

## Short-circuits in ES

### Short-circuit:

- cross fault, quick emergency change in ES
- the most often fault in ES
- transient events occur during short-circuits

### Short-circuit formation:

- fault connection between phases or between phase(s) and the ground in the system with the grounded neutral point

### Main causes:

- insulation defect caused by overvoltage
- direct lightning strike
- insulation aging
- direct damage of overhead lines or cables

### Short-circuit impacts:

- total impedance of the network affected part decreases
- currents are increasing => so called short-circuit currents  $I_k$
- the voltage decreases near the short-circuit
- $I_k$  impacts causes device heating and power strain
- problems with  $I_k$  disconnecting, electrical arc and overvoltage occurred during the short-circuit
- synchronism disruption of ES working in parallel
- communication line disturbing => induced voltages

Note: In short-circuit places transient resistances arise.

- transient resistance is a sum of electrical arc resistance and resistance of other  $I_k$  way parts (determination of exact resistances is difficult)
- current and electrical arc length is changing during short-circuit => resistance of electrical arc is also changing

- transient resistances are neglected for  $I_k$  calculation (dimensioning of electrical devices) → perfect short-circuits

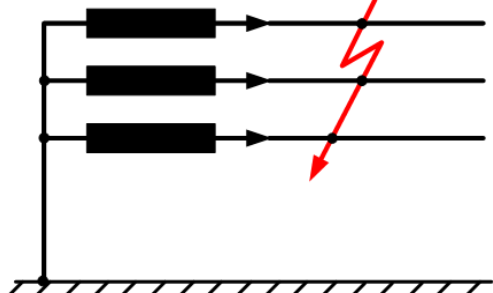
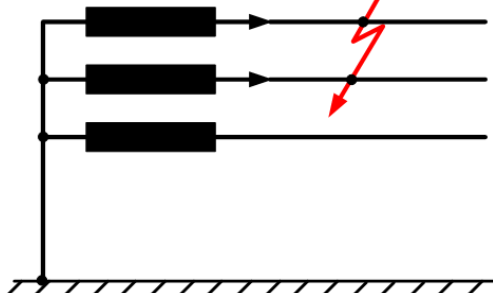
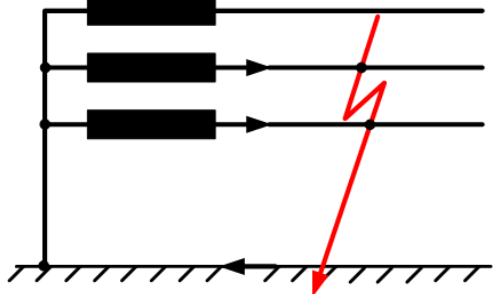
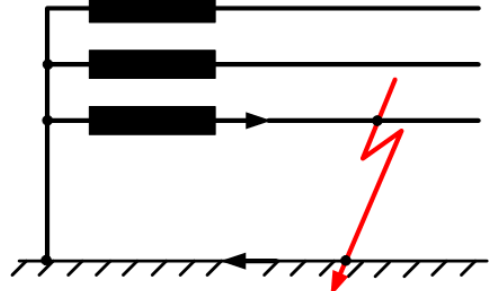
## Short-circuits types

### Symmetrical short-circuits:

- Three-phase short-circuit => all 3 phases are affected by short-circuit
  - little occurrence in the case of overhead lines
  - the most occurrences in the case of cable lines => other kinds of faults change to 3ph short-circuit due to electrical-arc impact

### Unbalanced (asymmetrical) short-circuits:

- phase-to-phase short-circuit
- double-phase-to-ground short-circuit
- single-phase-to-ground short-circuit:
  - in MV a different kind of fault => so called *ground fault*
  - in case of ground fault in MV (insulated or indirectly grounded neutral point) => no change in LV (grounded neutral point)

Short-circuit type	Diagram	Occurrence probability (%)		
		MV	110 kV	220 kV
3ph		5	0,6	0,9
2ph		10	4,8	0,6
2ph to ground		20	3,8	5,4
1ph		*	91	93,1

## Short-circuit current time behaviour

$$W_L = \frac{1}{2} Li^2$$

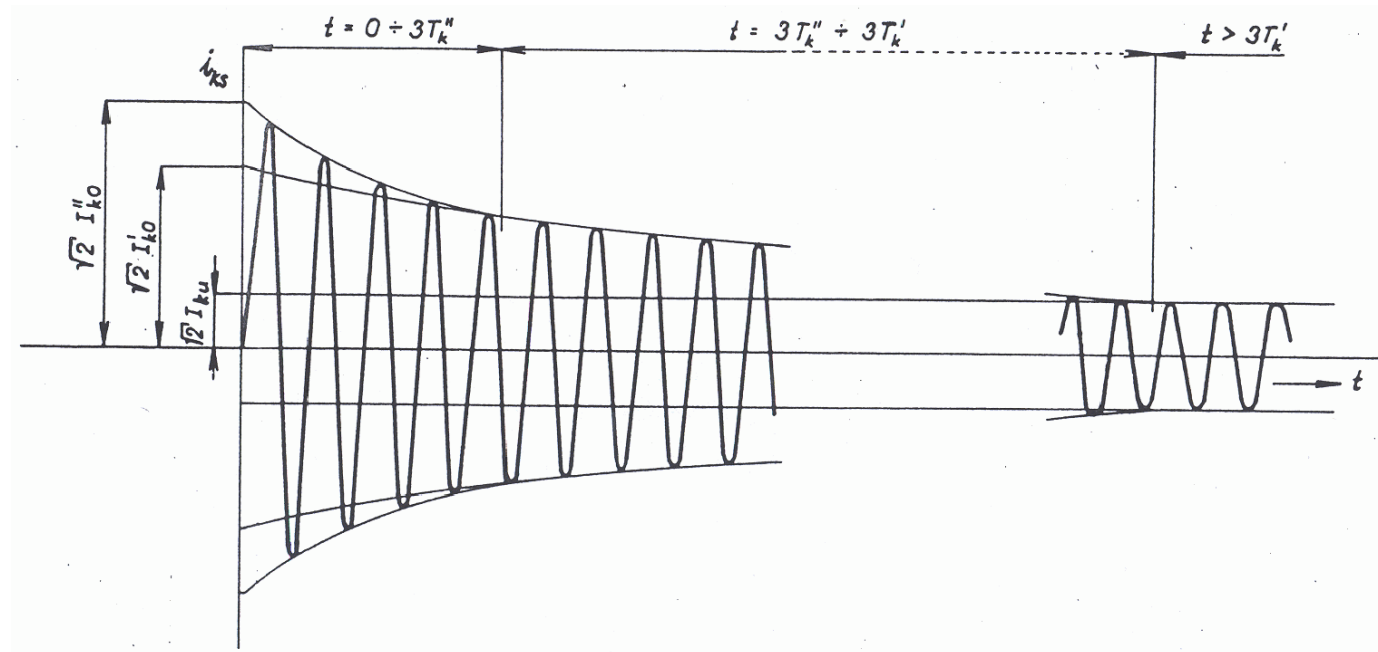
$$P = \frac{dW_L}{dt} < \infty \rightarrow \text{transient event}$$

Time behaviour: open-circuit, resistances neglected  
→ reactance, current of inductive character, higher  $I_k$  values

Impact of R on  $I_k$  attributes:

- finite R values decrease short-circuit impacts
- R neglecting results in time constants prolongation  $\tau = L/R$

