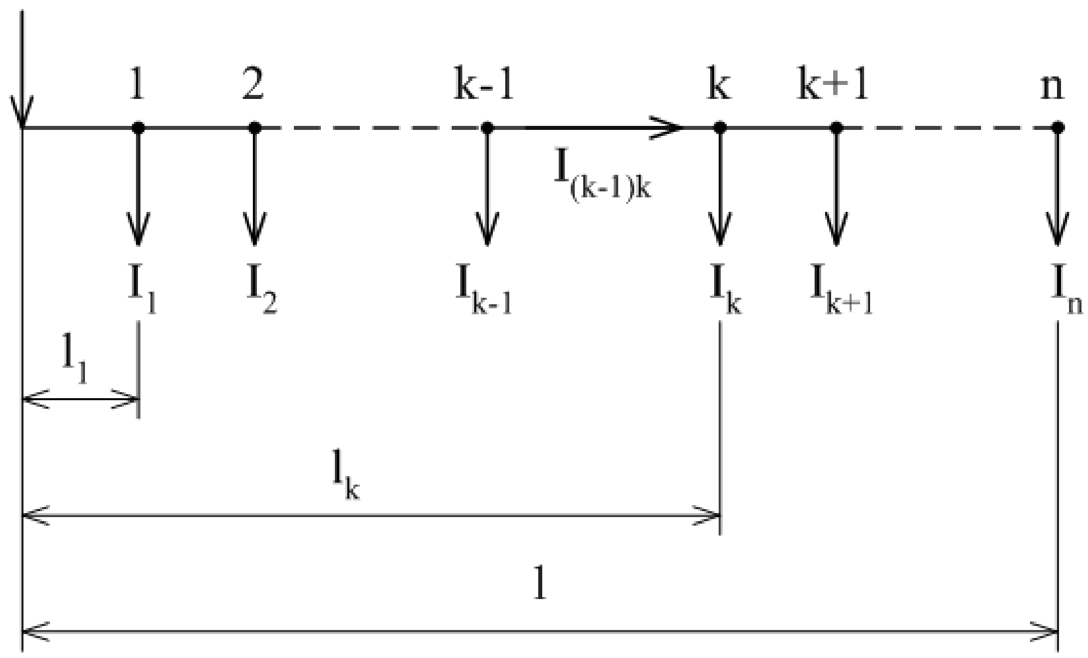
STEADY STATES (LOAD FLOW) CALCULATIONS IN POWER SYSTEMS - Current loads

Simple DC line (LV, MV)

Double-wire circuit. Assumption: constant cross-section and resistivity.

Single loads supplied from one side

Standard distribution lines.



a) addition method

It adds voltage drops along the power line sections.
(Voltage drops are always in both conductors in the section.)

*k*th section

$$U_{(k-1)} - U_k = \Delta U_{(k-1)k} = 2\frac{\rho}{S}\left(l_k - l_{(k-1)}\right)\cdot I_{(k-1)k} \quad (V; \Omega m, m^2, m, A)$$

Current in *k*th section

$$I_{(k-1)k} = \sum_{y=k}^{n} I_y$$

Maximum voltage drop

$$\Delta U_n = \sum_{k=1}^{n} \Delta U_{(k-1)k} = 2\frac{\rho}{S}\sum_{k=1}^{n}\left(l_k - l_{(k-1)}\right)\cdot \sum_{y=k}^{n} I_y$$

b) superposition method

It adds voltage drops for individual discrete loads:

$$\Delta U_n = 2\frac{\rho}{S}\sum_{k=1}^{n} l_k I_k$$

$l_k I_k$ …current moments to the feeder

Relative voltage drop:

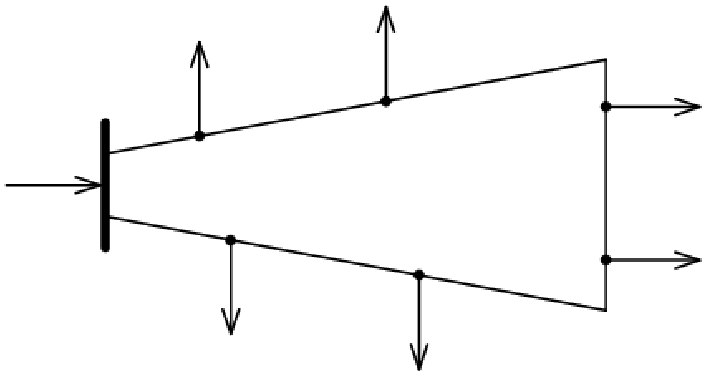
$$\varepsilon = \frac{\Delta U}{U_n} \quad (-;\, V,\, V)$$

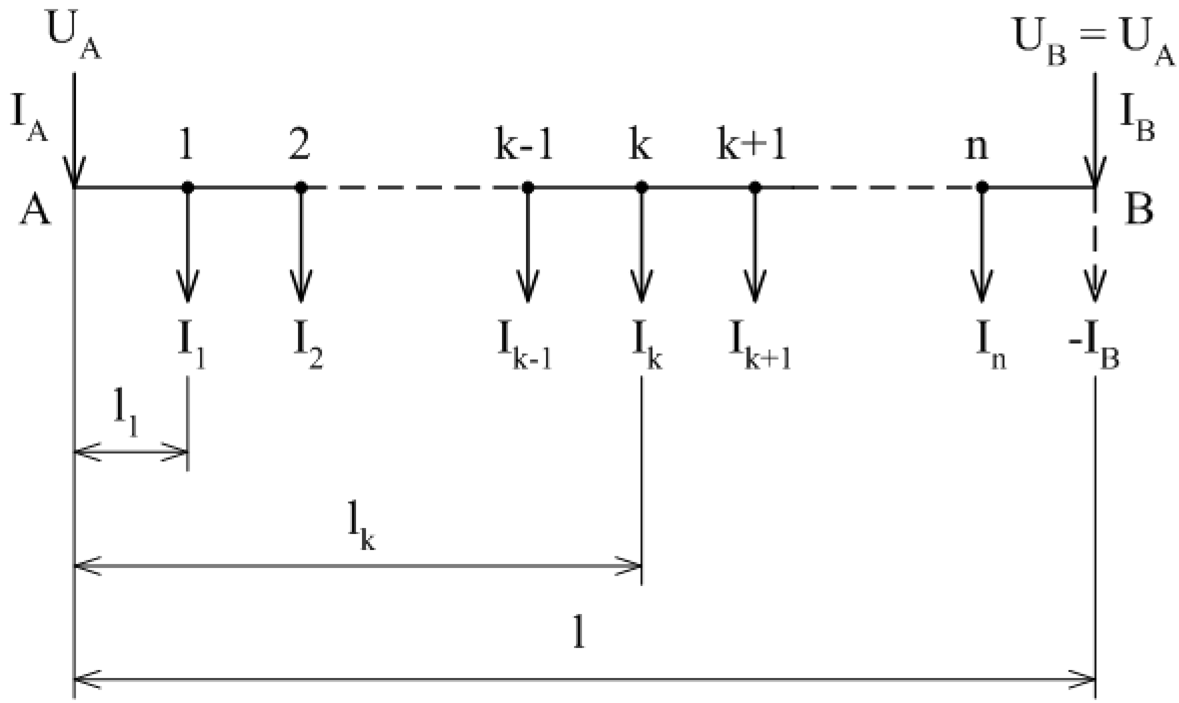
Note. Losses must be calculated only by means of the addition method!

$$\Delta P_{(k-1)k} = 2\frac{\rho}{S}\left(l_k - l_{(k-1)}\right)\cdot I_{(k-1)k}^2 \quad (W;\, \Omega m,\, m^2,\, m,\, A)$$

$$\Delta P = \sum_{k=1}^{n} \Delta P_{(k-1)k}$$

Single loads supplied from both sides – the same feeders voltages





- Ring grid, higher reliability of supply.
- Two one-feeder lines after a fault. More often also in standard operation mode.
- Calculation of current distribution and voltage drops.

Consider I_B as a negative load:

$$\Delta U_{AB} = U_A - U_B = 0 = 2\frac{\rho}{S}\sum_{k=1}^{n} l_k I_k - 2\frac{\rho}{S} l I_B$$

Hence (moment theorem)

$$I_B = \frac{\sum_{k=1}^{n} l_k I_k}{l}$$

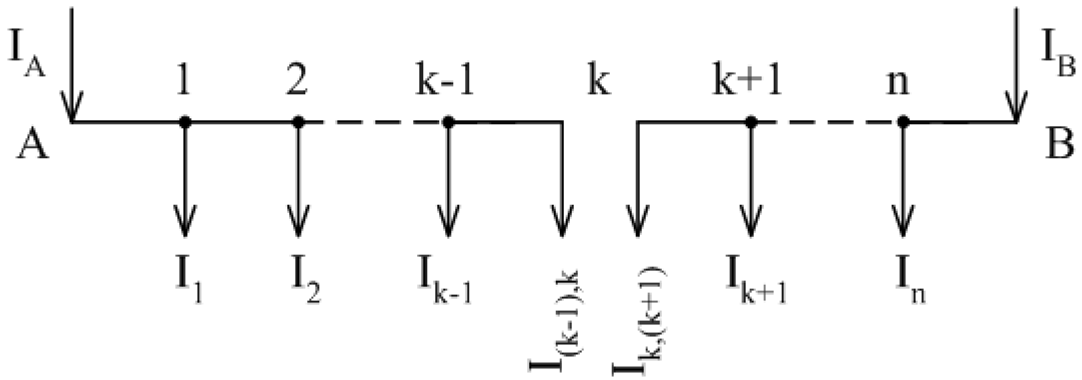
Analogous (current moments to other feeder)

$$I_A = \frac{\sum_{k=1}^{n} (l - l_k) I_k}{l}$$

Of course

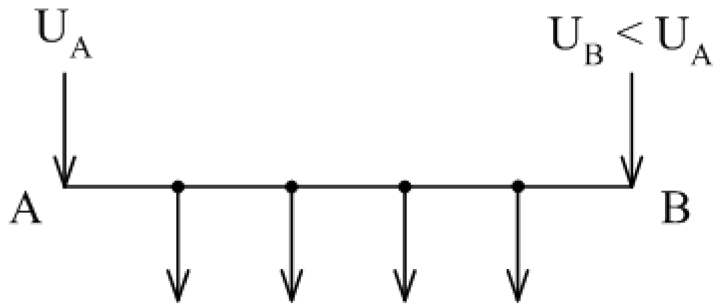
$$I_A + I_B = \sum_{y=1}^{n} I_y$$

Current distribution identifies the place with the biggest voltage drop = the place with feeder division → split-up into two one-feeder lines.



