_{\mathrm{q} 0}}$
shell

$$
\hat{z}_{q 0}=\hat{y}_{q}^{-1} \gg \hat{\mathrm{z}}_{\text {os }} \rightarrow \hat{\mathrm{z}}_{0} \approx \hat{\mathrm{z}}_{\text {ops }}=\hat{\mathrm{z}}_{1 \mathrm{k}}
$$

3-core

$$
\left|\hat{z}_{\mathrm{q} 0}\right|<\left|\hat{y}_{\mathrm{q}}^{-1}\right| \rightarrow\left|\hat{\mathrm{z}}_{0}\right| \approx(0,7 \div 0,9)\left|\hat{\mathrm{z}}_{\mathrm{Gps}}\right|
$$


d) $\mathbf{Y N} / \mathbf{Y}$

Zero sequence current can't flow through the secondary winding. Current io corresponds to the magnetization current.

$$
\begin{aligned}
& \mathrm{z}_{0 \mathrm{ps}} \rightarrow \infty \\
& \hat{\mathrm{z}}_{0}=\hat{\mathrm{z}}_{0 \mathrm{p} 0}=\hat{\mathrm{z}}_{\sigma \mathrm{p}}+\hat{\mathrm{z}}_{\mathrm{q} 0} \\
& \text { shell } \\
& \quad \hat{\mathrm{z}}_{\mathrm{q} 0}=\hat{\mathrm{y}}_{\mathrm{q}}^{-1} \rightarrow \mathrm{z}_{0} \rightarrow \infty
\end{aligned}
$$

3-core

$$
\left|\hat{\mathrm{z}}_{\mathrm{q} 0}\right|<\left|\hat{\mathrm{y}}_{\mathrm{q}}^{-1}\right| \rightarrow\left|\hat{\mathrm{z}}_{0}\right| \approx(0,3 \div 1)
$$


e)


## e) $\mathbf{Y N} / \mathbf{Y N}$

If element with YN or ZN behind TRF $\rightarrow$ points a-b are connected $\rightarrow$ as the positive sequence. If element with $\mathrm{Y}, \mathrm{Z}$ or D behind TRF $\rightarrow \mathrm{a}-\mathrm{b}$ are disconnected $\rightarrow$ as YN / Y.

Primary winding Secondary winding


## f) ZN / any connection

Currents $i_{0}$ induce mag. balance on the core themselves $\rightarrow$ only leakages between the halves of the windings.
$\mathrm{z}_{\text {0ps }} \rightarrow \infty$
$\hat{\mathrm{z}}_{0}=\hat{\mathrm{z}}_{0 \mathrm{p} 0} \approx(0,1 \div 0,3) \hat{\mathrm{z}}_{\text {ops }}$
$\mathrm{r}_{0}=\mathrm{r}_{\mathrm{p}}$

g) impedance in the neutral point

Current flowing through the neutral point is 3 i .
Voltage drop: $\quad \Delta \hat{\mathrm{u}}_{\mathrm{uz}}=\hat{\mathrm{z}}_{\mathrm{u}} \cdot 3 \hat{\mathrm{i}}_{0}=3 \hat{\mathrm{z}}_{\mathrm{u}} \cdot \hat{\mathrm{i}}_{0}$

## h) three-winding TRF



## System equivalent

Impedance (positive sequence) is given by the nominal voltage and shortcircuit current (power).
Three-phase (symmetrical) short-circuit: $\mathrm{S}_{\mathrm{k}}^{\prime \prime}$ (MVA), $\mathrm{I}_{\mathrm{k}}^{\prime \prime}(\mathrm{kA})$

$$
\begin{aligned}
& \mathrm{S}_{\mathrm{k}}^{\prime \prime}=\sqrt{3} \mathrm{U}_{\mathrm{n}} \mathrm{I}_{\mathrm{k}}^{\prime \prime} \\
& \mathrm{Z}_{\mathrm{s}}=\frac{\mathrm{U}_{\mathrm{n}}^{2}}{\mathrm{~S}_{\mathrm{k}}^{\prime \prime}}=\frac{\mathrm{U}_{\mathrm{n}}}{\sqrt{3} \cdot \mathrm{I}_{\mathrm{k}}^{\prime \prime}}
\end{aligned}
$$

CR: $\quad 400 \mathrm{kV} \quad \mathrm{S}_{\mathrm{k}}^{\prime \prime} \approx(6000 \div 30000) \mathrm{MVA} \quad \mathrm{I}_{\mathrm{k}}^{\prime \prime} \approx(9 \div 45) \mathrm{kA}$
$220 \mathrm{kV} \quad \mathrm{S}_{\mathrm{k}}^{\prime \prime} \approx(2000 \div 12000)$ MVA $\quad \mathrm{I}_{\mathrm{k}}^{\prime \prime} \approx(2 \div 30) \mathrm{kA}$
$110 \mathrm{kV} \quad \mathrm{S}_{\mathrm{k}}^{\prime \prime} \approx(100 \mathrm{x} \div 3000) \mathrm{MVA} \quad \mathrm{I}_{\mathrm{k}}^{\prime \prime} \approx(\mathrm{x} \div 15) \mathrm{kA}$