$U = U_{\max}$  in the short-circuit moment  $\rightarrow I_k$  starts from zero (min. value)

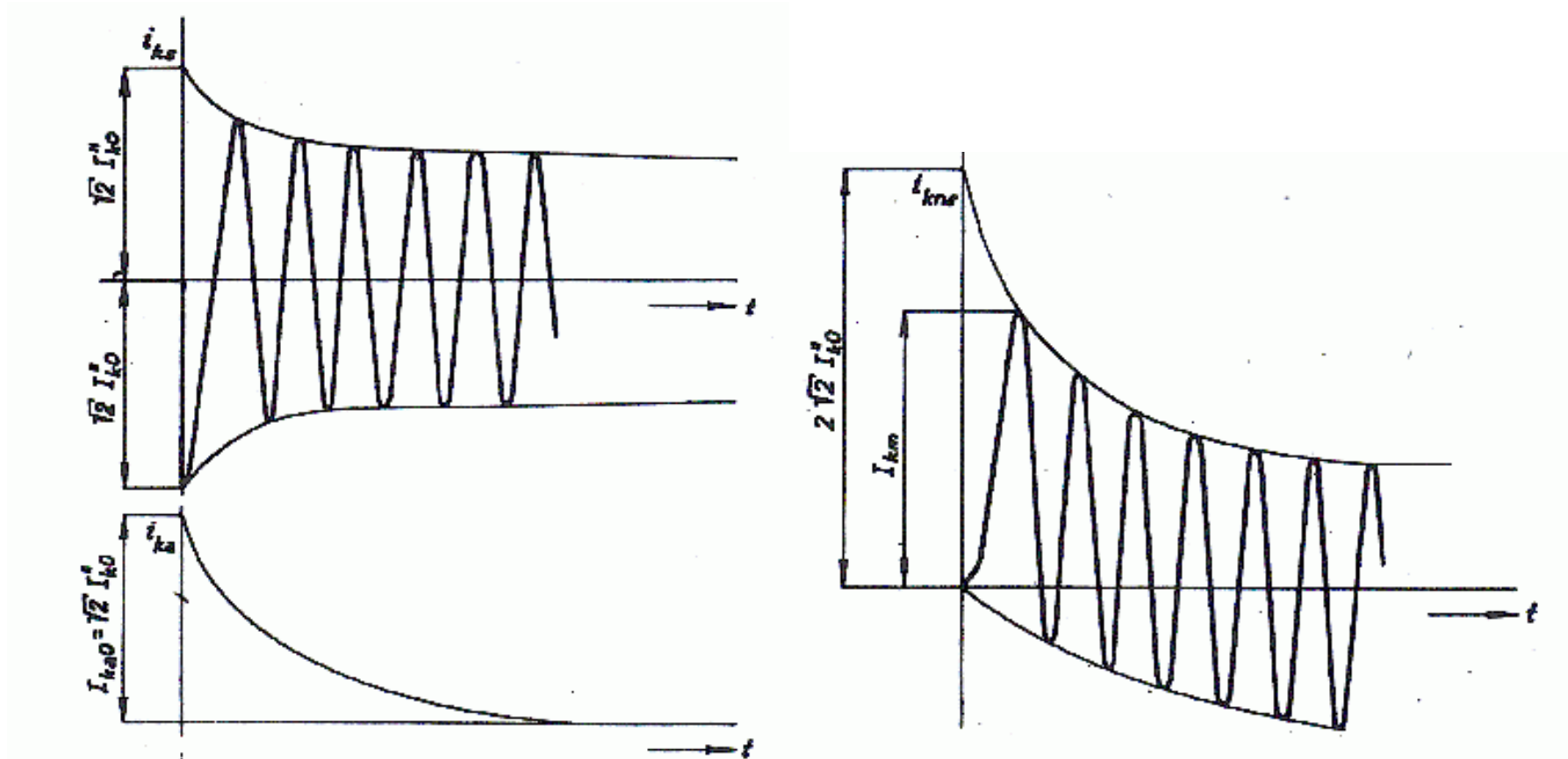


Short-circuit components ( $f = 50$  Hz):

- sub-transient – exponential envelope,  $T_k''$  (damping winding)
- transient – exponential envelope,  $T_k'$  (field winding)
- steady-state – constant magnitude

It is caused by synchronous machine behaviour during short-circuit  $\rightarrow$  more significant during short-circuits near the machine.

$U = 0$  in the short-circuit moment  $\rightarrow I_k$  starts from max. value



## Values

- symmetrical short-circuit current  $I_{ks}$  - steady-state, transient and sub-transient component sum, RMS value
- sub-transient short-circuit current  $I_k'' - I_{ks}$  RMS value in the period of sub-transient component  $t \doteq (0 \div 3T_k'')$
- initial sub-transient short-circuit current  $I_{k0}'' - I_k''$  value in the moment of short-circuit origin  $t = 0$
- DC component  $I_{ka}$  - disappears exponentially,  $T_{ka}$
- peak short-circuit current  $I_{km}$  - the first half-period magnitude during the maximal DC component



## Short-circuits in 3ph system

Conversion between phase values and symmetrical components

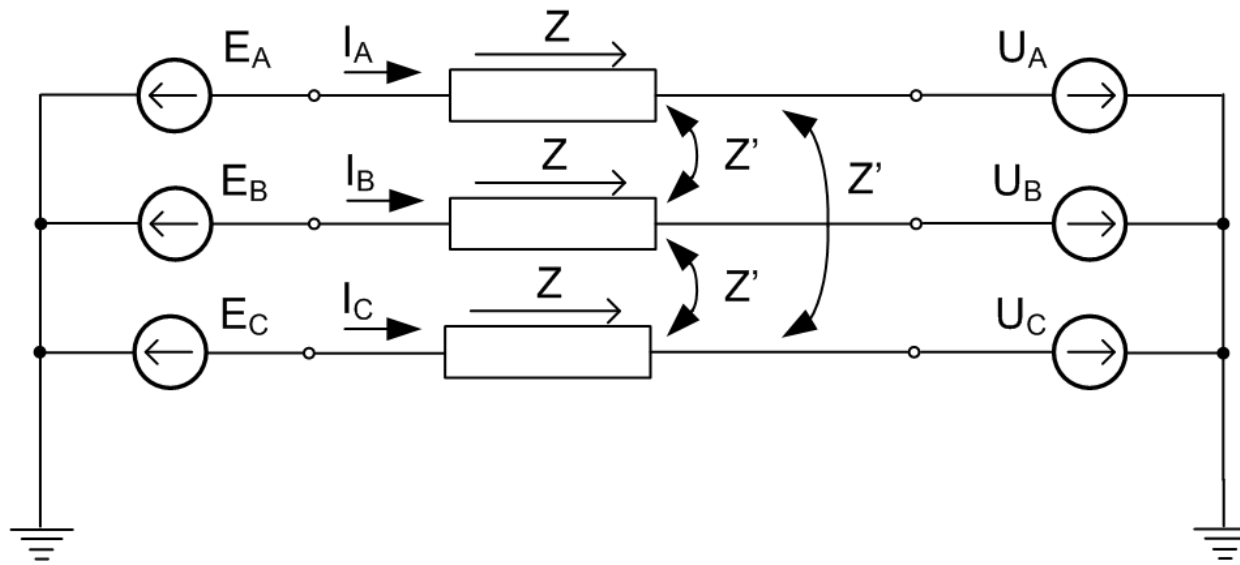
$$\begin{pmatrix} \hat{U}_{ABC} \end{pmatrix} = \begin{pmatrix} \hat{U}_A \\ \hat{U}_B \\ \hat{U}_C \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ \hat{a}^2 & \hat{a} & 1 \\ \hat{a} & \hat{a}^2 & 1 \end{pmatrix} \begin{pmatrix} \hat{U}_1 \\ \hat{U}_2 \\ \hat{U}_0 \end{pmatrix} = (\mathbf{T})(\mathbf{U}_{120})$$

