STEADY STATES CALCULATIONS IN POWER SYSTEMS

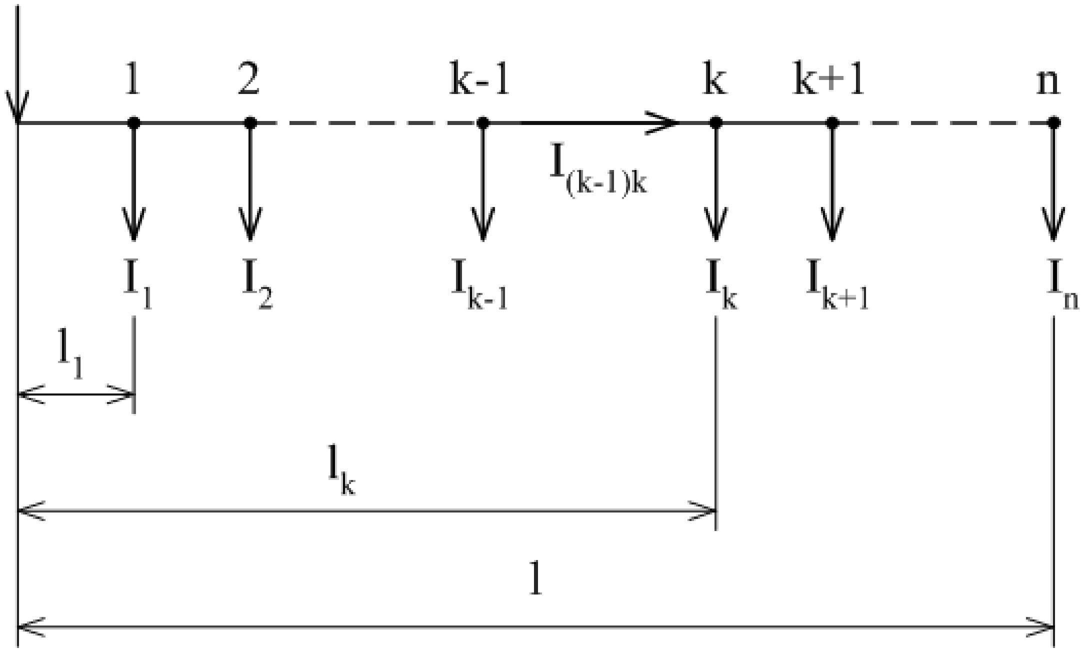
Current loads

Simple DC line (LV, MV)

Double-wire circuit. Assumption: constant cross-section and resistivity.

Single loads supplied from one side

Standard distribution lines.



a) addition method

It adds voltage drops along the power line sections.
(Voltage drops are always in both conductors in the section.)

k^{th} section

$$U_{(k-1)} - U_k = \Delta U_{(k-1)k} = 2\frac{\rho}{S}\left(l_k - l_{(k-1)}\right)\cdot I_{(k-1)k} \quad (\mathrm{V};\ \Omega\mathrm{m},\ \mathrm{m}^2,\ \mathrm{m},\ \mathrm{A})$$

Current in k^{th} section

$$I_{(k-1)k} = \sum_{y=k}^{n} I_y$$

Maximum voltage drop

$$\Delta U_n = \sum_{k=1}^{n} \Delta U_{(k-1)k} = 2\frac{\rho}{S}\sum_{k=1}^{n}\left(l_k - l_{(k-1)}\right)\cdot \sum_{y=k}^{n} I_y$$

b) superposition method

It adds voltage drops for individual discrete loads:

$$\Delta U_n = 2\frac{\rho}{S}\sum_{k=1}^{n} l_k I_k$$

$l_k I_k$ …current moments to the feeder

Relative voltage drop:

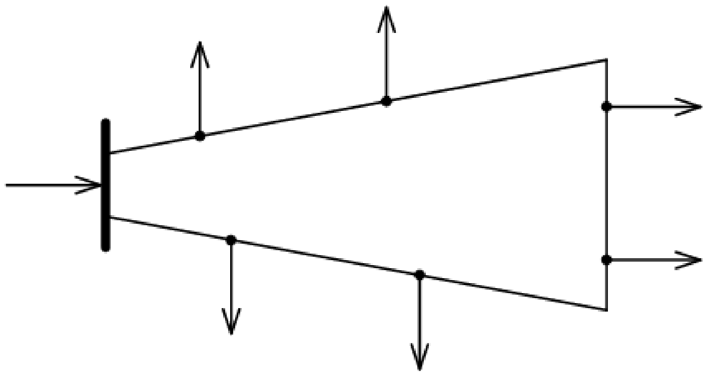
$$\varepsilon = \frac{\Delta U}{U_n} \quad (-;\ V,\ V)$$

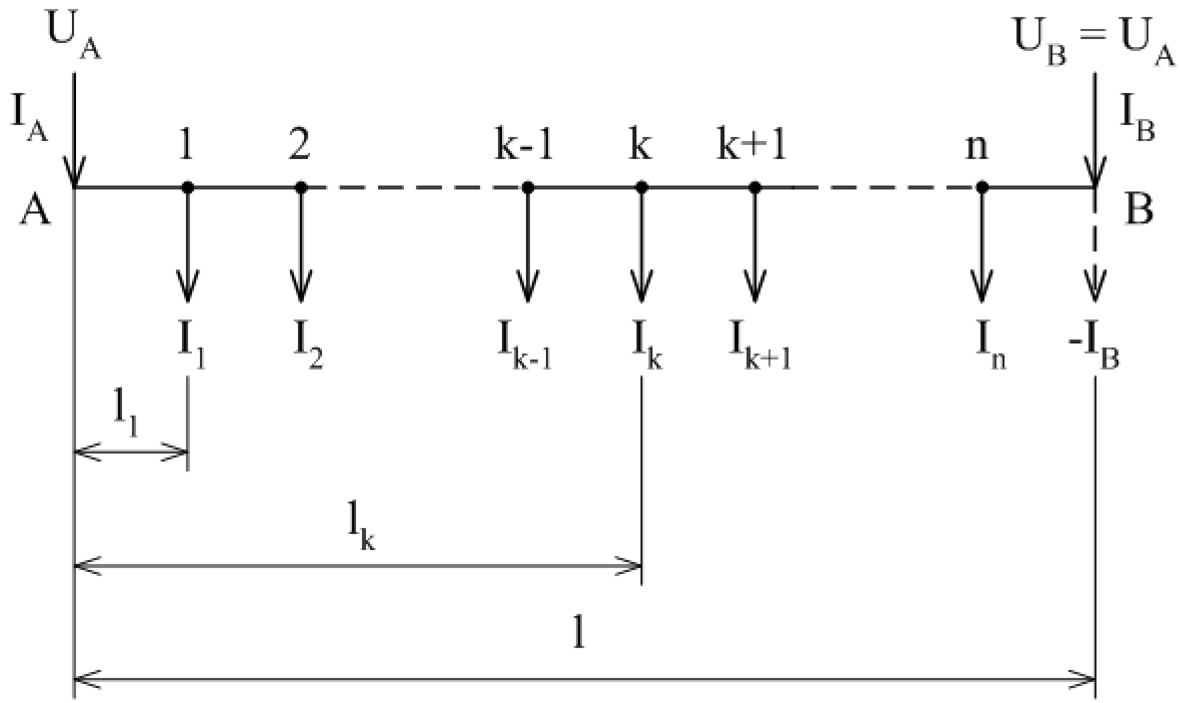
Note. Losses must be calculated only by means of the addition method!

$$\Delta P_{(k-1)k} = 2\frac{\rho}{S}\left(l_k - l_{(k-1)}\right)\cdot I_{(k-1)k}^2 \quad (W;\ \Omega m,\ m^2,\ m,\ A)$$

$$\Delta P = \sum_{k=1}^{n} \Delta P_{(k-1)k}$$

Single loads supplied from both sides – the same feeders voltages





- Ring grid, higher reliability of supply.
- Two one-feeder lines after a fault. More often also in standard operation mode.
- Calculation of current distribution and voltage drops.