Single loads supplied from both sides – different feeders voltages

Two different sources, meshed grid.



Superposition:

1) Current distribution with the same voltages.
2) Different voltages and zero loads → balancing current

$$I_v = \frac{U_A - U_B}{2\frac{\rho}{S}l}$$

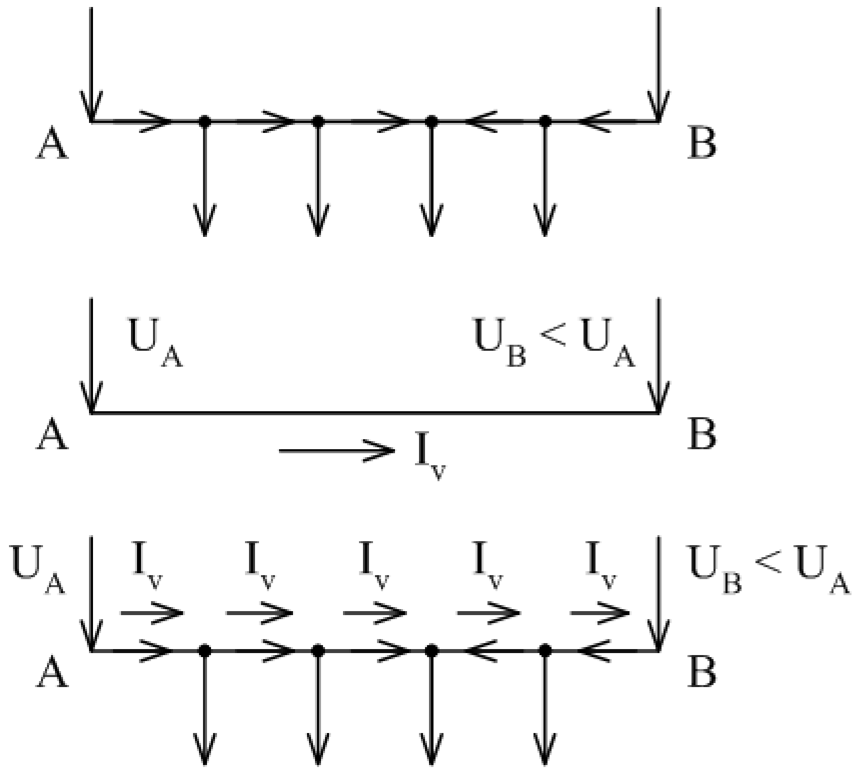
3) Sum of the solutions 1+2

Further calculation is the same.

Or directly:

$$U_A - U_B = 2\frac{\rho}{S}\sum_{k=1}^{n} l_k I_k - 2\frac{\rho}{S} l I_B$$

$$I_B = \frac{2\frac{\rho}{S}\sum_{k=1}^{n} l_k I_k}{2\frac{\rho}{S}l} - \frac{U_A - U_B}{2\frac{\rho}{S}l}$$

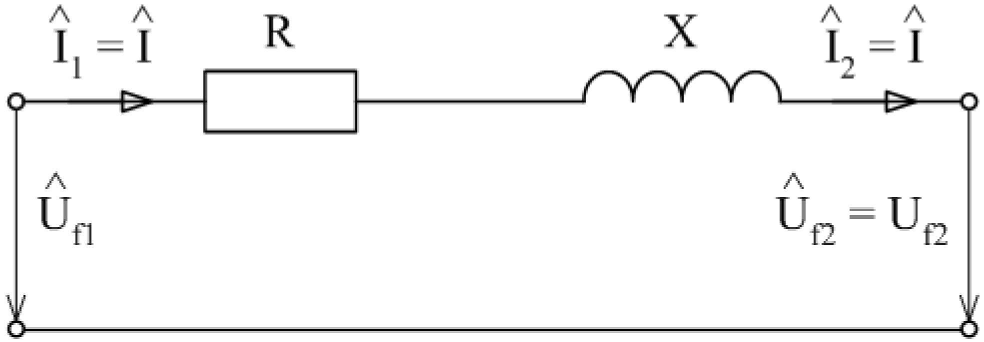


AC - 3 phase power lines LV, MV

Series parameters are applied, for LV $X \to 0$.

3 phase power line MV, 1 load at the end

Symmetrical load → 1 phase diagram, operational parameters.



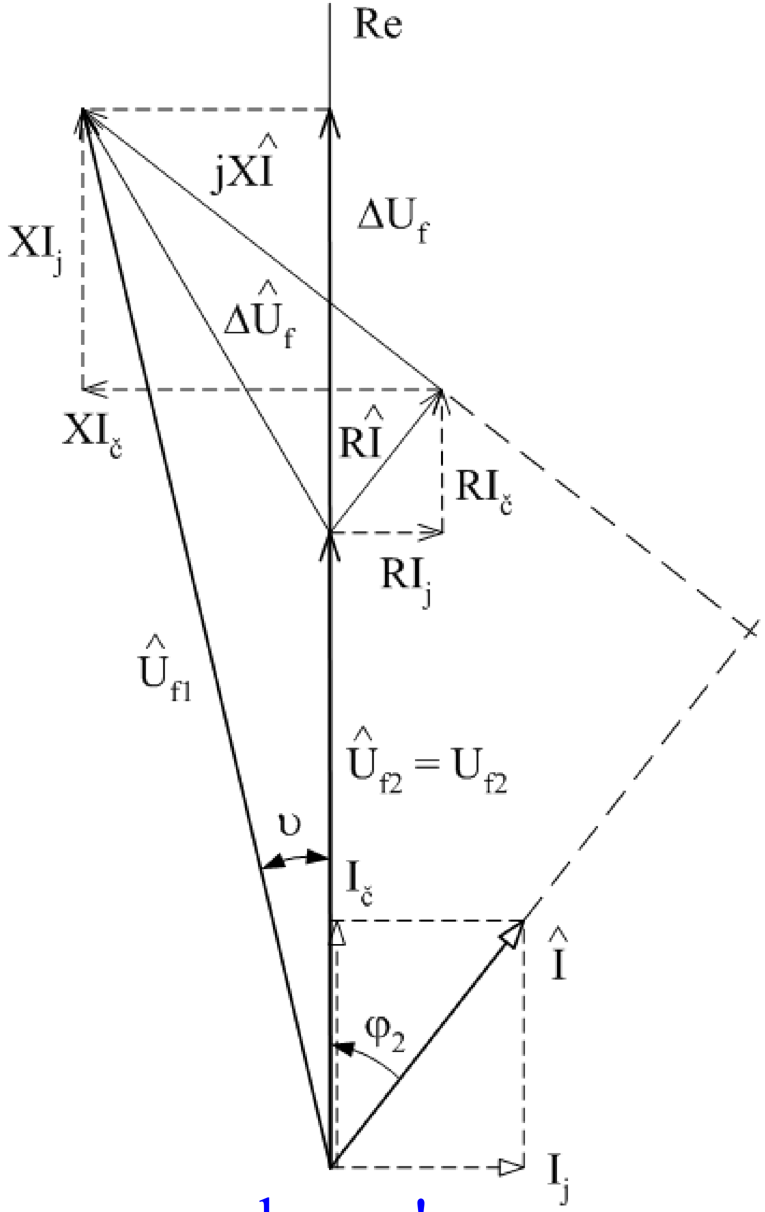
Complex voltage drop

$$\Delta\hat{U}_{ph} = \hat{Z}_l \hat{I} = (R + jX)(I_{re} \mp jI_{im}) \begin{matrix} \text{IND} \\ \text{CAP} \end{matrix}$$

$$\Delta\hat{U}_{ph} = RI_{re} \pm XI_{im} + j(XI_{re} \mp RI_{im}) \begin{matrix} \text{IND} \\ \text{CAP} \end{matrix}$$

magnitude phase

Phasor diagram (input U_{ph2}, I, φ_2)
(angle υ usually small, up to 3°)



Imagin. part neglecting and modifications

$$\Delta U_{ph} = \frac{R3U_{ph}I_{re} \pm X3U_{ph}I_{im}}{3U_{ph}} = \frac{RP \pm XQ}{3U_{ph}}$$

Percentage voltage drop

$$\varepsilon = \frac{\Delta U_{ph}}{U_{ph}} = \frac{RP \pm XQ}{3U_{ph}^2} = \frac{RP \pm XQ}{U^2}$$