$$\begin{pmatrix} \mathbf{U}_{120} \end{pmatrix} = \begin{pmatrix} \hat{U}_1 \\ \hat{U}_2 \\ \hat{U}_0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & \hat{a} & \hat{a}^2 \\ 1 & \hat{a}^2 & \hat{a} \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \hat{U}_A \\ \hat{U}_B \\ \hat{U}_C \end{pmatrix} = (\mathbf{T}^{-1})(\mathbf{U}_{ABC})$$

Impedance matrix in symmetrical components (for series sym. segment)

$$\begin{pmatrix} \hat{Z}_{120} \end{pmatrix} = \begin{pmatrix} \hat{Z}_1 & 0 & 0 \\ 0 & \hat{Z}_2 & 0 \\ 0 & 0 & \hat{Z}_0 \end{pmatrix} = \begin{pmatrix} \hat{Z} - \hat{Z}' & 0 & 0 \\ 0 & \hat{Z} - \hat{Z}' & 0 \\ 0 & 0 & \hat{Z} + 2\hat{Z}' \end{pmatrix}$$

### 3ph system during short-circuit – internal generator voltage E (or U<sub>i</sub>)



$$\begin{pmatrix} E_{ABC} \end{pmatrix} = \begin{pmatrix} Z_{ABC} \end{pmatrix} \begin{pmatrix} I_{ABC} \end{pmatrix} + \begin{pmatrix} U_{ABC} \end{pmatrix}$$

### Symmetrical system (independent systems 1, 2, 0)

$$\hat{E}_1 = \hat{Z}_1 \hat{I}_1 + \hat{U}_1$$

$$\hat{E}_2 = \hat{Z}_2 \hat{I}_2 + \hat{U}_2$$

$$\hat{E}_0 = \hat{Z}_0 \hat{I}_0 + \hat{U}_0$$

$$\begin{pmatrix} E_{120} \end{pmatrix} = \begin{pmatrix} Z_{120} \end{pmatrix} \begin{pmatrix} I_{120} \end{pmatrix} + \begin{pmatrix} U_{120} \end{pmatrix}$$

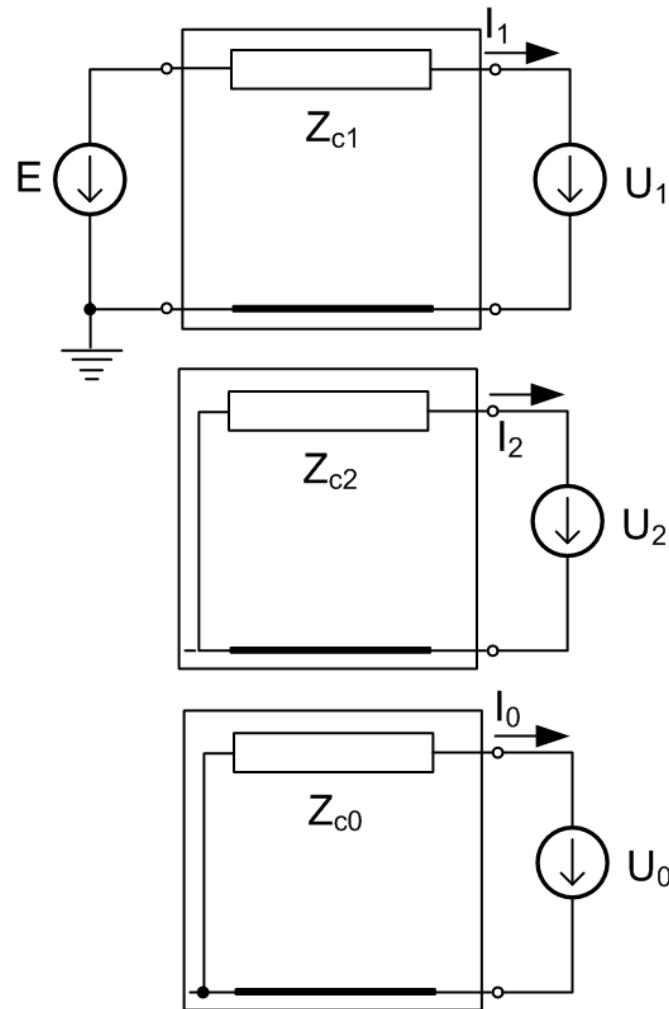
Generator symmetrical voltage → only positive sequence component  
 Reference phase A:

$$\begin{pmatrix} \hat{E}_{120} \end{pmatrix} = (\mathbf{T}^{-1})(\mathbf{E}_{ABC}) = \frac{1}{3} \begin{pmatrix} 1 & \hat{a} & \hat{a}^2 \\ 1 & \hat{a}^2 & \hat{a} \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \hat{E}_A \\ \hat{a}^2 \hat{E}_A \\ \hat{a} \hat{E}_A \end{pmatrix} = \begin{pmatrix} \hat{E}_A \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \hat{E} \\ 0 \\ 0 \end{pmatrix}$$

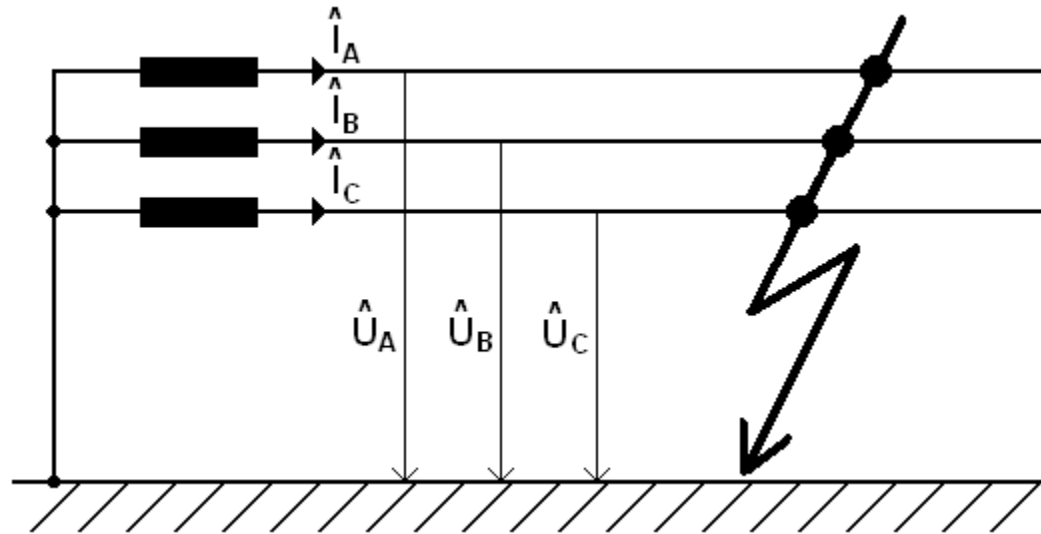
$$\begin{pmatrix} \hat{E}_1 \\ \hat{E}_2 \\ \hat{E}_0 \end{pmatrix} = \begin{pmatrix} \hat{E} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \hat{Z}_1 & 0 & 0 \\ 0 & \hat{Z}_2 & 0 \\ 0 & 0 & \hat{Z}_0 \end{pmatrix} \begin{pmatrix} \hat{I}_1 \\ \hat{I}_2 \\ \hat{I}_0 \end{pmatrix} + \begin{pmatrix} \hat{U}_1 \\ \hat{U}_2 \\ \hat{U}_0 \end{pmatrix}$$

Negative and zero sequence are caused by voltage unbalance in the faulted place.