Consider I_B as a negative load:

$$\Delta U_{AB} = U_A - U_B = 0 = 2\frac{\rho}{S}\sum_{k=1}^{n} l_k I_k - 2\frac{\rho}{S} l I_B$$

Hence (moment theorem)

$$I_B = \frac{\sum_{k=1}^{n} l_k I_k}{l}$$

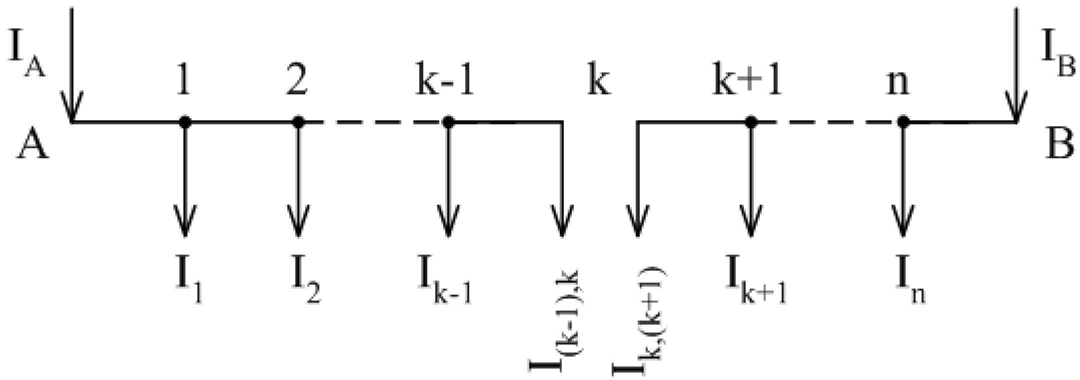
Analogous (current moments to other feeder)

$$I_A = \frac{\sum_{k=1}^{n} (l - l_k) I_k}{l}$$

Of course

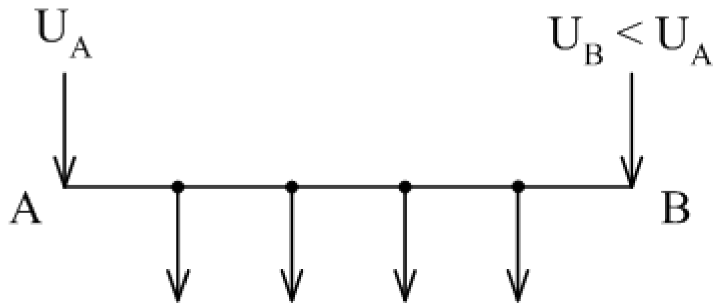
$$I_A + I_B = \sum_{y=1}^{n} I_y$$

Current distribution identifies the place with the biggest voltage drop = the place with feeder division → split-up into two one-feeder lines.



Single loads supplied from both sides – different feeders voltages

Two different sources, meshed grid.



Superposition:

1) Current distribution with the same voltages.
2) Different voltages and zero loads → balancing current

$$I_v = \frac{U_A - U_B}{2\frac{\rho}{S}l}$$

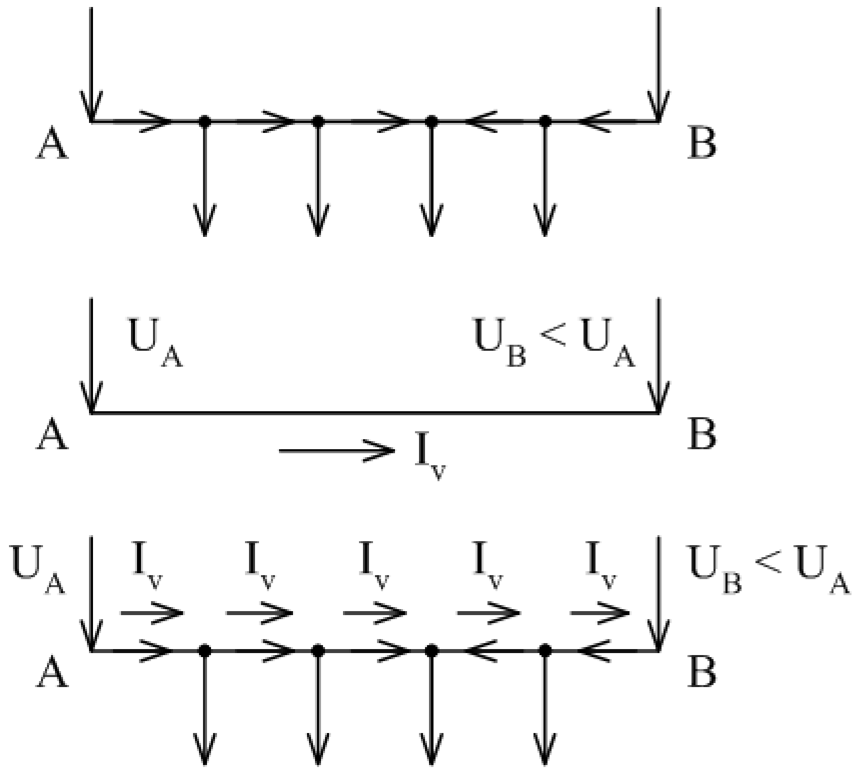
3) Sum of the solutions 1+2

Further calculation is the same.

Or directly:

$$U_A - U_B = 2\frac{\rho}{S}\sum_{k=1}^{n} l_k I_k - 2\frac{\rho}{S} l I_B$$

$$I_B = \frac{2\frac{\rho}{S}\sum_{k=1}^{n} l_k I_k}{2\frac{\rho}{S}l} - \frac{U_A - U_B}{2\frac{\rho}{S}l}$$

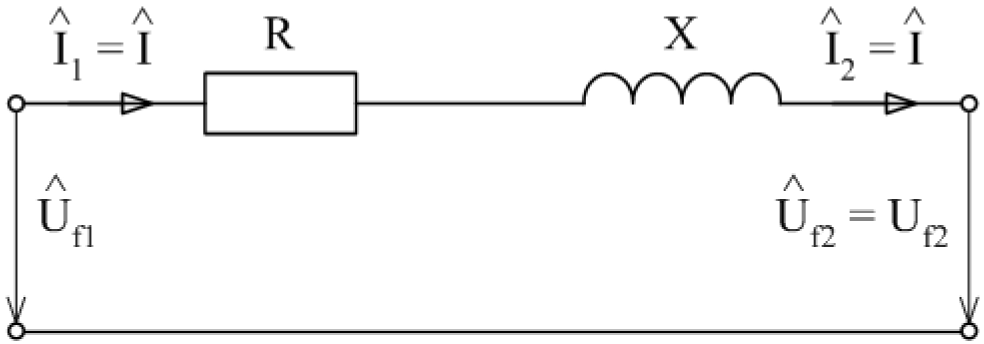


AC - 3 phase power lines LV, MV

Series parameters are applied, for LV X→0.

3 phase power line MV, 1 load at the end

Symmetrical load → 1 phase diagram, operational parameters.