3 phase active power losses

$$\Delta \hat{S} = 3\Delta \hat{U}_{ph}\hat{I}^* = 3\hat{Z}_1 \hat{I} \cdot \hat{I}^* = 3\hat{Z}_1 I^2 =$$

$$= 3(R + jX)I^2 = 3RI^2 + j3XI^2$$

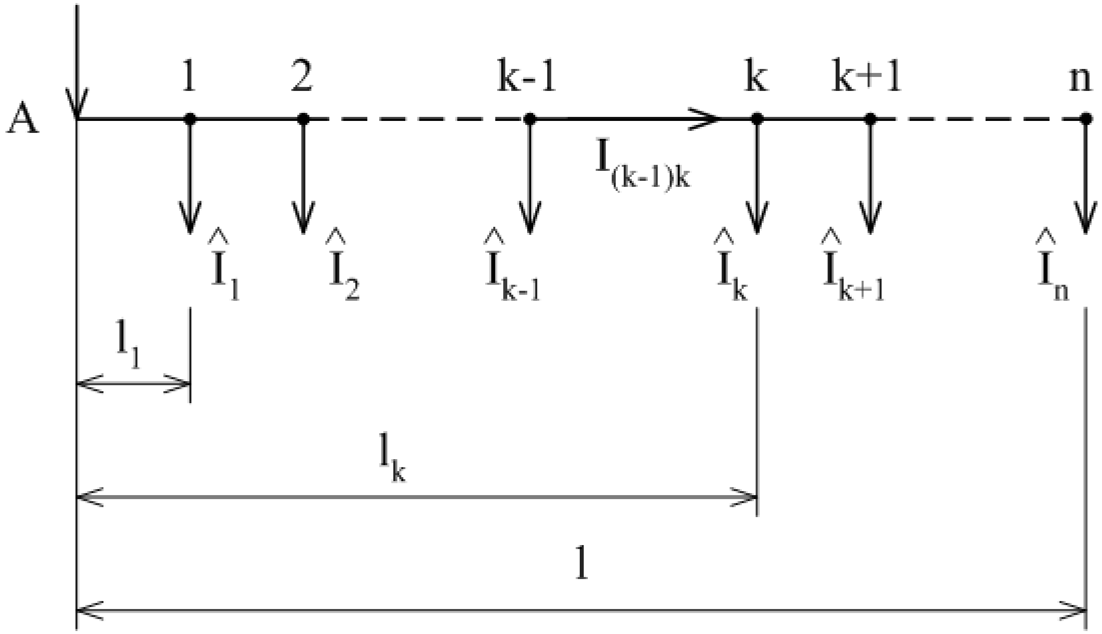
$$\Delta P = 3RI^2 = 3R\left(I_{re}^2 + I_{im}^2\right) \quad (W; \Omega, A)$$

! Even the reactive current (power) causes active power losses!

3 phase MV power line supplied from one side

Constant series impedance

$$\hat{Z}_{l_1} = R_1 + jX_1 \quad (\Omega / km)$$



Voltage drop at the end (needn't be the highest one, it depends on load character)

- superposition $$\Delta\hat{U}_{phAn} = \hat{Z}_{l_1} \sum_{k=1}^{n} l_k \hat{I}_k$$
- addition $$\Delta\hat{U}_{phAn} = \hat{Z}_{l_1} \sum_{k=1}^{n} \left(l_k - l_{(k-1)}\right) \cdot \sum_{y=k}^{n} \hat{I}_y$$

After imaginary part neglecting (addition)

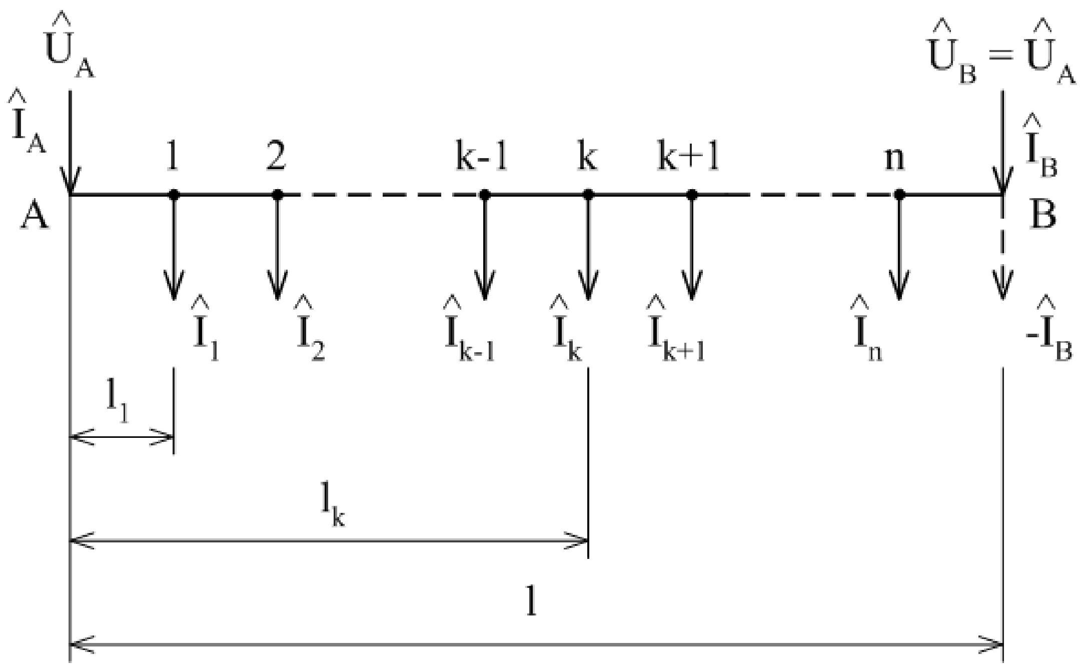
$$\Delta U_{phAn} \doteq R_1 \sum_{k=1}^{n} \left(l_k - l_{(k-1)}\right) \cdot \sum_{y=k}^{n} I_{rek} \pm X_1 \sum_{k=1}^{n} \left(l_k - l_{(k-1)}\right) \cdot \sum_{y=k}^{n} I_{imk} \quad \begin{matrix} \text{IND} \\ \text{CAP} \end{matrix}$$

$$\Delta U_{phAn} \doteq \frac{R_1 \sum_{k=1}^{n} \left(l_k - l_{(k-1)}\right) \cdot \sum_{y=k}^{n} P_k \pm X_1 \sum_{k=1}^{n} \left(l_k - l_{(k-1)}\right) \cdot \sum_{y=k}^{n} Q_k}{3U_{ph}} \quad \begin{matrix} \text{IND} \\ \text{CAP} \end{matrix}$$

Voltage drop up to the point X (not end)

- superposition $$\Delta\hat{U}_{phAX} = \hat{Z}_{l_1}\sum_{k=1}^{X} l_k \hat{I}_k + \hat{Z}_{l_1} l_{AX} \sum_{k=X+1}^{n} \hat{I}_k$$
- addition $$\Delta\hat{U}_{phAX} = \hat{Z}_{l_1}\sum_{k=1}^{X}\left(l_k - l_{(k-1)}\right)\cdot\sum_{y=k}^{n}\hat{I}_y$$

3 phase MV power line supplied from both sides



Calculation as for DC line (feeder is a negative load, zero voltage drop).

$$\Delta \hat{U}_{phAB} = 0 = \hat{Z}_{l_1} \sum_{k=1}^{n} l_k \hat{I}_k - \hat{Z}_{l_1} l \cdot \hat{I}_B$$

Moment theorems

$$\hat{I}_B = \frac{\sum_{k=1}^{n} l_k \hat{I}_k}{l} \qquad \hat{I}_A = \frac{\sum_{k=1}^{n} (l - l_k) \hat{I}_k}{l} \qquad \hat{I}_A + \hat{I}_B = \sum_{y=1}^{n} \hat{I}_y$$

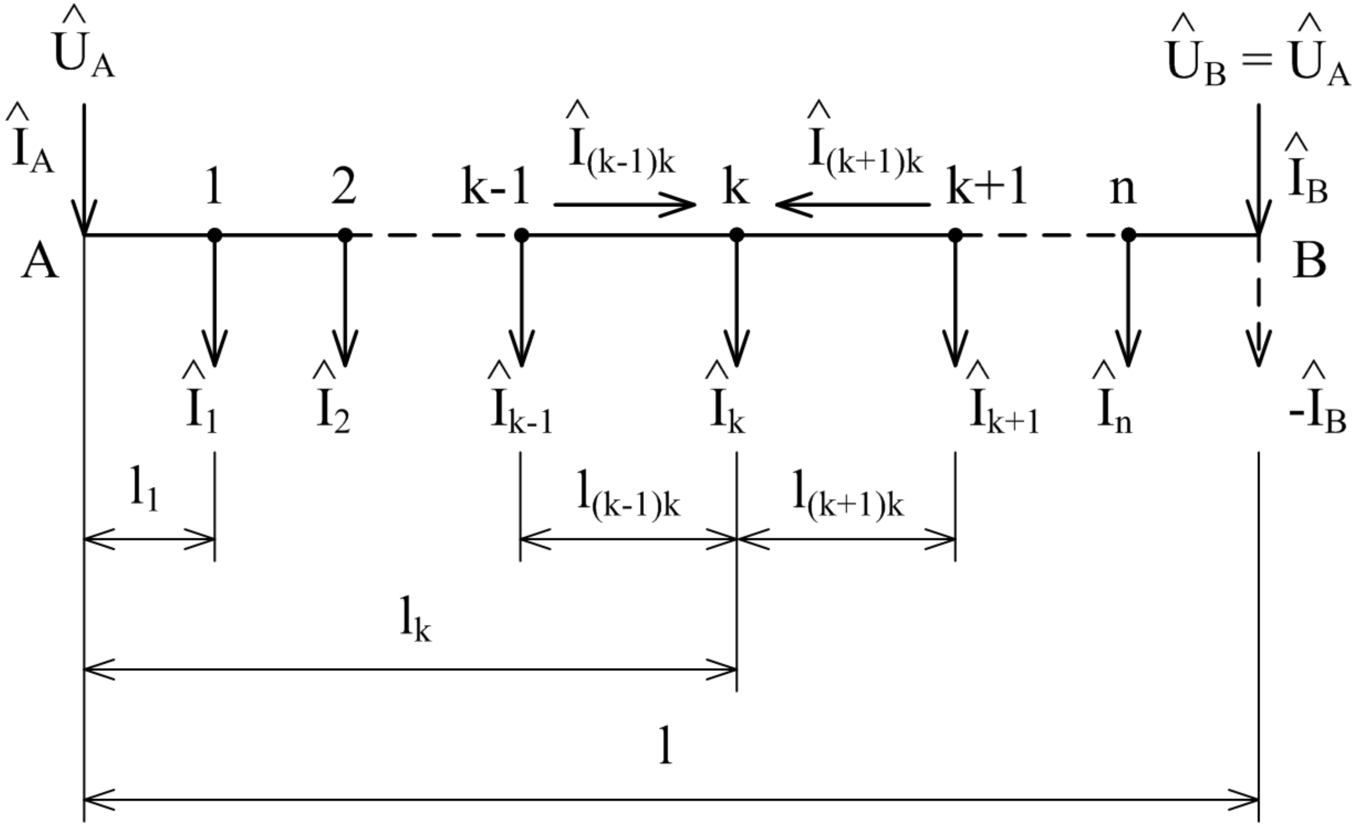
(In principle it is the current divider for each load.)