In the fault point 6 quantities ( $U_{120}, I_{120}$ )  $\rightarrow$  3 equations necessary to be added by other 3 equations according to the short-circuit type (local unbalance description).



## Three-phase (to-ground) short-circuit



3 char. equations

$$\hat{U}_A = \hat{U}_B = \hat{U}_C = 0$$

## Components

$$(U_{120}) = (T^{-1})(U_{ABC}) = \frac{1}{3} \begin{pmatrix} 1 & \hat{a} & \hat{a}^2 \\ 1 & \hat{a}^2 & \hat{a} \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\hat{U}_1 = \hat{U}_2 = \hat{U}_0 = 0$$

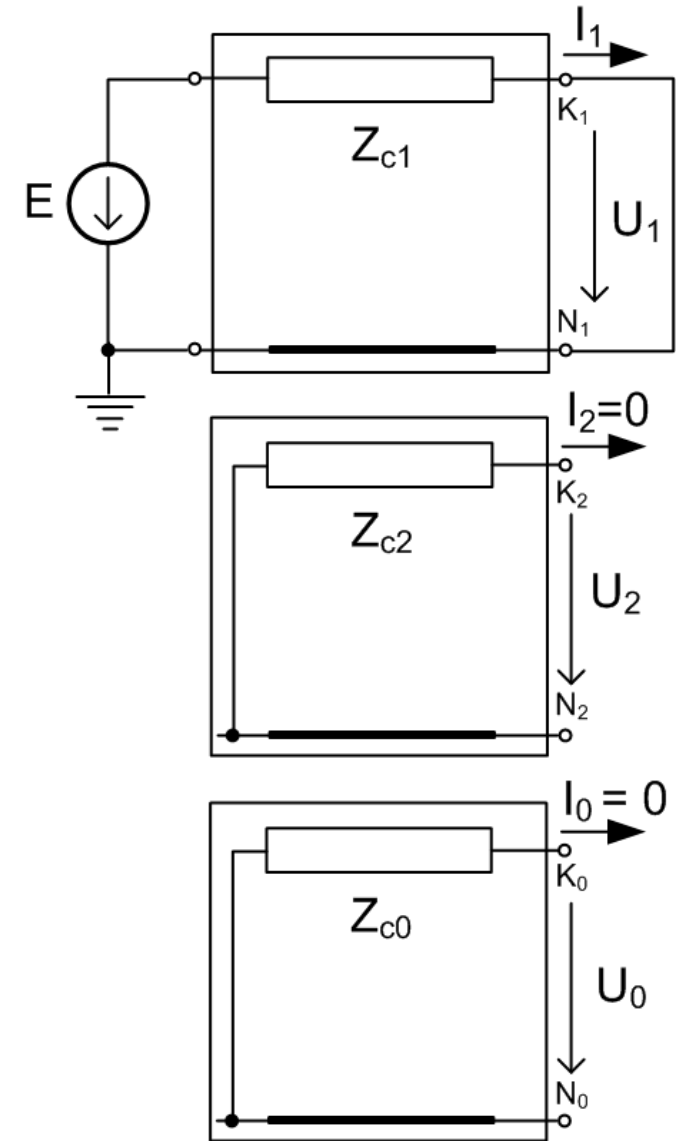
$$\hat{I}_1 = \frac{\hat{E}}{\hat{Z}_1}; \quad \hat{I}_2 = 0; \quad \hat{I}_0 = 0$$

## Phases

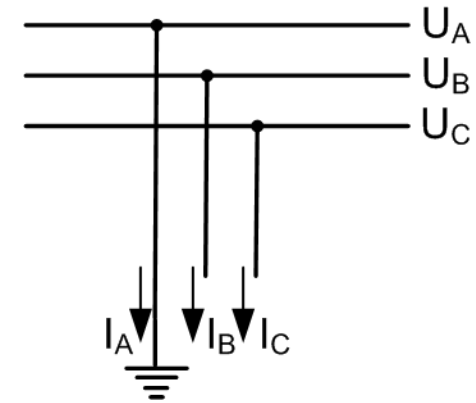
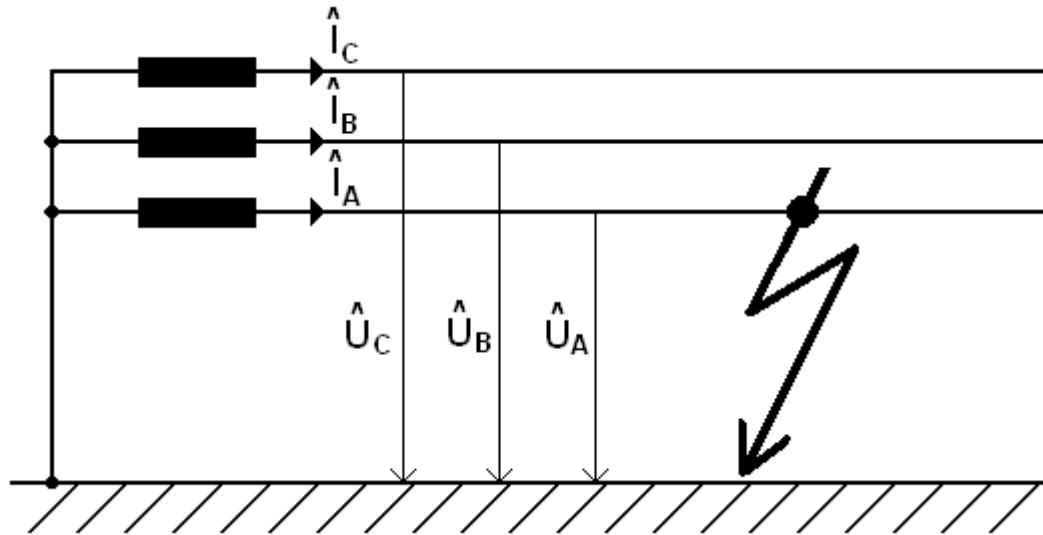
$$(I_{ABC}) = (T)(I_{120})$$

$$\hat{I}_A = \frac{\hat{E}}{\hat{Z}_1}; \quad \hat{I}_B = \hat{a}^2 \frac{\hat{E}}{\hat{Z}_1}; \quad \hat{I}_C = \hat{a} \frac{\hat{E}}{\hat{Z}_1}$$

Only the positive-sequence component included.