Complex voltage drop

$$\Delta\hat{U}_{ph} = \hat{Z}_l\hat{I} = (R + jX)(I_{re} \mp jI_{im}) \begin{matrix} \text{IND} \\ \text{CAP} \end{matrix}$$

$$\Delta\hat{U}_{ph} = RI_{re} \pm XI_{im} + j(XI_{re} \mp RI_{im}) \begin{matrix} \text{IND} \\ \text{CAP} \end{matrix}$$

magnitude phase

Phasor diagram (input U_{ph2}, I, φ_2)
(angle υ usually small, up to 3°)

Imagin. part neglecting and modifications

$$\Delta U_{ph} = \frac{R3U_{ph}I_{re} \pm X3U_{ph}I_{im}}{3U_{ph}} = \frac{RP \pm XQ}{3U_{ph}}$$

Percentage voltage drop

$$\varepsilon = \frac{\Delta U_{ph}}{U_{ph}} = \frac{RP \pm XQ}{3U_{ph}^2} = \frac{RP \pm XQ}{U^2}$$

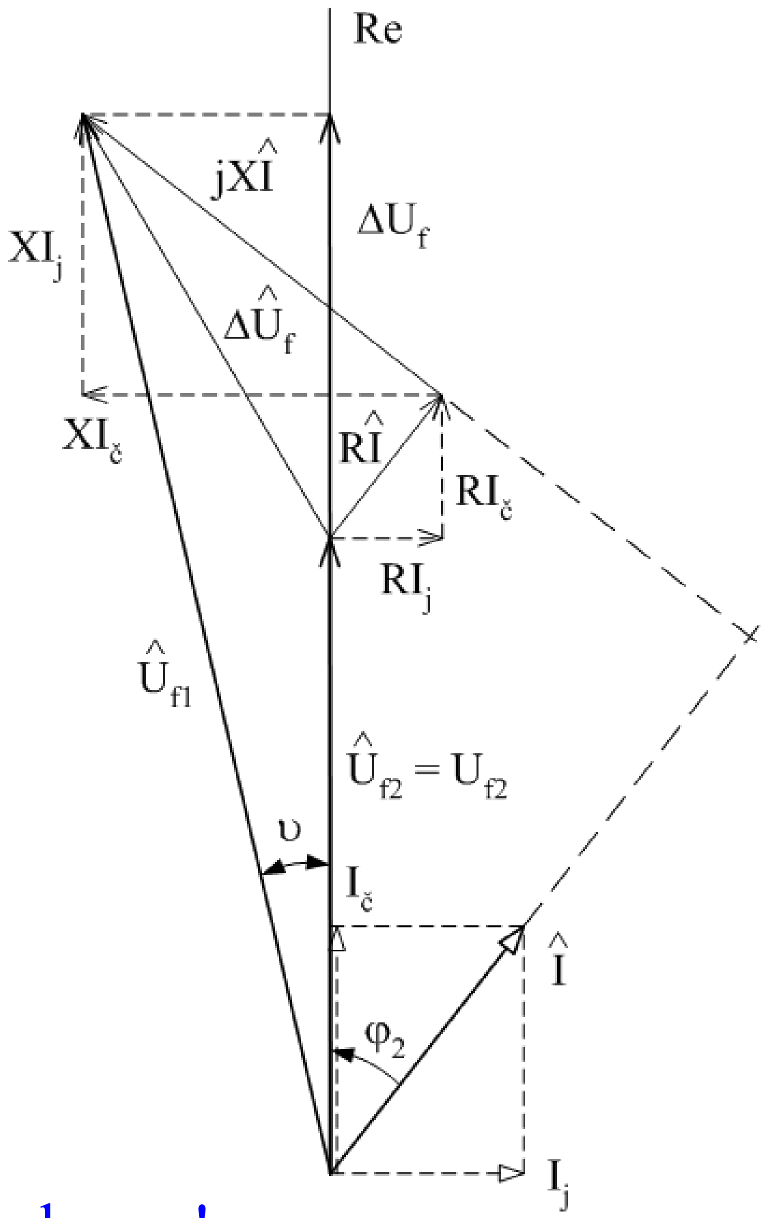
3 phase active power losses

$$\Delta\hat{S} = 3\Delta\hat{U}_{ph}\hat{I}^* = 3\hat{Z}_1\hat{I}\cdot\hat{I}^* = 3\hat{Z}_1 I^2 =$$

$$= 3(R + jX)I^2 = 3RI^2 + j3XI^2$$

$$\Delta P = 3RI^2 = 3R\left(I_{re}^2 + I_{im}^2\right) \quad (W; \Omega, A)$$

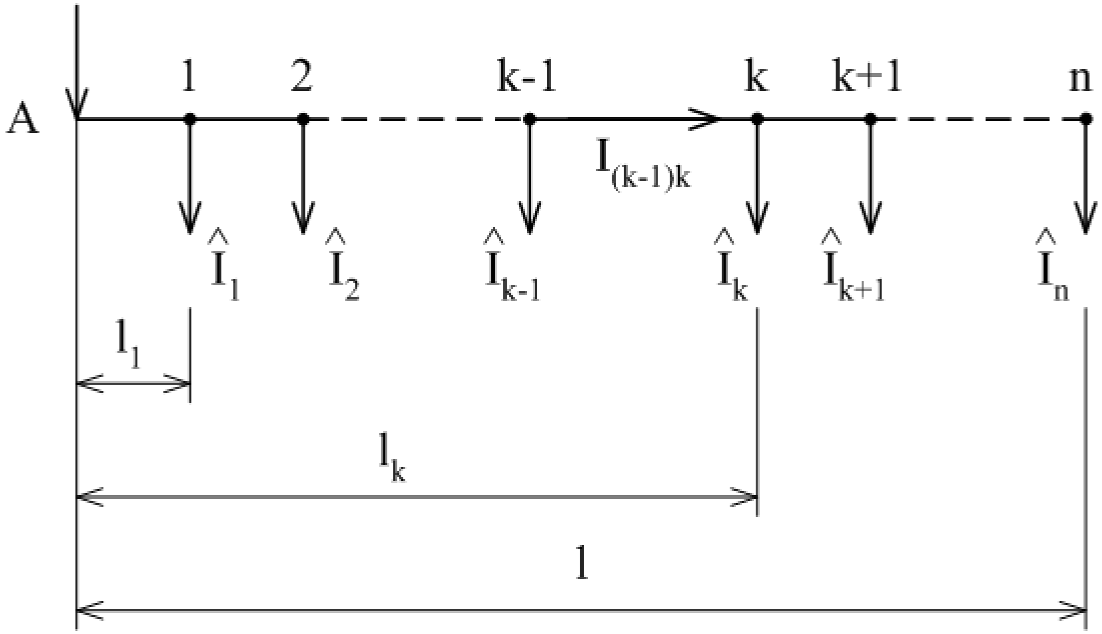
! Even the reactive current causes active power losses!



3 phase MV power line supplied from one side

Constant series impedance

$$\hat{Z}_{l_1} = R_1 + jX_1 \quad (\Omega / km)$$



Voltage drop at the end (needn't be the highest one, it depends on load character)

- superposition $$\Delta\hat{U}_{phAn} = \hat{Z}_{l_1}\sum_{k=1}^{n} l_k \hat{I}_k$$
- addition $$\Delta\hat{U}_{phAn} = \hat{Z}_{l_1}\sum_{k=1}^{n}\left(l_k - l_{(k-1)}\right)\cdot\sum_{y=k}^{n}\hat{I}_y$$

After imaginary part neglecting

$$\Delta U_{phAn} \doteq R_1\sum_{k=1}^{n}\left(l_k - l_{(k-1)}\right)\cdot\sum_{y=k}^{n} I_{rek} \pm X_1\sum_{k=1}^{n}\left(l_k - l_{(k-1)}\right)\cdot\sum_{y=k}^{n} I_{imk} \quad \begin{matrix}\text{IND}\\ \text{CAP}\end{matrix}$$

$$\Delta U_{phAn} \doteq \frac{R_1\sum_{k=1}^{n}\left(l_k - l_{(k-1)}\right)\cdot\sum_{y=k}^{n} P_k \pm X_1\sum_{k=1}^{n}\left(l_k - l_{(k-1)}\right)\cdot\sum_{y=k}^{n} Q_k}{3U_{ph}} \quad \begin{matrix}\text{IND}\\ \text{CAP}\end{matrix}$$