Active and reactive current sign change could be in different nodes → maximum voltage drop should be checked in all grid points.

- addition

$$\Delta\hat{U}_{phAX} = \hat{Z}_{l_1} \sum_{k=1}^{X} \left(l_k - l_{(k-1)} \right) \cdot \hat{I}_{(k-1)k} = \hat{Z}_{l_1} \sum_{k=1}^{X} l_{(k-1)k} \cdot \hat{I}_{(k-1)k}$$

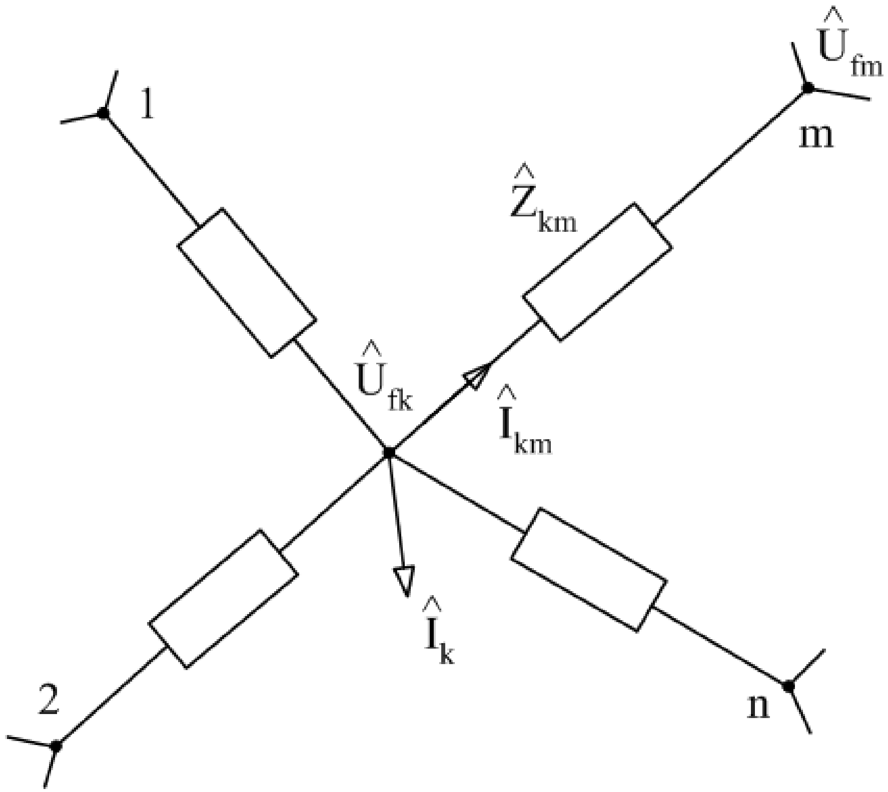
$$\Delta\hat{U}_{phBX} = \hat{Z}_{l_1} \sum_{k=X}^{n} \left(l_{(k+1)} - l_k \right) \cdot \hat{I}_{(k+1)k} = \hat{Z}_{l_1} \sum_{k=X}^{n} l_{(k+1)k} \cdot \hat{I}_{(k+1)k}$$



Meshed grids MV

Bus voltage method

Grid with *n* nodes. Set series branch parameters $\hat{Z}_{km}$, load currents (bus currents) $\hat{I}_k$, min. 1 bus voltage $\hat{U}_{phk}$ (between the bus and the ground).



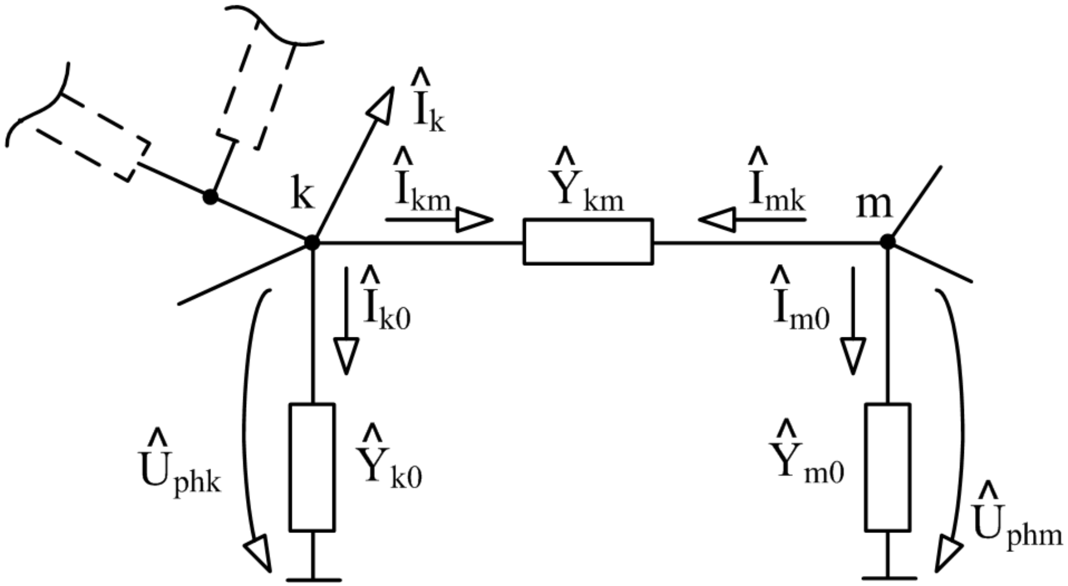
Calculation with series admittances

$$\hat{Y}_{km} = \hat{Z}_{km}^{-1} = \frac{1}{R_{km} + jX_{km}}$$

Node *k*

$$\hat{I}_k + \sum_{\substack{m=1 \\ m \neq k}}^{n} \hat{I}_{km} + \hat{I}_{k0} = 0$$

$$\hat{I}_{k0} = \hat{U}_{phk} \hat{Y}_{k0}$$



Branches *k, m*

$$\hat{I}_{km} = \left(\hat{U}_{phk} - \hat{U}_{phm}\right)\hat{Y}_{km}$$

After modifications:

$$\hat{I}_k = -\sum_{\substack{m=1 \\ m \neq k}}^{n} \left(\hat{U}_{phk} - \hat{U}_{phm}\right)\hat{Y}_{km} - \hat{U}_{phk}\hat{Y}_{k0}$$

$$\hat{I}_k = -\hat{U}_{phk}\left(\sum_{\substack{m=1\\ m\neq k}}^{n} \hat{Y}_{km} + \hat{Y}_{k0}\right) + \sum_{\substack{m=1\\ m\neq k}}^{n} \hat{U}_{phm}\hat{Y}_{km}$$

Admittance matrix parameters definition:

Bus self-admittance (diagonal element)

$$\hat{Y}_{(k,k)} = -\sum_{\substack{m=1\\ m\neq k}}^{n} \hat{Y}_{km} - \hat{Y}_{k0}$$

Between buses admittance (non-diagonal element)

$$\hat{Y}_{(k,m)} = \hat{Y}_{(m,k)} = \hat{Y}_{km} \quad \text{for } m \neq k$$

(for non-connected buses $\hat{Y}_{(k,m)} = 0$)

Hence

$$\hat{I}_k = \sum_{m=1}^{n} \hat{Y}_{(k,m)}\hat{U}_{fm}$$

Matrix form

$$\left(\hat{I}\right)=\left(\hat{Y}\right)\left(\hat{U}_{ph}\right)$$

Set voltages at buses 1 to *k* (x), currents at buses *k+1* to *n* (y)

$$\begin{pmatrix}\left(\hat{I}_x\right)\\\left(\hat{I}_y\right)\end{pmatrix}=\begin{pmatrix}\left(\hat{Y}_{xx}\right) & \left(\hat{Y}_{xy}\right)\\\left(\hat{Y}_{xy}\right)^T & \left(\hat{Y}_{yy}\right)\end{pmatrix}\begin{pmatrix}\left(\hat{U}_{phx}\right)\\\left(\hat{U}_{phy}\right)\end{pmatrix}$$

Hence

$$\left(\hat{I}_x\right)=\left(\hat{Y}_{xx}\right)\left(\hat{U}_{phx}\right)+\left(\hat{Y}_{xy}\right)\left(\hat{U}_{phy}\right)$$

$$\left(\hat{I}_y\right)=\left(\hat{Y}_{xy}\right)^T\left(\hat{U}_{phx}\right)+\left(\hat{Y}_{yy}\right)\left(\hat{U}_{phy}\right)$$

Calculate $\left(\hat{I}_x\right),\left(\hat{U}_{phy}\right)$

$$\left(\hat{U}_{phy}\right)=\left(\hat{Y}_{yy}\right)^{-1}\left(\hat{I}_y\right)-\left(\hat{Y}_{yy}\right)^{-1}\left(\hat{Y}_{xy}\right)^T\left(\hat{U}_{phx}\right)$$

If some nodes are connected to the ground (through an admittance), then the admittance matrix is regular → to set all nodal current is enough.

$$\left(\hat{U}_f\right) = \left(\hat{Y}\right)^{-1}\left(\hat{I}\right)$$

Note 1: Similar for DC grid.

$$\left(I\right) = \left(G\right)\left(U\right)$$

Note 2: For power engineering – powers are set, currents are calculated from the powers.

$$\hat{I} = \left(\frac{\hat{S}}{\sqrt{3}\hat{U}}\right)^{*}$$

Results are not precise if nominal voltages are used → iteration methods.