## Single-phase-to-ground short-circuit



3 char. equations

$$\hat{U}_A = 0; \hat{I}_B = \hat{I}_C = 0$$

## Components

$$(\mathbf{I}_{120}) = (\mathbf{T}^{-1})(\mathbf{I}_{ABC}) = \frac{1}{3} \begin{pmatrix} 1 & \hat{a} & \hat{a}^2 \\ 1 & \hat{a}^2 & \hat{a} \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \hat{I}_A \\ 0 \\ 0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} \hat{I}_A \\ \hat{I}_A \\ \hat{I}_A \end{pmatrix}$$

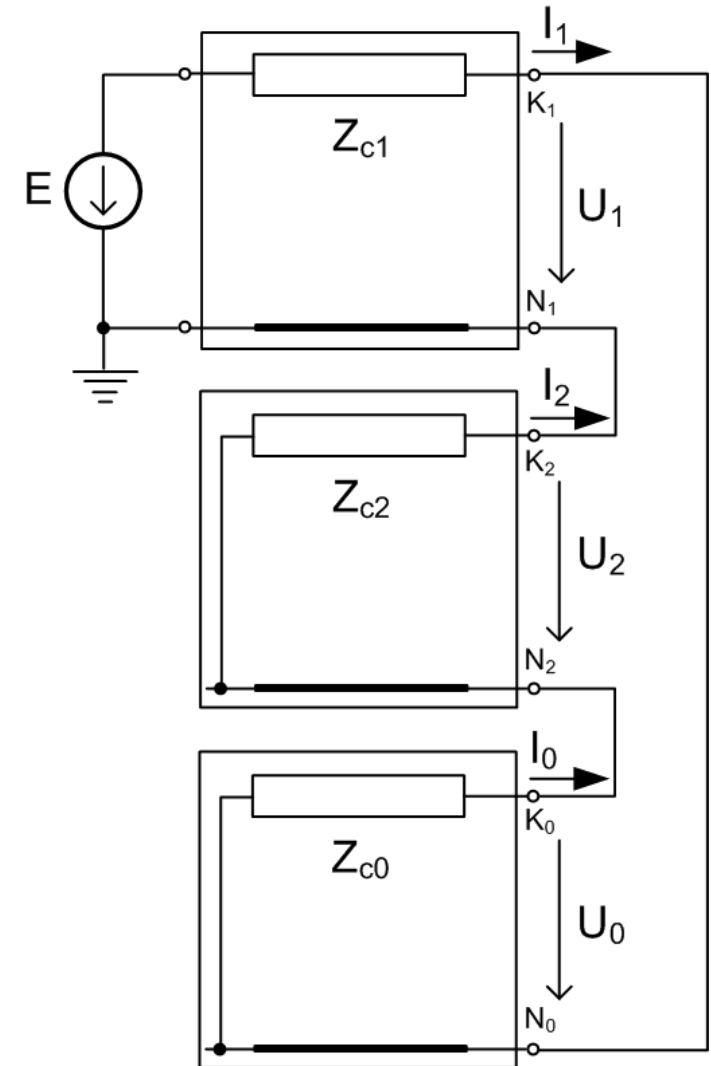
$$\hat{I}_1 = \hat{I}_2 = \hat{I}_0 = \frac{\hat{E}}{\hat{Z}_1 + \hat{Z}_2 + \hat{Z}_0}$$

$$\hat{U}_1 = (\hat{Z}_0 + \hat{Z}_2)\hat{I}_1$$

$$\hat{U}_2 = -\hat{Z}_2\hat{I}_1$$

$$\hat{U}_0 = -\hat{Z}_0\hat{I}_1$$

All three components are in series.





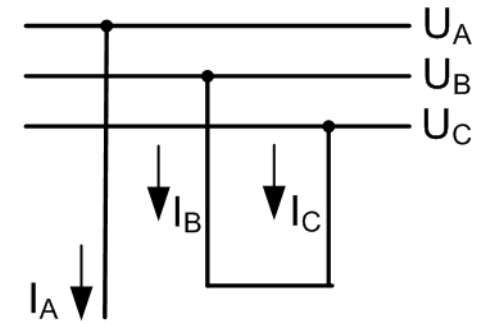
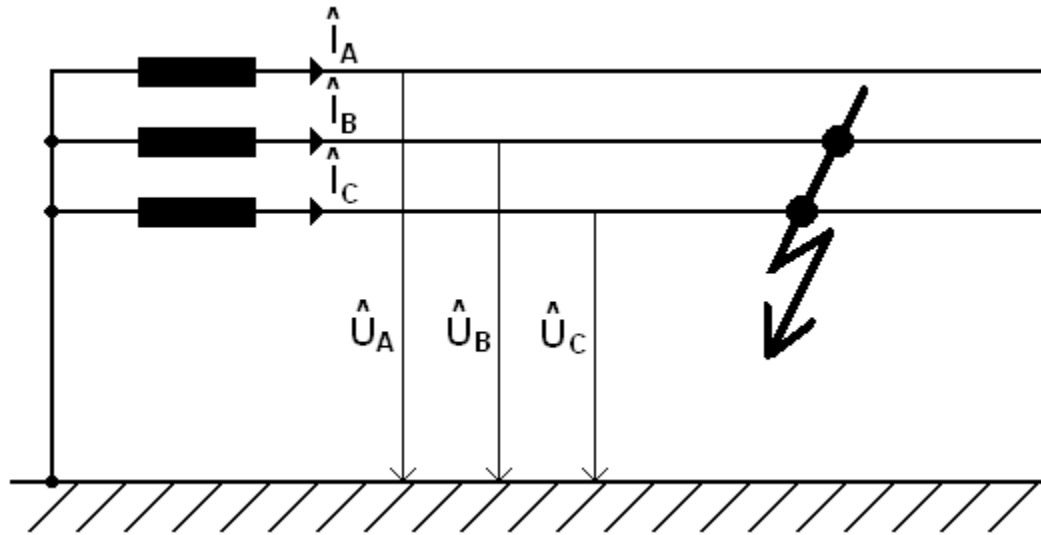
## Phases

$$(\mathbf{I}_{ABC}) = (\mathbf{T})(\mathbf{I}_{120}) = \begin{pmatrix} 1 & 1 & 1 \\ \hat{a}^2 & \hat{a} & 1 \\ \hat{a} & \hat{a}^2 & 1 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{I}}_1 \\ \hat{\mathbf{I}}_1 \\ \hat{\mathbf{I}}_1 \end{pmatrix} = \begin{pmatrix} 3\hat{\mathbf{I}}_1 \\ 0 \\ 0 \end{pmatrix}$$

$$\hat{\mathbf{I}}_A = \frac{3\hat{\mathbf{E}}}{\hat{\mathbf{Z}}_1 + \hat{\mathbf{Z}}_2 + \hat{\mathbf{Z}}_0}; \quad \hat{\mathbf{I}}_B = 0; \quad \hat{\mathbf{I}}_C = 0$$

$$(\mathbf{U}_{ABC}) = (\mathbf{T})(\mathbf{U}_{120}) = \begin{pmatrix} 1 & 1 & 1 \\ \hat{a}^2 & \hat{a} & 1 \\ \hat{a} & \hat{a}^2 & 1 \end{pmatrix} \begin{pmatrix} (\hat{\mathbf{Z}}_0 + \hat{\mathbf{Z}}_2)\hat{\mathbf{I}}_1 \\ -\hat{\mathbf{Z}}_2\hat{\mathbf{I}}_1 \\ -\hat{\mathbf{Z}}_0\hat{\mathbf{I}}_1 \end{pmatrix} = \begin{pmatrix} 0 \\ (\hat{a}^2 - \hat{a})\hat{\mathbf{Z}}_2 + (\hat{a}^2 - 1)\hat{\mathbf{Z}}_0 \\ (\hat{a} - \hat{a}^2)\hat{\mathbf{Z}}_2 + (\hat{a} - 1)\hat{\mathbf{Z}}_0 \end{pmatrix} \hat{\mathbf{I}}_1$$