Voltage drop up to the point X (not end)

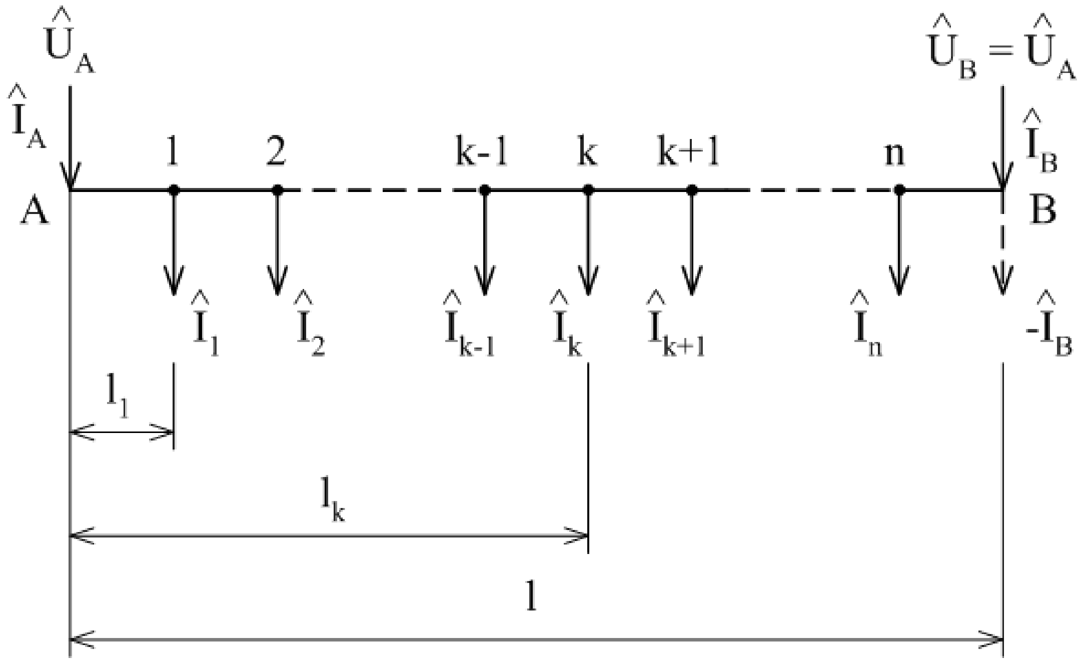
- superposition

$$\Delta\hat{U}_{phAX} = \hat{Z}_{l_1}\sum_{k=1}^{X} l_k \hat{I}_k + \hat{Z}_{l_1} l_{AX} \sum_{k=X+1}^{n} \hat{I}_k$$

- addition

$$\Delta\hat{U}_{phAX} = \hat{Z}_{l_1}\sum_{k=1}^{X} \left(l_k - l_{(k-1)}\right) \cdot \sum_{y=k}^{n} \hat{I}_y$$

<u>3 phase MV power line supplied from both sides</u>



Calculation as for DC line (feeder is a negative load, zero voltage drop).

$$\Delta\hat{U}_{phAB} = 0 = \hat{Z}_{l_1}\sum_{k=1}^{n} l_k \hat{I}_k - \hat{Z}_{l_1} l \cdot \hat{I}_B$$

Moment theorems

$$\hat{I}_B = \frac{\sum_{k=1}^{n} l_k \hat{I}_k}{l} \qquad \hat{I}_A = \frac{\sum_{k=1}^{n} (l - l_k)\hat{I}_k}{l} \qquad \hat{I}_A + \hat{I}_B = \sum_{y=1}^{n} \hat{I}_y$$

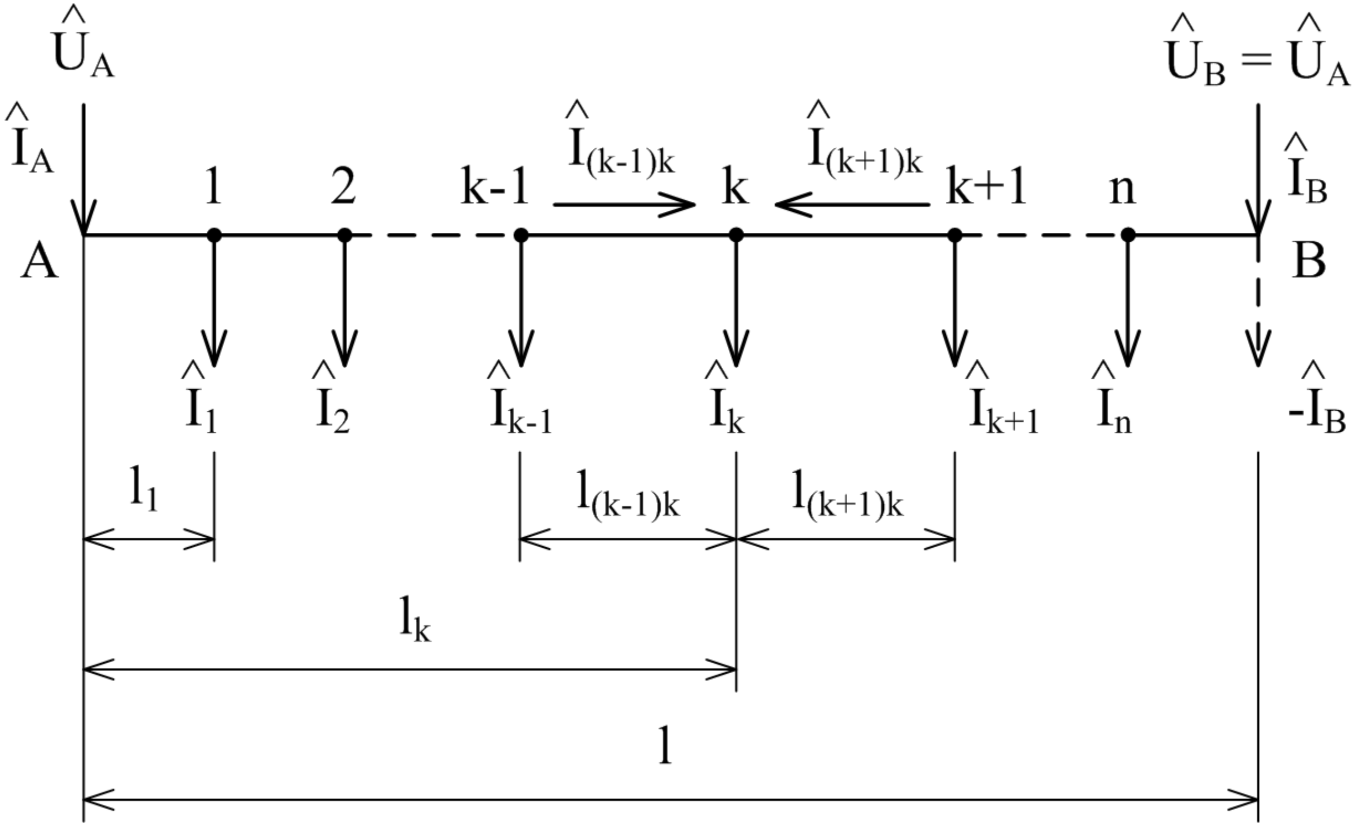
(In principle it is the current divider for each load.)

Active and reactive current sign change could be in different nodes → maximum voltage drop should be checked in all grid points.