HV lines

No load points.

Open-circuit

$$\hat{I}_2 = 0$$

$$\hat{U}_{f10} = \hat{U}_{f2} \cosh \hat{\gamma} l$$

$$\hat{I}_{10} = \frac{\hat{U}_{f2}}{\hat{Z}_v} \sinh \hat{\gamma} l$$

For ideal line

$$\hat{U}_{f10} = \hat{U}_{f2} \cos \beta l$$

$$\hat{I}_{10} = j \frac{\hat{U}_{f2}}{Z_v} \sin \beta l$$

It is valid $U_{f10} \leq U_{f2}$ $\rightarrow$ Ferranti effect

Line character is like capacity.

Short-circuit

$$\hat{U}_{f2} = 0$$

$$\hat{U}_{f1} = \hat{Z}_v \hat{I}_2 \sinh \hat{\gamma} l$$

$$\hat{I}_1 = \hat{I}_2 \cosh \hat{\gamma} l$$

For ideal line

$$\hat{U}_{f1} = jZ_v \hat{I}_2 \sin \beta l$$

$$\hat{I}_1 = \hat{I}_2 \cos \beta l$$

Voltage decreases from the beginning to the end.
Line character is like inductance.

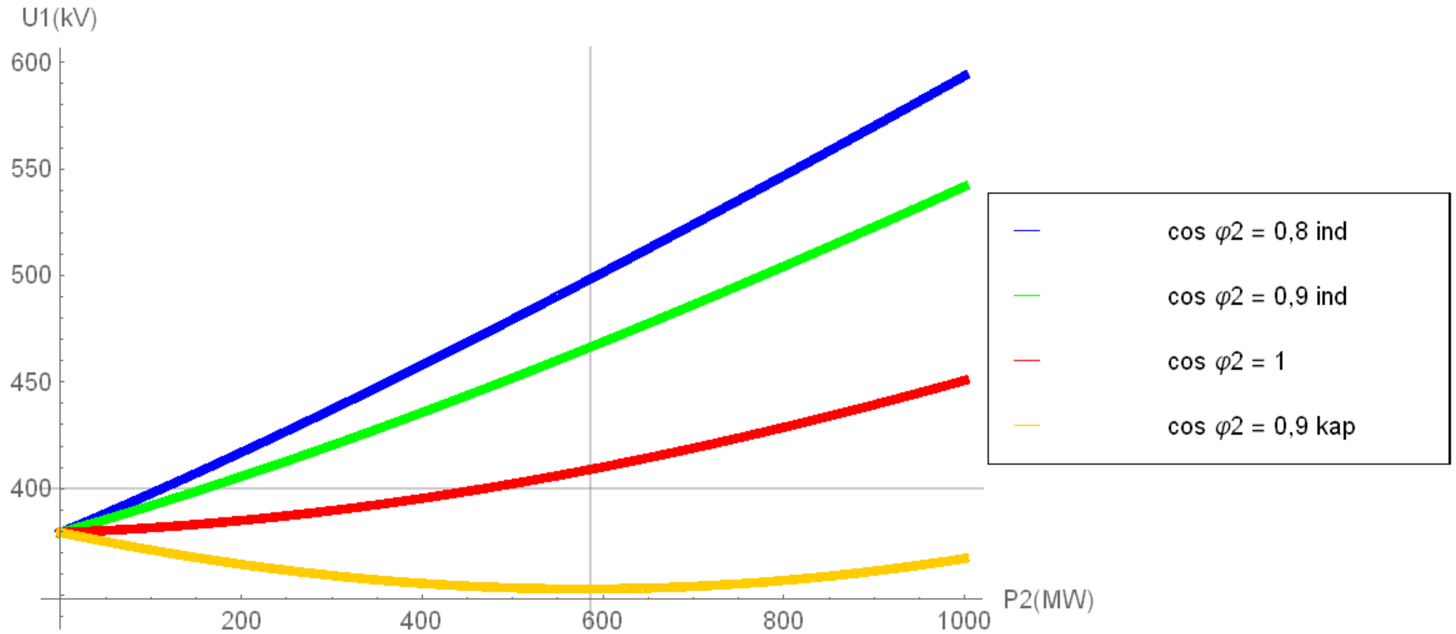
Example:

line 1 x 400 kV with two ground wires

phase conductor: 3xACSR 450/52, ground wire: ACSR 185/31, l = 300 km

$R_1 = 0{,}021\,\Omega/\text{km};\ X_1 = 0{,}293\,\Omega/\text{km};\ G_1 = 2\cdot10^{-8}\ \text{S/km};\ B_1 = 3{,}9\cdot10^{-6}\ \text{S/km}$

Voltage level (U_2 = 400 kV)