## Phase-to-phase short-circuit



3 char. equations

$$\hat{U}_B = \hat{U}_C; \hat{I}_B = -\hat{I}_C; \hat{I}_A = 0$$

## Components

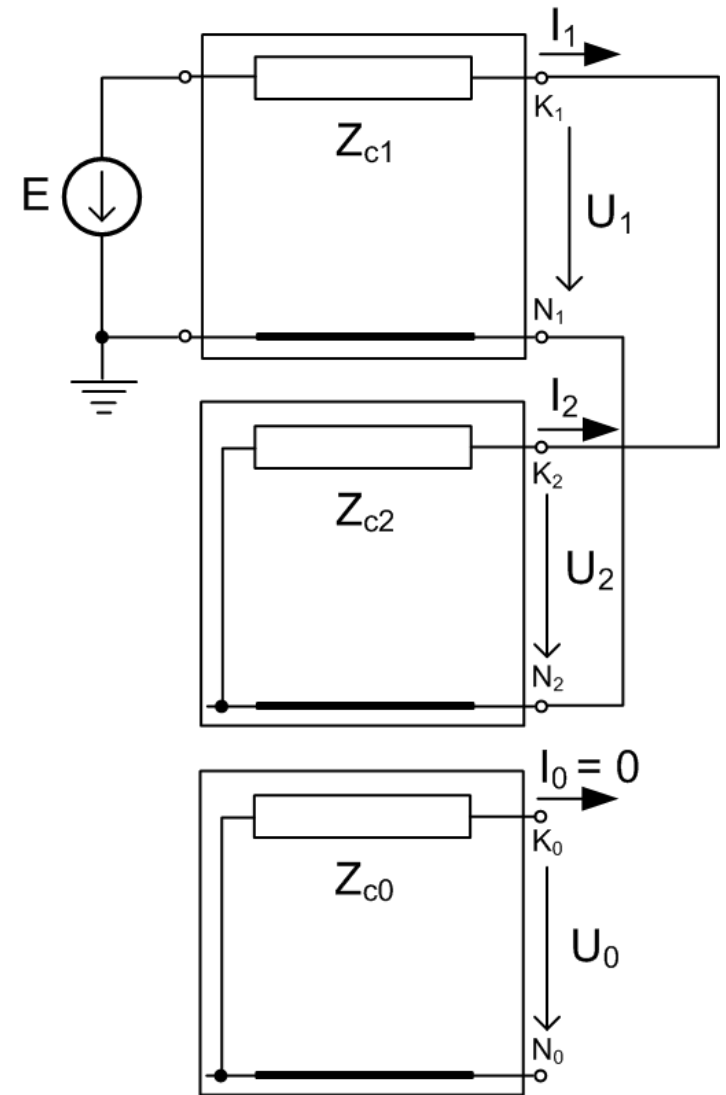
$$(\mathbf{I}_{120}) = \frac{1}{3} \begin{pmatrix} 1 & \hat{a} & \hat{a}^2 \\ 1 & \hat{a}^2 & \hat{a} \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \hat{I}_B \\ -\hat{I}_B \end{pmatrix} = \frac{1}{3} \begin{pmatrix} j\sqrt{3}\hat{I}_B \\ -j\sqrt{3}\hat{I}_B \\ 0 \end{pmatrix}$$

$$\hat{I}_1 = \frac{\hat{E}}{\hat{Z}_1 + \hat{Z}_2}; \quad \hat{I}_2 = -\hat{I}_1; \quad \hat{I}_0 = 0$$

$$\hat{U}_1 = \hat{U}_2 = \frac{\hat{Z}_2 \cdot \hat{E}}{\hat{Z}_1 + \hat{Z}_2} = \hat{Z}_2 \cdot \hat{I}_1$$

$$\hat{U}_0 = 0$$

Positive and negative components in parallel.



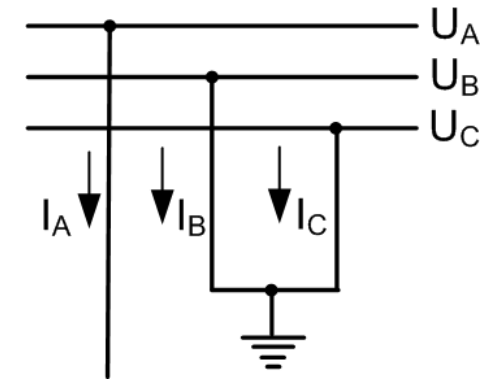
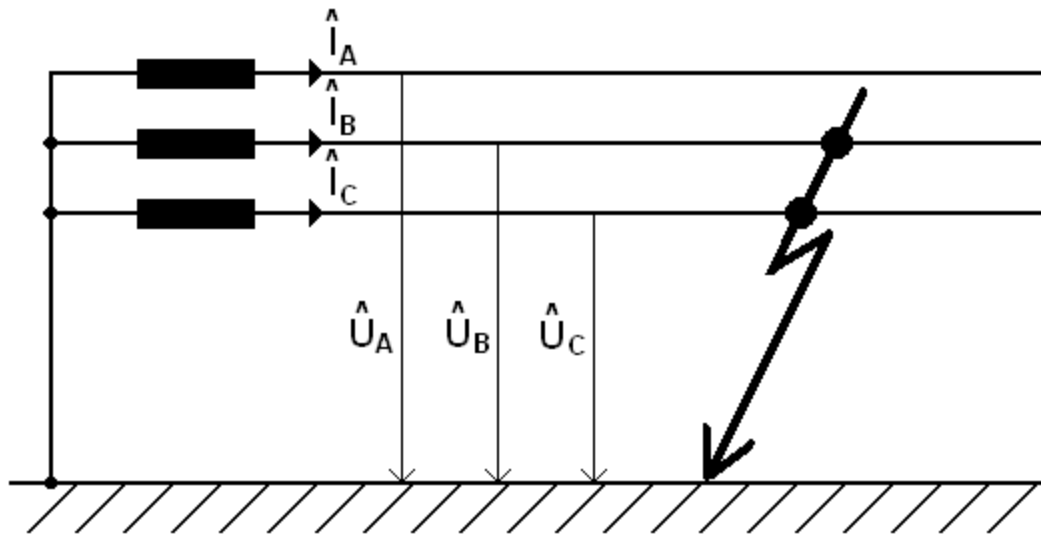
## Phases

$$(\mathbf{I}_{ABC}) = (\mathbf{T})(\mathbf{I}_{120}) = \begin{pmatrix} 1 & 1 & 1 \\ \hat{a}^2 & \hat{a} & 1 \\ \hat{a} & \hat{a}^2 & 1 \end{pmatrix} \begin{pmatrix} \hat{I}_1 \\ -\hat{I}_1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -j\sqrt{3}\hat{I}_1 \\ j\sqrt{3}\hat{I}_1 \end{pmatrix}$$

$$\hat{I}_A = 0; \quad \hat{I}_B = \frac{-j\sqrt{3}\hat{E}}{\hat{Z}_1 + \hat{Z}_2}; \quad \hat{I}_C = \frac{j\sqrt{3}\hat{E}}{\hat{Z}_1 + \hat{Z}_2}$$

$$(\mathbf{U}_{ABC}) = (\mathbf{T})(\mathbf{U}_{120}) = \begin{pmatrix} 1 & 1 & 1 \\ \hat{a}^2 & \hat{a} & 1 \\ \hat{a} & \hat{a}^2 & 1 \end{pmatrix} \begin{pmatrix} \hat{U}_1 \\ \hat{U}_1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\hat{U}_1 \\ -\hat{U}_1 \\ -\hat{U}_1 \end{pmatrix} = \begin{pmatrix} 2\hat{Z}_2 \cdot \hat{I}_1 \\ -\hat{Z}_2 \cdot \hat{I}_1 \\ -\hat{Z}_2 \cdot \hat{I}_1 \end{pmatrix}$$