- addition

$$\Delta\hat{U}_{phAX} = \hat{Z}_{l_1} \sum_{k=1}^{X} \left(l_k - l_{(k-1)}\right) \cdot \hat{I}_{(k-1)k} = \hat{Z}_{l_1} \sum_{k=1}^{X} l_{(k-1)k} \cdot \hat{I}_{(k-1)k}$$

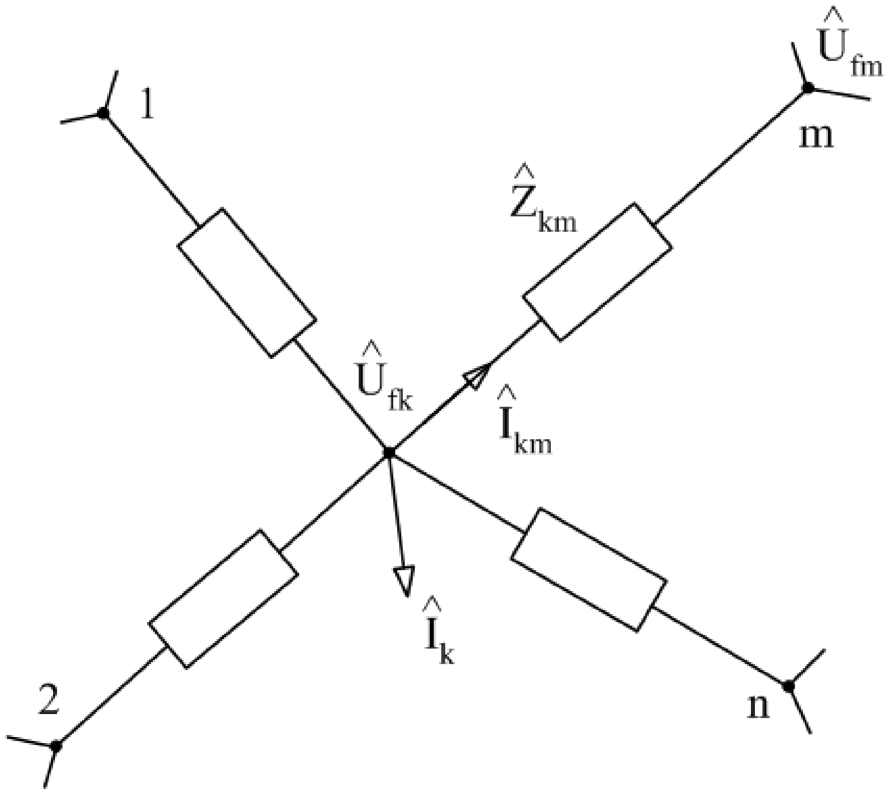
$$\Delta\hat{U}_{phBX} = \hat{Z}_{l_1} \sum_{k=X}^{n} \left(l_{(k+1)} - l_k\right) \cdot \hat{I}_{(k+1)k} = \hat{Z}_{l_1} \sum_{k=X}^{n} l_{(k+1)k} \cdot \hat{I}_{(k+1)k}$$



Meshed grids MV

Bus voltage method

Grid with *n* nodes. Set series branch parameters $\hat{Z}_{km}$, load currents (bus currents) $\hat{I}_k$, min. 1 bus voltage $\hat{U}_{phk}$ (between the bus and the ground).



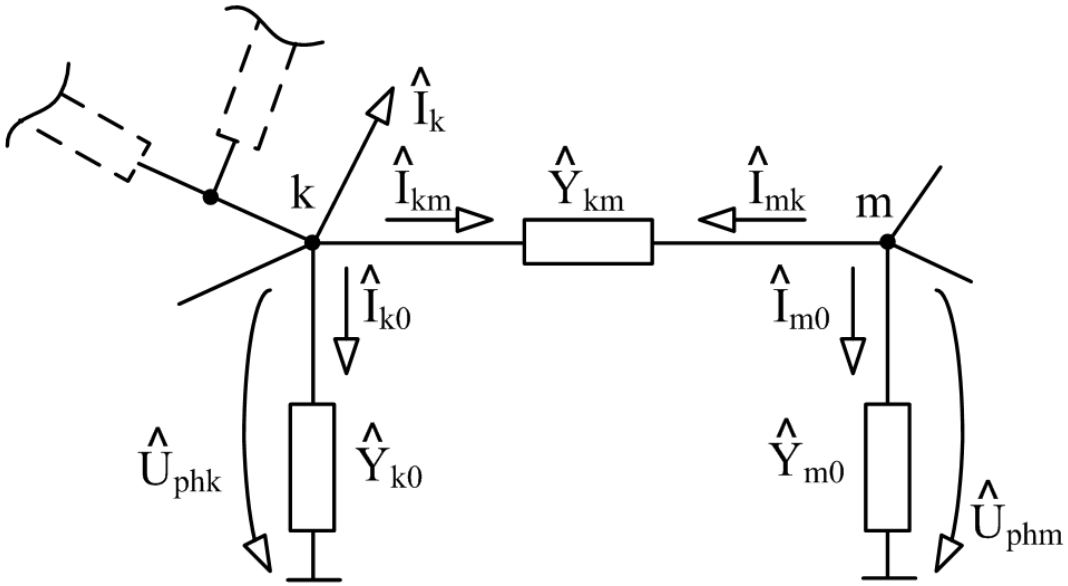
Calculation with series admittances

$$\hat{Y}_{km} = \hat{Z}_{km}^{-1} = \frac{1}{R_{km} + jX_{km}}$$

Node *k*

$$\hat{I}_k + \sum_{\substack{m=1 \\ m \neq k}}^{n} \hat{I}_{km} + \hat{I}_{k0} = 0$$

$$\hat{I}_{k0} = \hat{U}_{phk} \hat{Y}_{k0}$$



Branches *k, m*

$$\hat{I}_{km} = \left(\hat{U}_{phk} - \hat{U}_{phm}\right)\hat{Y}_{km}$$

After modifications:

$$\hat{I}_k = -\sum_{\substack{m=1 \\ m \neq k}}^{n} \left(\hat{U}_{phk} - \hat{U}_{phm}\right)\hat{Y}_{km} - \hat{U}_{phk} \hat{Y}_{k0}$$

$$\hat{I}_k = -\hat{U}_{phk}\left(\sum_{\substack{m=1\\ m\neq k}}^{n} \hat{Y}_{km} + \hat{Y}_{k0}\right) + \sum_{\substack{m=1\\ m\neq k}}^{n} \hat{U}_{phm}\hat{Y}_{km}$$

Admittance matrix parameters definition:

Bus self-admittance (diagonal element)

$$\hat{Y}_{(k,k)} = -\sum_{\substack{m=1\\ m\neq k}}^{n} \hat{Y}_{km} - \hat{Y}_{k0}$$

Between buses admittance (non-diagonal element)

$$\hat{Y}_{(k,m)} = \hat{Y}_{(m,k)} = \hat{Y}_{km} \quad \text{for } m \neq k$$

(for non-connected buses $\hat{Y}_{(k,m)} = 0$)

Hence

$$\hat{I}_k = \sum_{m=1}^{n} \hat{Y}_{(k,m)}\hat{U}_{fm}$$

Matrix form

$$\left(\hat{\mathrm{I}}\right) = \left(\hat{\mathrm{Y}}\right)\left(\hat{\mathrm{U}}_{\mathrm{ph}}\right)$$