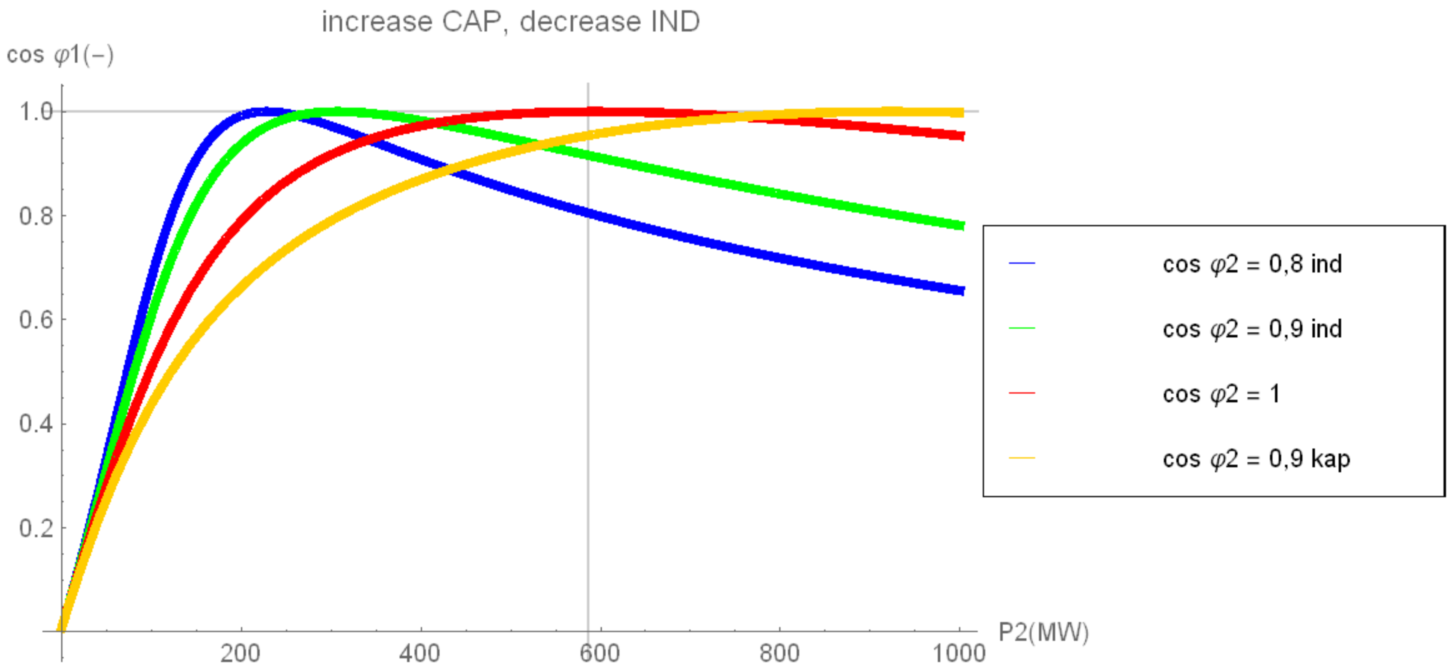


$U_1 < U_n$: Ferranti effect

$U_1 \sim U_n$ for S_p area and cos φ = 1

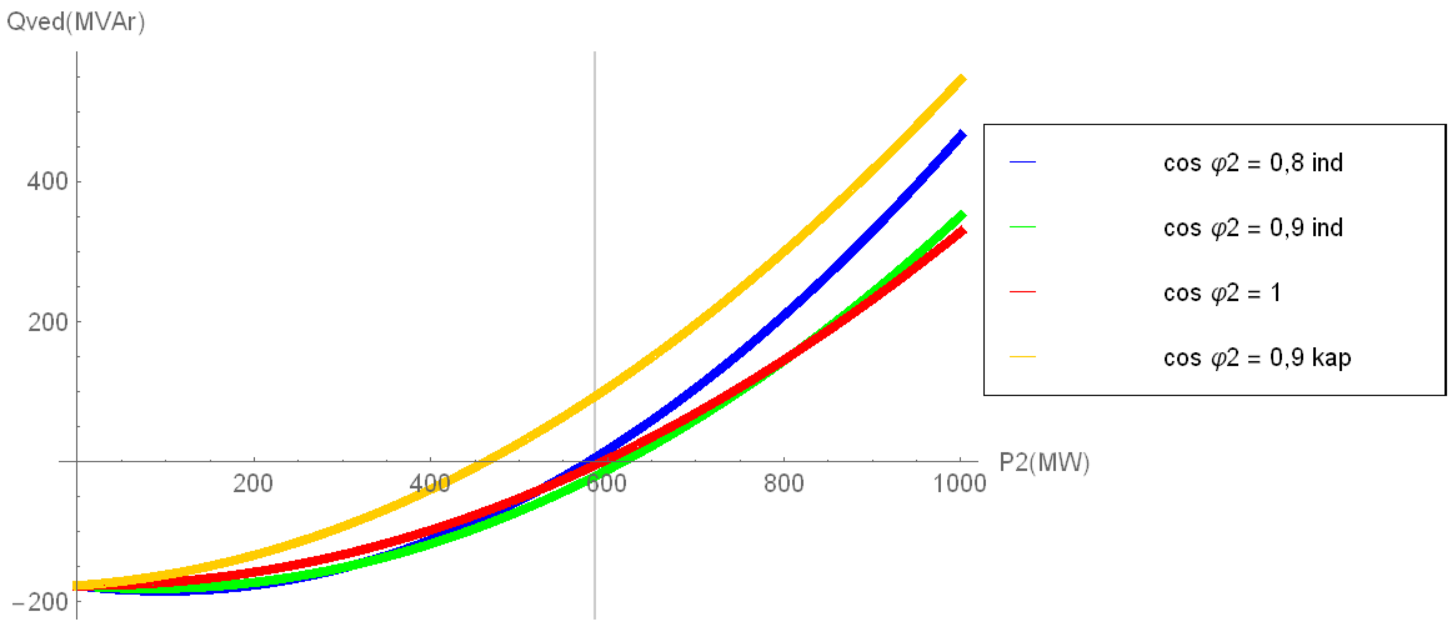
Transmission power factor

$$\cos\varphi_1 = \frac{P_1}{S_1}$$

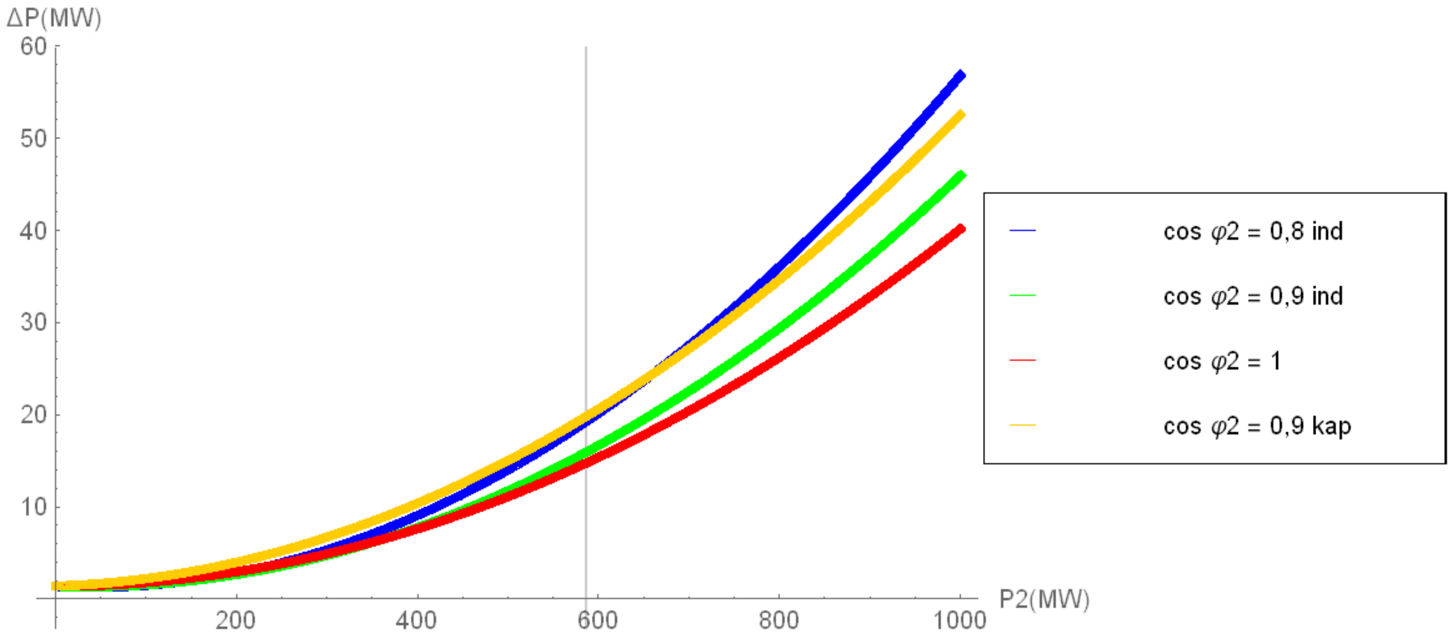


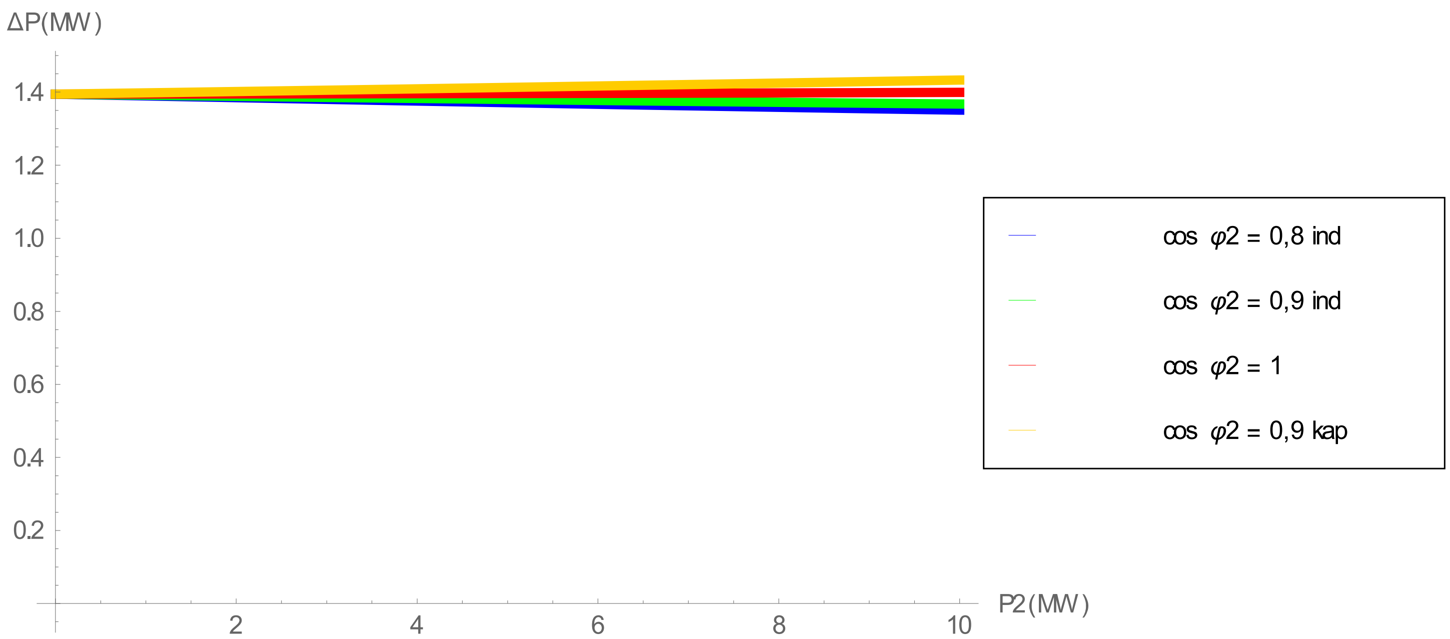
open-circuit → line is like capacitive load
higher power → line „self-compensation“

Line reactive power



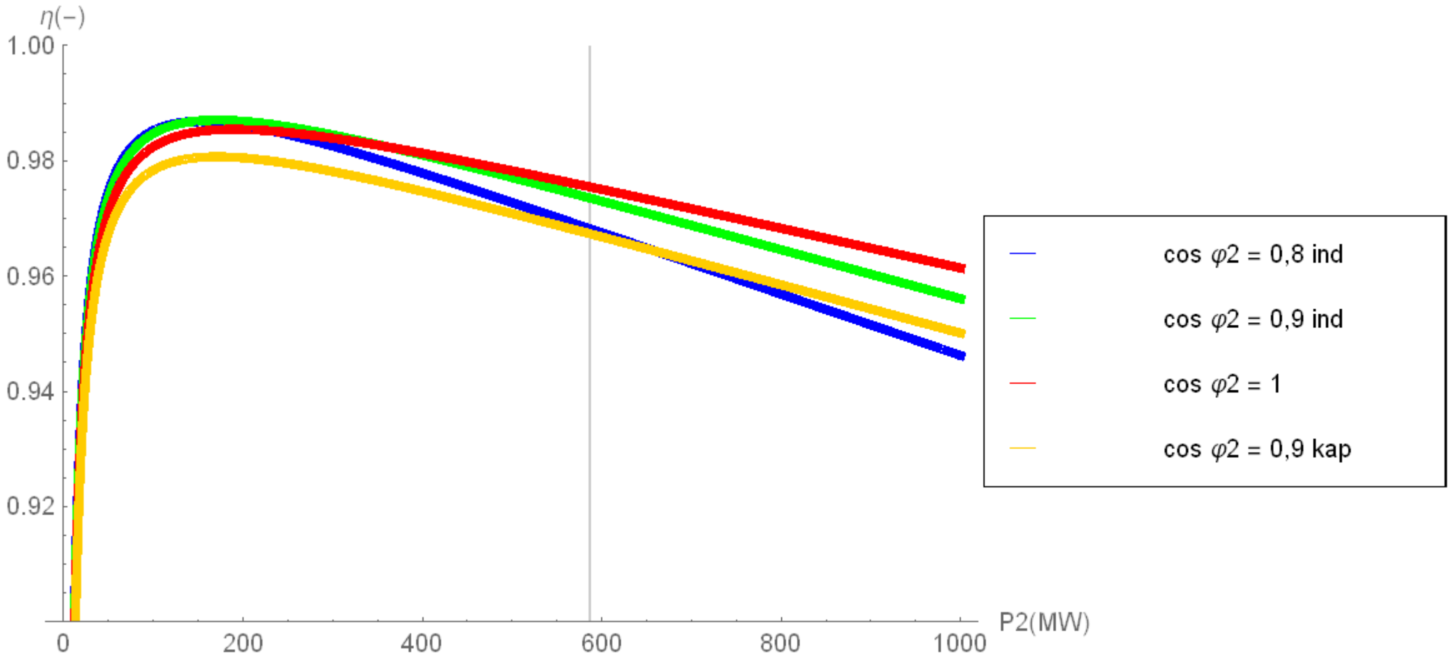
Line losses
= open-circuit ~ U^2 + load ~ I^2