## Double-phase-to-ground short-circuit



3 char. equations

$$\hat{U}_B = \hat{U}_C = 0; \hat{I}_A = 0$$

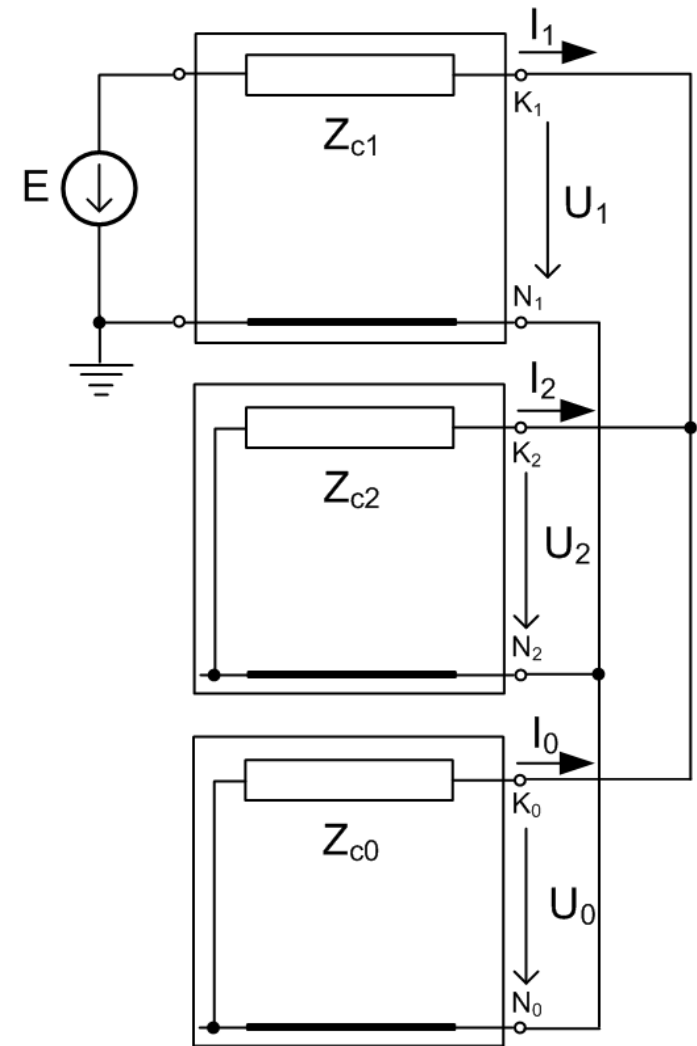
## Components

$$(\mathbf{U}_{120}) = \frac{1}{3} \begin{pmatrix} 1 & \hat{a} & \hat{a}^2 \\ 1 & \hat{a}^2 & \hat{a} \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \hat{U}_A \\ 0 \\ 0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} \hat{U}_A \\ \hat{U}_A \\ \hat{U}_A \end{pmatrix}$$

$$\hat{I}_1 = \frac{\hat{E}}{\hat{Z}_1 + \frac{\hat{Z}_0 \hat{Z}_2}{\hat{Z}_0 + \hat{Z}_2}}$$

$$\hat{I}_2 = -\frac{\hat{Z}_0}{\hat{Z}_0 + \hat{Z}_2} \hat{I}_1; \quad \hat{I}_0 = -\frac{\hat{Z}_2}{\hat{Z}_0 + \hat{Z}_2} \hat{I}_1$$

$$\hat{U}_1 = \hat{U}_2 = \hat{U}_0 = \frac{\hat{E} \frac{\hat{Z}_0 \hat{Z}_2}{\hat{Z}_0 + \hat{Z}_2}}{\hat{Z}_1 + \frac{\hat{Z}_0 \hat{Z}_2}{\hat{Z}_0 + \hat{Z}_2}}$$



All three components are in parallel.

Phases

$$(\mathbf{I}_{ABC}) = (\mathbf{T})(\mathbf{I}_{120})$$

$$\hat{\mathbf{I}}_B = \frac{\hat{\mathbf{E}}(\hat{\mathbf{Z}}_0(\hat{\mathbf{a}}^2 - \hat{\mathbf{a}}) + \hat{\mathbf{Z}}_2(\hat{\mathbf{a}}^2 - 1))}{\hat{\mathbf{Z}}_1\hat{\mathbf{Z}}_2 + \hat{\mathbf{Z}}_0\hat{\mathbf{Z}}_1 + \hat{\mathbf{Z}}_0\hat{\mathbf{Z}}_2}$$

$$\hat{\mathbf{I}}_C = \frac{\hat{\mathbf{E}}(\hat{\mathbf{Z}}_0(\hat{\mathbf{a}} - \hat{\mathbf{a}}^2) + \hat{\mathbf{Z}}_2(\hat{\mathbf{a}} - 1))}{\hat{\mathbf{Z}}_1\hat{\mathbf{Z}}_2 + \hat{\mathbf{Z}}_0\hat{\mathbf{Z}}_1 + \hat{\mathbf{Z}}_0\hat{\mathbf{Z}}_2}$$

$$(\mathbf{U}_{ABC}) = (\mathbf{T})(\mathbf{U}_{120}) = \begin{pmatrix} 1 & 1 & 1 \\ \hat{\mathbf{a}}^2 & \hat{\mathbf{a}} & 1 \\ \hat{\mathbf{a}} & \hat{\mathbf{a}}^2 & 1 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{U}}_1 \\ \hat{\mathbf{U}}_1 \\ \hat{\mathbf{U}}_1 \end{pmatrix} = \begin{pmatrix} 3\hat{\mathbf{U}}_1 \\ 0 \\ 0 \end{pmatrix}$$

## Components during short-circuit:

3ph	positive
2ph	positive, negative
2ph ground	positive, negative, zero
1ph	positive, negative, zero



## Short-circuits calculation by means of relative values

Relative values – related to a defined base.

base power (3ph)	$S_v$ (VA)
base voltage (phase-to-phase)	$U_v$ (V)
base current	$I_v$ (A)
base impedance	$Z_v$ ( $\Omega$ )

$$S_v = \sqrt{3}U_v I_v$$

$$Z_v = \frac{U_{vf}}{I_v}$$

Relative impedance

$$z = \frac{Z}{Z_v} = \frac{Z}{\frac{U_{vf}}{I_v}} = Z \frac{I_v}{U_{vf}} \frac{3U_{vf}}{3U_{vf}} = Z \frac{S_v}{3U_{vf}^2} = Z \frac{S_v}{U_v^2}$$