Set voltages at buses 1 to *k* (x), currents at buses *k+1* to *n* (y)

$$\begin{pmatrix} \left(\hat{\mathrm{I}}_{\mathrm{x}}\right) \\ \left(\hat{\mathrm{I}}_{\mathrm{y}}\right) \end{pmatrix} = \begin{pmatrix} \left(\hat{\mathrm{Y}}_{\mathrm{xx}}\right) & \left(\hat{\mathrm{Y}}_{\mathrm{xy}}\right) \\ \left(\hat{\mathrm{Y}}_{\mathrm{xy}}\right)^{\mathrm{T}} & \left(\hat{\mathrm{Y}}_{\mathrm{yy}}\right) \end{pmatrix} \begin{pmatrix} \left(\hat{\mathrm{U}}_{\mathrm{phx}}\right) \\ \left(\hat{\mathrm{U}}_{\mathrm{phy}}\right) \end{pmatrix}$$

Hence

$$\left(\hat{\mathrm{I}}_{\mathrm{x}}\right) = \left(\hat{\mathrm{Y}}_{\mathrm{xx}}\right)\left(\hat{\mathrm{U}}_{\mathrm{phx}}\right) + \left(\hat{\mathrm{Y}}_{\mathrm{xy}}\right)\left(\hat{\mathrm{U}}_{\mathrm{phy}}\right)$$

$$\left(\hat{\mathrm{I}}_{\mathrm{y}}\right) = \left(\hat{\mathrm{Y}}_{\mathrm{xy}}\right)^{\mathrm{T}}\left(\hat{\mathrm{U}}_{\mathrm{phx}}\right) + \left(\hat{\mathrm{Y}}_{\mathrm{yy}}\right)\left(\hat{\mathrm{U}}_{\mathrm{phy}}\right)$$

Calculate $\left(\hat{\mathrm{I}}_{\mathrm{x}}\right), \left(\hat{\mathrm{U}}_{\mathrm{phy}}\right)$

$$\left(\hat{\mathrm{U}}_{\mathrm{phy}}\right) = \left(\hat{\mathrm{Y}}_{\mathrm{yy}}\right)^{-1}\left(\hat{\mathrm{I}}_{\mathrm{y}}\right) - \left(\hat{\mathrm{Y}}_{\mathrm{yy}}\right)^{-1}\left(\hat{\mathrm{Y}}_{\mathrm{xy}}\right)^{\mathrm{T}}\left(\hat{\mathrm{U}}_{\mathrm{phx}}\right)$$

If some nodes are connected to the ground (through an admittance), then the admittance matrix is regular → to set all nodal current is enough.

$$\left(\hat{U}_f\right) = \left(\hat{Y}\right)^{-1}\left(\hat{I}\right)$$

Note 1: Similar for DC grid.

$$(I) = (G)(U)$$

Note 2: For power engineering – powers are set, currents are calculated from the powers.

$$\hat{I} = \left(\frac{\hat{S}}{\sqrt{3}\hat{U}}\right)^{*}$$

Results are not precise if nominal voltages are used → iteration methods.

HV lines

No load points.

Open-circuit

$$\hat{I}_2 = 0$$

$$\hat{U}_{f10} = \hat{U}_{f2} \cosh \hat{\gamma} l$$

$$\hat{I}_{10} = \frac{\hat{U}_{f2}}{\hat{Z}_v} \sinh \hat{\gamma} l$$

For ideal line

$$\hat{U}_{f10} = \hat{U}_{f2} \cos \beta l$$

$$\hat{I}_{10} = j \frac{\hat{U}_{f2}}{Z_v} \sin \beta l$$

It is valid $U_{f10} \leq U_{f2}$ $\longrightarrow$ Ferranti effect

Line character is like capacity.

Short-circuit

$$\hat{U}_{f2} = 0$$

$$\hat{U}_{f1} = \hat{Z}_v \hat{I}_2 \sinh \hat{\gamma} l$$

$$\hat{I}_1 = \hat{I}_2 \cosh \hat{\gamma} l$$

For ideal line

$$\hat{U}_{f1} = jZ_v \hat{I}_2 \sin \beta l$$

$$\hat{I}_1 = \hat{I}_2 \cos \beta l$$

Voltage decreases from the beginning to the end.

Line character is like inductance.

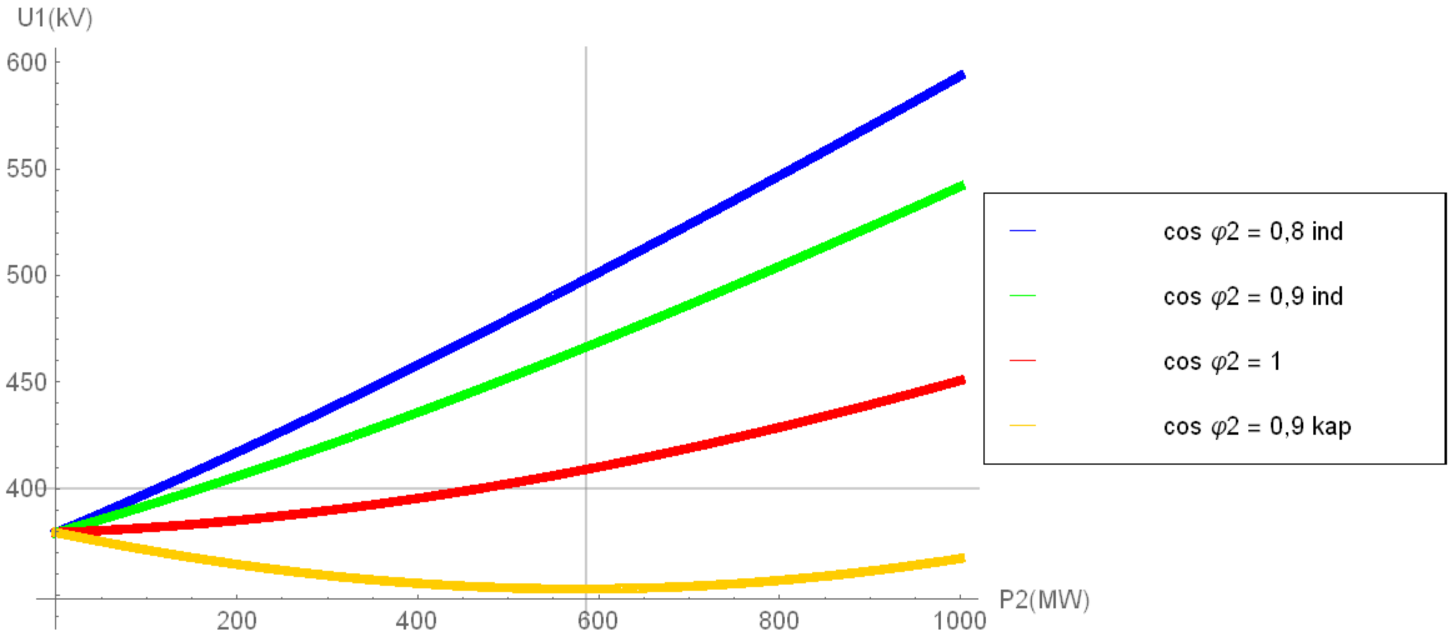
Example:

line 1 x 400 kV with two ground wires

phase conductor: 3xACSR 450/52, ground wire: ACSR 185/31, l = 300 km

$$R_1 = 0{,}021\,\Omega/\mathrm{km};\ X_1 = 0{,}293\,\Omega/\mathrm{km};\ G_1 = 2\cdot 10^{-8}\ \mathrm{S/km};\ B_1 = 3{,}9\cdot 10^{-6}\ \mathrm{S/km}$$

Voltage level (U_2 = 400 kV)