ΔP(MW)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
2
4
6
8
10
P2(MW)
cos φ2 = 0,8 ind
cos φ2 = 0,9 ind
cos φ2 = 1
cos φ2 = 0,9 kap

Transmission efficiency

$$\eta = \frac{P_2}{P_1}$$



maximum for low powers
for higher powers a flat curve

Newton-Raphson method

- the most often method for non-linear equations
- it uses Taylor polynomial
- it converts non-linear equations solution to linear equations solution, gradually higher precision of the estimation

Basic idea

$$f(x) = c$$

If $x^{(0)}$ is the initial estimation and $\Delta x^{(0)}$ is the difference from the right solution, then

$$f(x^{(0)} + \Delta x^{(0)}) = c$$

Taylor series

$$f(x)\Big|_{x_0} = \sum_{k=0}^{\infty} \frac{\left(\frac{df(x_0)}{dx}\right)^{(k)}}{k!} (x - x_o)^k$$

Expansion to the Taylor series

$$f(x^{(0)}) + \left(\frac{df}{dx}\right)^{(0)} \Delta x^{(0)} + \frac{1}{2!}\left(\frac{d^2 f}{dx^2}\right)^{(0)} (\Delta x^{(0)})^2 + \ldots = c$$

Higher orders neglecting (linearization)

$$\Delta c^{(0)} \approx \left(\frac{df}{dx}\right)^{(0)} \Delta x^{(0)}$$

where

$$\Delta c^{(0)} = c - f(x^{(0)})$$

is called “defect”.

Adding $\Delta x^{(0)}$ to the initial estimation gives the second approximation

$$x^{(1)} = x^{(0)} + \frac{\Delta c^{(0)}}{\left(\dfrac{df}{dx}\right)^{(0)}}$$

(Note: impossible if the derivative equals zero)

The same relations in the next steps give the method algorithm:

$$\Delta c^{(k)} = c - f(x^{(k)})$$

$$\Delta x^{(k)} = \frac{\Delta c^{(k)}}{\left(\dfrac{df}{dx}\right)^{(k)}}$$

$$x^{(k+1)} = x^{(k)} + \Delta x^{(k)}$$

$$\Delta c^{(k+1)} = c - f(x^{(k+1)})$$

The system of n equations with n unknowns

$$f_1(x_1, x_2, ..., x_n) = c_1$$

$$f_2(x_1, x_2, ..., x_n) = c_2$$

....

$$f_n(x_1, x_2, ..., x_n) = c_n$$

Expansion to the Taylor series

$$(f_1)^{(0)} + \left(\frac{\partial f_1}{\partial x_1}\right)^{(0)} \Delta x_1^{(0)} + \left(\frac{\partial f_1}{\partial x_2}\right)^{(0)} \Delta x_2^{(0)} + \ldots + \left(\frac{\partial f_1}{\partial x_n}\right)^{(0)} \Delta x_n^{(0)} = c_1$$

$$(f_2)^{(0)} + \left(\frac{\partial f_2}{\partial x_1}\right)^{(0)} \Delta x_1^{(0)} + \left(\frac{\partial f_2}{\partial x_2}\right)^{(0)} \Delta x_2^{(0)} + \ldots + \left(\frac{\partial f_2}{\partial x_n}\right)^{(0)} \Delta x_n^{(0)} = c_2$$

....

$$(f_n)^{(0)} + \left(\frac{\partial f_n}{\partial x_1}\right)^{(0)} \Delta x_1^{(0)} + \left(\frac{\partial f_n}{\partial x_2}\right)^{(0)} \Delta x_2^{(0)} + \ldots + \left(\frac{\partial f_n}{\partial x_n}\right)^{(0)} \Delta x_n^{(0)} = c_n$$

Matrix expression

$$\begin{pmatrix} c_1 - (f_1^{(0)}) \\ c_2 - (f_2^{(0)}) \\ \vdots \\ c_n - (f_n^{(0)}) \end{pmatrix} = \begin{pmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^{(0)} & \left(\frac{\partial f_1}{\partial x_2}\right)^{(0)} & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^{(0)} \\ \left(\frac{\partial f_2}{\partial x_1}\right)^{(0)} & \left(\frac{\partial f_2}{\partial x_2}\right)^{(0)} & \cdots\cdot & \left(\frac{\partial f_2}{\partial x_n}\right)^{(0)} \\ \vdots & \vdots & \vdots & \vdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^{(0)} & \left(\frac{\partial f_n}{\partial x_2}\right)^{(0)} & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^{(0)} \end{pmatrix} \cdot \begin{pmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \\ \vdots \\ \Delta x_n^{(0)} \end{pmatrix}$$

in short

$$\left(\Delta C^{(0)}\right) = \left(J^{(0)}\right) \cdot \left(\Delta X^{(0)}\right)$$

Hence

$$\left(\Delta X^{(0)}\right) = \left(J^{(0)}\right)^{-1} \cdot \left(\Delta C^{(0)}\right)$$

The method algorithm:

$$\left(\Delta C^{(k)}\right) = \begin{pmatrix} c_1 - (f_1^{(k)}) \\ c_2 - (f_2^{(k)}) \\ \vdots \\ c_n - (f_n^{(k)}) \end{pmatrix}$$

$$\left(\Delta X^{(k)}\right) = \left(J^{(k)}\right)^{-1} \cdot \left(\Delta C^{(k)}\right)$$

$$\left(X^{(k+1)}\right) = \left(X^{(k)}\right) + \left(\Delta X^{(k)}\right)$$

$$\left(\Delta C^{(k+1)}\right)=\begin{pmatrix} c_1-(f_1^{(k+1)}) \\ c_2-(f_2^{(k+1)}) \\ \vdots \\ c_n-(f_n^{(k+1)}) \end{pmatrix}$$

where

$$\left(\Delta X^{(k)}\right)=\begin{pmatrix} \Delta x_1^{(k)} \\ \Delta x_2^{(k)} \\ \vdots \\ \Delta x_n^{(k)} \end{pmatrix}$$

$$\left(J^{(k)}\right)=\begin{pmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_1}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^{(k)} \\ \left(\frac{\partial f_2}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_2}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^{(k)} \\ \vdots & \vdots & \vdots & \vdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_n}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^{(k)} \end{pmatrix}$$

$(J^{(k)})$ – Jakobi matrix, regularity assumption

Load Flow solution

U-I equations system can be extended to voltage-power dependence

$$\hat{I}_k = \sum_{m=1}^{n} \hat{U}_{fm} \hat{Y}_{(k,m)}$$

$$\hat{S}_k = 3\hat{S}_{fk} = 3\hat{U}_{fk}\hat{I}_k^* = 3\hat{U}_{fk} \sum_{m=1}^{n} \hat{U}_{fm}^* \hat{Y}_{(k,m)}^*$$

$$\hat{S}_k = \hat{U}_k \sum_{m=1}^{n} \hat{U}_m^* \hat{Y}_{(k,m)}^*$$

$$\left(\hat{S}\right) = \left(\hat{U}_{diag}\right)\left(\hat{Y}^*\right)\left(\hat{U}^*\right)$$

$$\begin{pmatrix} \hat{S}_1 \\ \dots \\ \hat{S}_k \\ \dots \\ \hat{S}_n \end{pmatrix} = \begin{pmatrix} \hat{U}_1 & 0 & 0 & 0 & 0 \\ 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & \hat{U}_k & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \hat{U}_n \end{pmatrix} \cdot \begin{pmatrix} \hat{Y}^*_{(1,1)} & \dots & \dots & \dots & \hat{Y}^*_{(1,n)} \\ \dots & \dots & \dots & \dots & \dots \\ \hat{Y}^*_{(k,1)} & \dots & \hat{Y}^*_{(k,k)} & \dots & \hat{Y}^*_{(k,n)} \\ \dots & \dots & \dots & \dots & \dots \\ \hat{Y}^*_{(n,1)} & \dots & \dots & \dots & \hat{Y}^*_{(n,n)} \end{pmatrix} \cdot \begin{pmatrix} \hat{U}^*_1 \\ \dots \\ \hat{U}^*_k \\ \dots \\ \hat{U}^*_n \end{pmatrix}$$

- powers defined → nonlinearity

Aim: to calculate **P, Q, U, δ** in nodes and branches

Note: Assumption of symmetrical system and its loading → single phase models.

Bus types

Bus power		Bus voltage phasor components	
defined	to be calculated	defined	to be calculated
–	P, Q	U, δ	–
P, Q	–	–	U, δ
P	Q	U	δ
Q	P	δ	U

slack (swing bus) – „balance bus“, balance P, Q for losses, as a huge system, large generation

PQ – loads

PU – generators, controlled voltage

Quantities

- fixed – requirements (P, Q for loads; P for generators)
- state – independent variables (U, δ for loads; δ for generators)
- control – here no changes (U for slack and generators), they change in optimization procedures

Calculations in relative values

Denominated values

$$\hat{S} = 3\hat{U}_f\hat{I}^* = \sqrt{3}\hat{U}\hat{I}^* \qquad \hat{U}_f = \hat{Z}\hat{I} \qquad \hat{U} = \sqrt{3}\hat{Z}\hat{I}$$

Base values

$$\hat{S}_v = \sqrt{3}\hat{U}_v\hat{I}_v^*$$

$$\hat{Z}_v = \frac{\hat{U}_v}{\sqrt{3}\hat{I}_v} = \frac{\hat{U}_v}{\sqrt{3}\left(\dfrac{\hat{S}_v}{\sqrt{3}\hat{U}_v}\right)^*} = \frac{U_v^2}{\hat{S}_v^*}$$

Relative values

$$\hat{s} \cdot S_v = \sqrt{3} \cdot \hat{u} \cdot U_v \cdot \hat{i}^* \cdot I_v^* \qquad \hat{u} \cdot U_v = \sqrt{3} \cdot \hat{z} \cdot Z_v \cdot \hat{i} \cdot I_v$$

$$\underline{\hat{s} = \hat{u} \cdot \hat{i}^*} \qquad \underline{\hat{u} = \hat{z} \cdot \hat{i}}$$

Bus current (single phase)

$$\hat{I}_k = \hat{U}_{fk}\left(\sum_{\substack{m=1\\m\neq k}}^{n}\hat{Y}_{km} + \hat{Y}_{k0}\right) - \sum_{\substack{m=1\\m\neq k}}^{n}\hat{U}_{fm}\hat{Y}_{km}$$

$$\hat{i}_i = \hat{u}_i\sum_{\substack{j=0\\j\neq i}}^{n}\hat{y}_{ij} - \sum_{\substack{j=1\\j\neq i}}^{n}\hat{y}_{ij}\hat{u}_j$$

Bus power

$$p_i + jq_i = \hat{u}_i \cdot \hat{i}_i^*$$

$$\hat{i}_i = \frac{p_i - jq_i}{\hat{u}_i^*}$$

hence

$$\frac{p_i - jq_i}{\hat{u}_i^*} = \hat{u}_i\sum_{\substack{j=0\\j\neq i}}^{n}\hat{y}_{ij} - \sum_{\substack{j=1\\j\neq i}}^{n}\hat{y}_{ij}\hat{u}_j$$

Newton-Raphson Power Flow Solution

$$\hat{S}_i = \hat{U}_i \sum_{j=1}^{n} \hat{U}_j^* \hat{Y}_{(i,j)}^* = U_i^2 \hat{Y}_{(i,i)}^* + \hat{U}_i \sum_{\substack{j=1 \\ j \neq i}}^{n} \hat{U}_j^* \hat{Y}_{(i,j)}^*$$

$$\hat{S}_i = f_i\left(\left(\hat{U}\right)\right)$$

Exponential form

$$\hat{S}_i = P_i + jQ_i \qquad \hat{U}_i = U_i e^{j\delta_i} \qquad \hat{Y}_{(i,j)} = Y_{(i,j)} e^{j\theta_{(i,j)}}$$

$$\hat{S}_i = U_i e^{j\delta_i} \sum_{j=1}^{n} U_j Y_{(i,j)} e^{-j\left(\delta_j + \theta_{(i,j)}\right)}$$

Power separated into the real and imaginary part

$$P_i = \sum_{j=1}^{n} U_i U_j Y_{(i,j)} \cos\left(\delta_i - \delta_j - \theta_{(i,j)}\right)$$

$$Q_i = \sum_{j=1}^{n} U_i U_j Y_{(i,j)} \sin\left(\delta_i - \delta_j - \theta_{(i,j)}\right)$$

→ 2 equations for each PQ bus, 1 equation for each PU bus

The power changes are expressed (linearization)

$$\Delta\hat{S}_i = \sum_{j=1}^{n} \left(\frac{\partial \hat{S}_i}{\partial \delta_j} \Delta\delta_j + \frac{\partial \hat{S}_i}{\partial U_j} \Delta U_j \right)$$

$$\Delta P_i = \sum_{j=1}^{n} \left(\frac{\partial P_i}{\partial \delta_j} \Delta\delta_j + \frac{\partial P_i}{\partial U_j} \Delta U_j \right)$$

$$\Delta Q_i = \sum_{j=1}^{n} \left(\frac{\partial Q_i}{\partial \delta_j} \Delta\delta_j + \frac{\partial Q_i}{\partial U_j} \Delta U_j \right)$$

Complete equation description

$$\begin{pmatrix} \Delta P_2^{(k)} \\ \cdots \\ \Delta P_n^{(k)} \\ \Delta Q_2^{(k)} \\ \cdots \\ \Delta Q_n^{(k)} \end{pmatrix} = \begin{pmatrix} \frac{\partial P_2}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial P_2}{\partial \delta_n}^{(k)} & \frac{\partial P_2}{\partial U_2}^{(k)} & \cdots & \frac{\partial P_2}{\partial U_n}^{(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial P_n}{\partial \delta_n}^{(k)} & \frac{\partial P_n}{\partial U_2}^{(k)} & \cdots & \frac{\partial P_n}{\partial U_n}^{(k)} \\ \frac{\partial Q_2}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial Q_2}{\partial \delta_n}^{(k)} & \frac{\partial Q_2}{\partial U_2}^{(k)} & \cdots & \frac{\partial Q_2}{\partial U_n}^{(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial Q_n}{\partial \delta_n}^{(k)} & \frac{\partial Q_n}{\partial U_2}^{(k)} & \cdots & \frac{\partial Q_n}{\partial U_n}^{(k)} \end{pmatrix} \cdot \begin{pmatrix} \Delta \delta_2^{(k)} \\ \cdots \\ \Delta \delta_n^{(k)} \\ \Delta U_2^{(k)} \\ \cdots \\ \Delta U_n^{(k)} \end{pmatrix}$$

More compact equations form

$$\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} = \begin{pmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial U} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial U} \end{pmatrix} \begin{pmatrix} \Delta \delta \\ \Delta U \end{pmatrix}$$

$$(J) = \begin{pmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial U} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial U} \end{pmatrix} = \begin{pmatrix} J_1 & J_2 \\ J_3 & J_4 \end{pmatrix}$$

Equations number for *n* buses, *s* slacks, *m* PU buses, *p* PQ buses (n = s + m + p):

ΔP x (n-s), ΔQ x (n-s-m)

$$P_i = \sum_{j=1}^{n} U_i U_j Y_{(i,j)} \cos\left(\delta_i - \delta_j - \theta_{(i,j)}\right)$$

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ j\neq i}}^{n} U_i U_j Y_{(i,j)} \sin\left(-\delta_i + \delta_j + \theta_{(i,j)}\right)$$

$$\frac{\partial P_i}{\partial \delta_j} = U_i U_j Y_{(i,j)} \sin\left(\delta_i - \delta_j - \theta_{(i,j)}\right) \qquad j \neq i$$

$$\frac{\partial P_i}{\partial U_i} = 2U_i Y_{(i,i)} \cos\left(\theta_{(i,i)}\right) + \sum_{\substack{j=1 \\ j\neq i}}^{n} U_j Y_{(i,j)} \cos\left(\delta_i - \delta_j - \theta_{(i,j)}\right)$$

$$\frac{\partial P_i}{\partial U_j} = U_i Y_{(i,j)} \cos\left(\delta_i - \delta_j - \theta_{(i,j)}\right) \qquad j \neq i$$

$$Q_i = \sum_{j=1}^{n} U_i U_j Y_{(i,j)} \sin\left(\delta_i - \delta_j - \theta_{(i,j)}\right)$$

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} U_i U_j Y_{(i,j)} \cos\left(\delta_i - \delta_j - \theta_{(i,j)}\right)$$

$$\frac{\partial Q_i}{\partial \delta_j} = -U_i U_j Y_{(i,j)} \cos\left(\delta_i - \delta_j - \theta_{(i,j)}\right) \qquad j \neq i$$

$$\frac{\partial Q_i}{\partial U_i} = -2 U_i Y_{(i,i)} \sin\left(\theta_{(i,i)}\right) + \sum_{\substack{j=1 \\ j \neq i}}^{n} U_j Y_{(i,j)} \sin\left(\delta_i - \delta_j - \theta_{(i,j)}\right)$$

$$\frac{\partial Q_i}{\partial U_j} = U_i Y_{(i,j)} \sin\left(\delta_i - \delta_j - \theta_{(i,j)}\right) \qquad j \neq i$$

Iterative solution idea

$$\begin{pmatrix} \delta \\ U \end{pmatrix}_k$$

defect $\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix}$

$$\begin{pmatrix} \Delta\delta \\ \Delta U \end{pmatrix} = (J)^{-1} \begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix}$$

$$\begin{pmatrix} \delta \\ U \end{pmatrix}_{k+1} = \begin{pmatrix} \delta \\ U \end{pmatrix}_k + \begin{pmatrix} \Delta\delta \\ \Delta U \end{pmatrix}$$