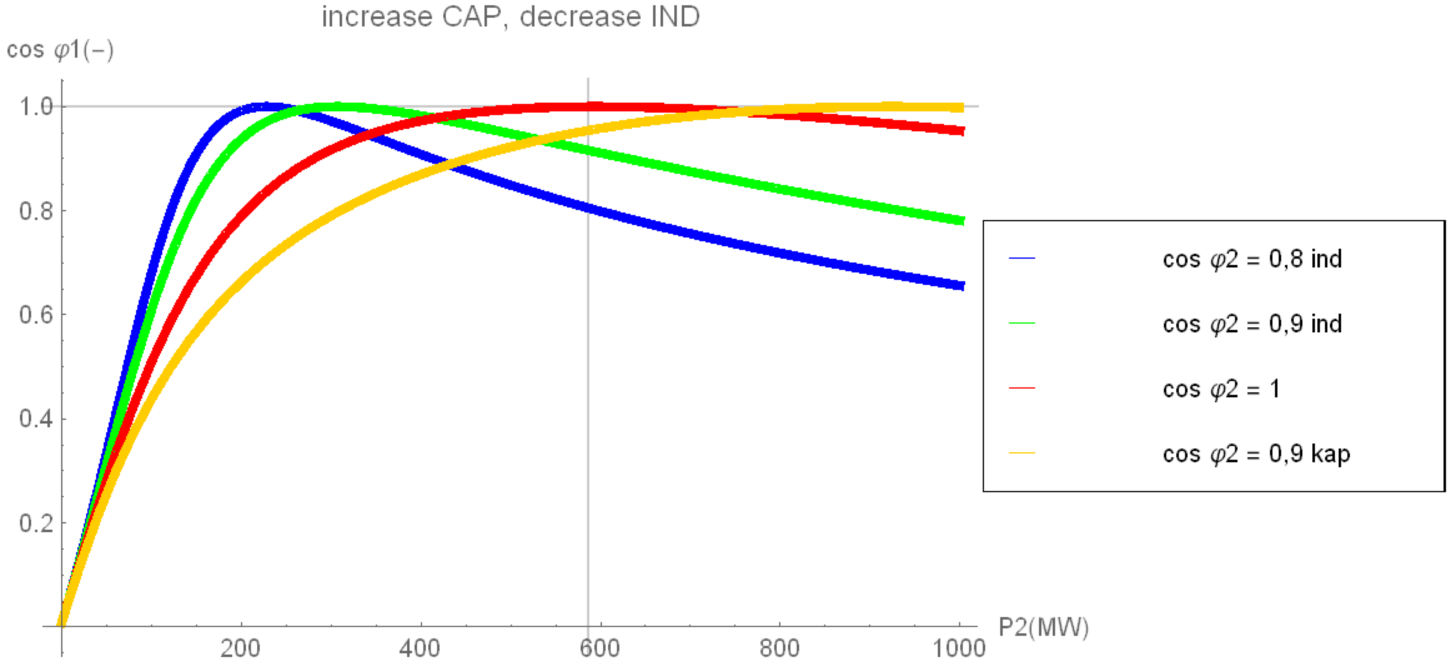


$U_1 < U_n$: Ferranti effect

$U_1 \sim U_n$ for S_p area and $\cos\varphi = 1$

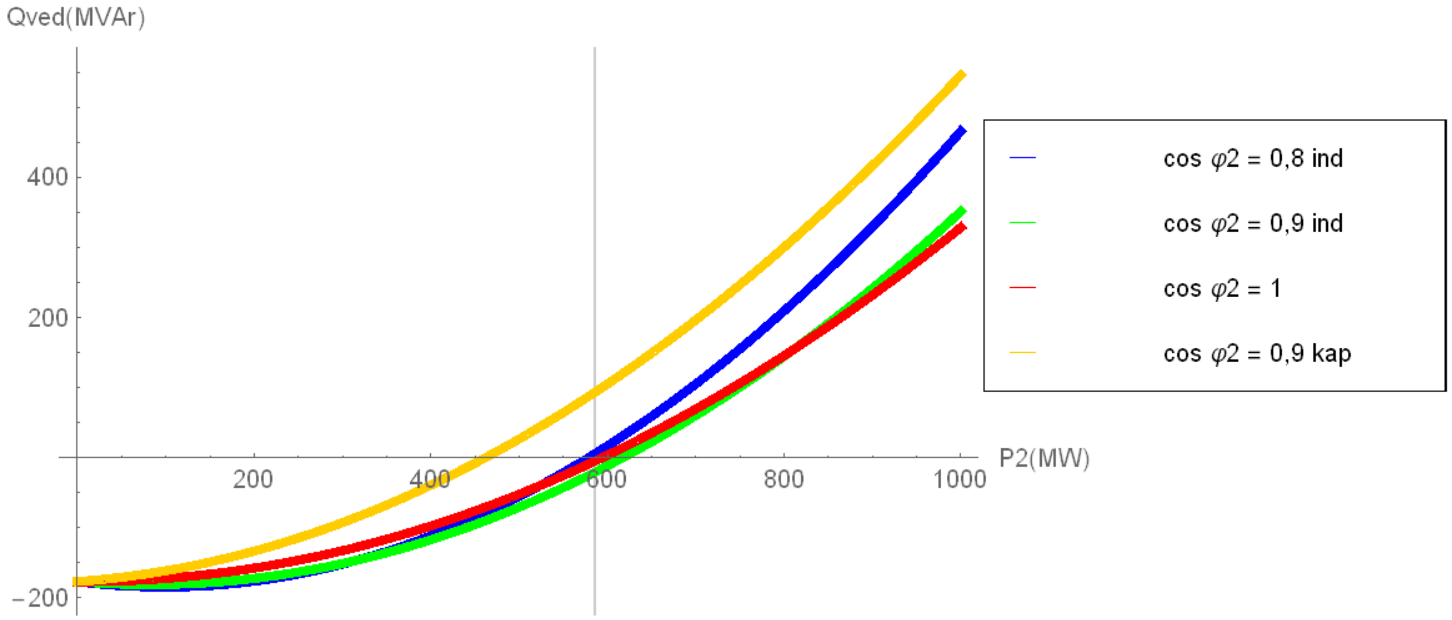
Transmission power factor

$$\cos\varphi_1 = \frac{P_1}{S_1}$$

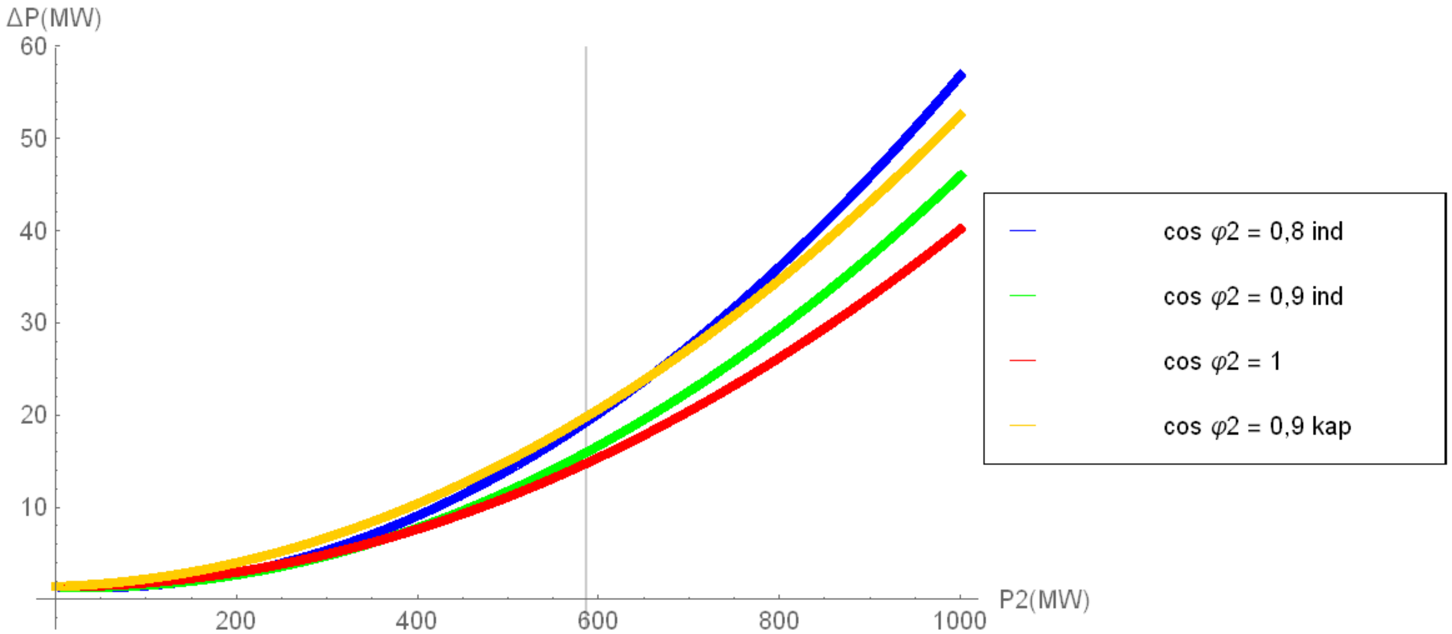


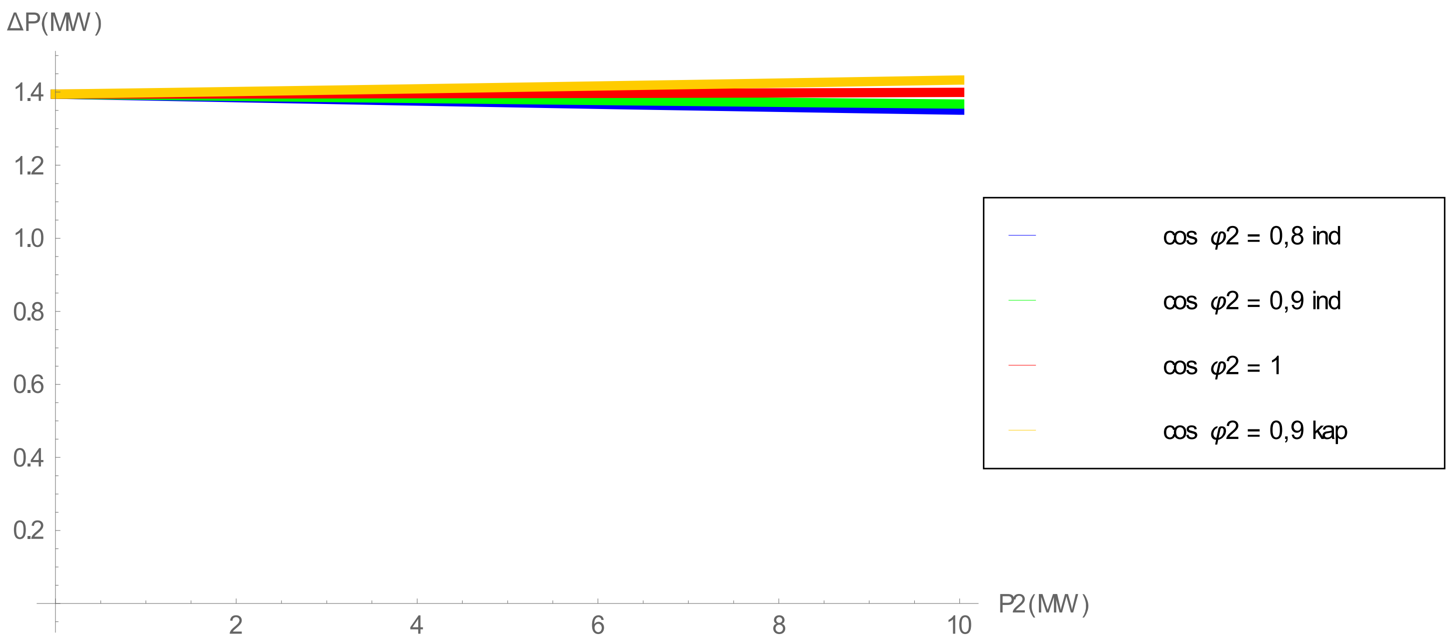
open-circuit → line is like capacitive load
higher power → line „self-compensation“

Line reactive power



Line losses
= open-circuit ~ U^2 + load ~ I^2

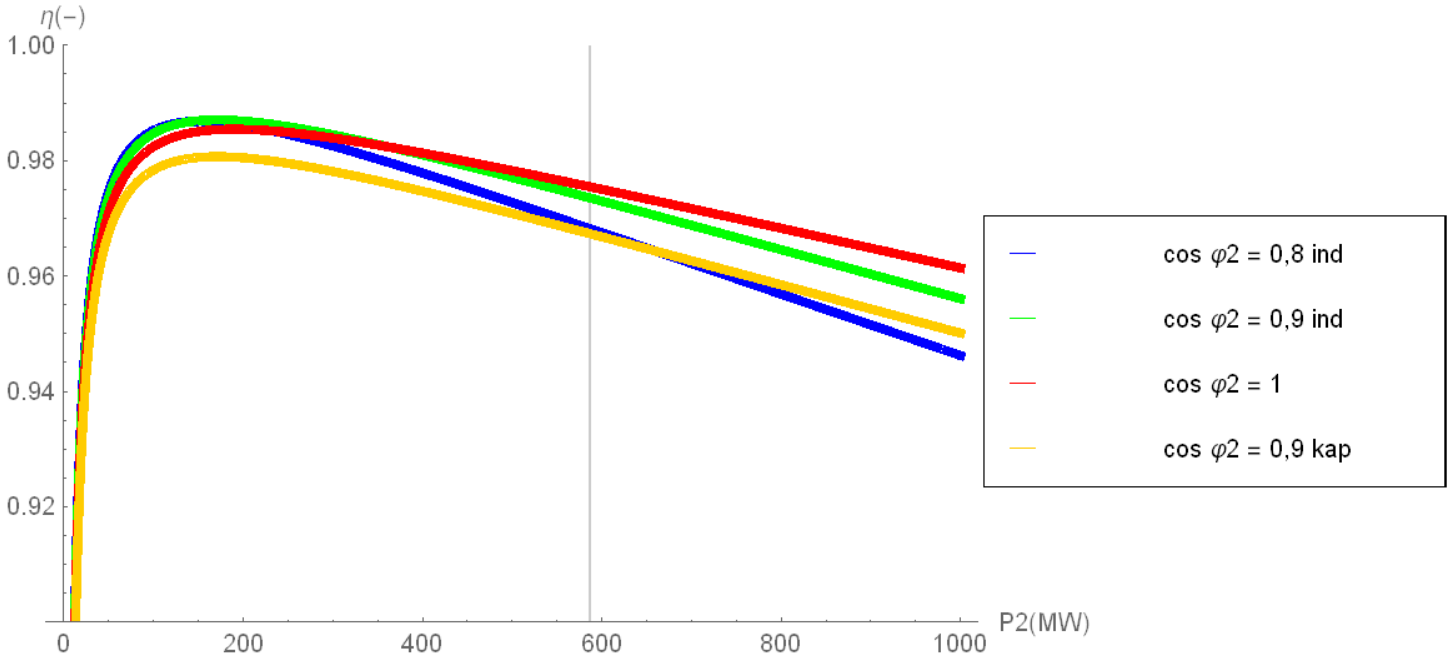




ΔP(MW)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
2
4
6
8
10
P2(MW)
cos φ2 = 0,8 ind
cos φ2 = 0,9 ind
cos φ2 = 1
cos φ2 = 0,9 kap

Transmission efficiency

$$\eta = \frac{P_2}{P_1}$$



maximum for low powers
for higher powers a flat curve