## Initial sub-transient short-circuit current (3ph short-circuit)

$$I''_{k0} = |\hat{I}_A| = \frac{|\hat{U}_f|}{|\hat{Z}_1|}$$

$$Z_1 = z_1 \frac{U_v^2}{S_v}$$

$$I''_{k0} = \frac{\frac{U_v}{\sqrt{3}}}{z_1 \frac{U_v^2}{S_v}} = \frac{1}{z_1} \frac{S_v}{\sqrt{3}U_v} = \frac{1}{z_1} I_v$$

## Initial sub-transient short-circuit power

$$S''_{k0} = \sqrt{3}U_v I''_{k0} = \sqrt{3}U_v \frac{I_v}{z_1} = \frac{1}{z_1} S_v$$

Similarly for

1ph short-circuit

$$I_{k0}''^{(1)} = \frac{3}{Z_1 + Z_2 + Z_0} I_v$$

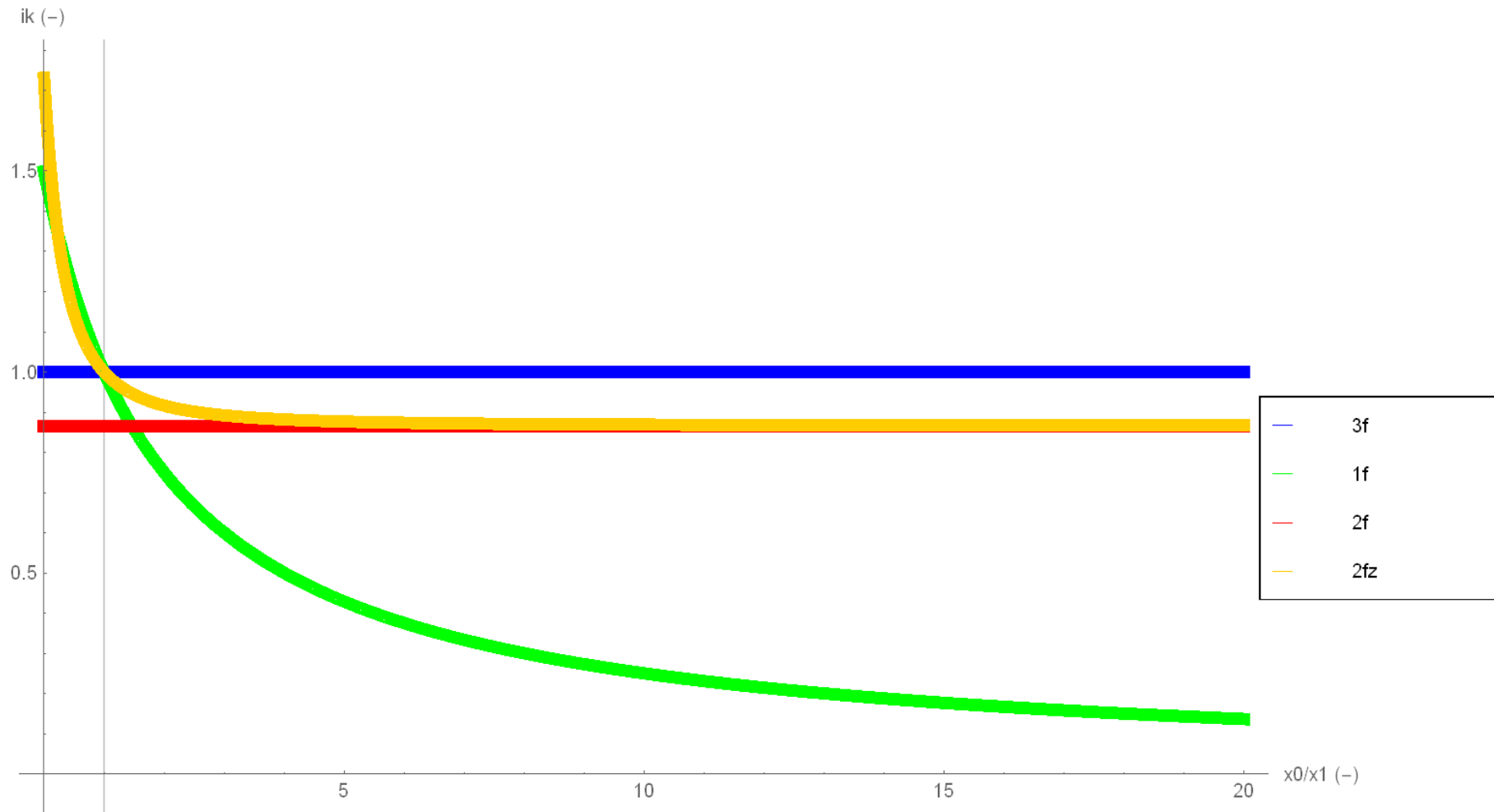
2ph short-circuit

$$I_{k0}''^{(2)} = \frac{\sqrt{3}}{Z_1 + Z_2} I_v$$

Note: Sometimes it is respected generator loading, more precisely higher internal generator voltage than nominal one.

$$I_{k0}'' = k \frac{1}{Z_1} I_v$$

$$k > 1$$



$$I_k^{(1)} = (0 \div 1,5) I_k^{(3)} \quad I_k^{(2)} = \frac{\sqrt{3}}{2} I_k^{(3)} \quad I_k^{(2z)} = \left( \frac{\sqrt{3}}{2} \div \sqrt{3} \right) I_k^{(3)}$$

## Short-circuit currents impacts

### Mechanical impacts

Influence mainly at tightly placed stiff conductors, supporting insulators, disconnectors, construction elements,...

Forces frequency  $2f$  at AC  $\rightarrow$  dynamic strain.

Force on the conductor in magnetic field

$$F = B \cdot I \cdot l \cdot \sin \alpha \quad (\text{N})$$

$$B = \mu \cdot H \quad (\text{T})$$

$$\mu_0 = 4\pi \cdot 10^{-7} \quad (\text{H/m})$$

$\alpha$  – angle between mag. induction vector and the conductor axis  
(current direction)

Magnetic field intensity in the distance  $a$  from the conductor

$$H = \frac{I}{2\pi a} \quad (\text{A/m})$$

2 parallel conductors → force perpendicular to the conductor axis  
( $\sin \alpha = 1$ ) → it is the biggest

$$F = 4\pi \cdot 10^{-7} \frac{I}{2\pi a} I \cdot 1 = 2 \cdot 10^{-7} \frac{I^2}{a} \quad (\text{N})$$

The highest force corresponds to the highest immediate current value  
→ peak short-circuit current  $I_{km}$  (1<sup>st</sup> magnitude after s.-c. origin)

$$\underline{I_{km} = \sqrt{2} I''_{k0} \left(1 + e^{-0,01/T_k}\right) = \kappa \sqrt{2} I''_{k0} \quad (\text{A})}$$

$\kappa$  – peak coefficient according to grid type ( $\kappa_{LV} = 1,8$ ;  $\kappa_{HV} = 1,7$ )  
theoretical range  $\kappa = 1 \div 2$

$T_k$  – time constant of equivalent short-circuit loop ( $L_e/R_e$ )  
i.e. for DC component of short-circuit current

$I''_{k0}$  - initial sub-transient short-circuit current

Real value differs according to the short-circuit origin moment.  
AC component decreasing slower than for DC therefore neglected.

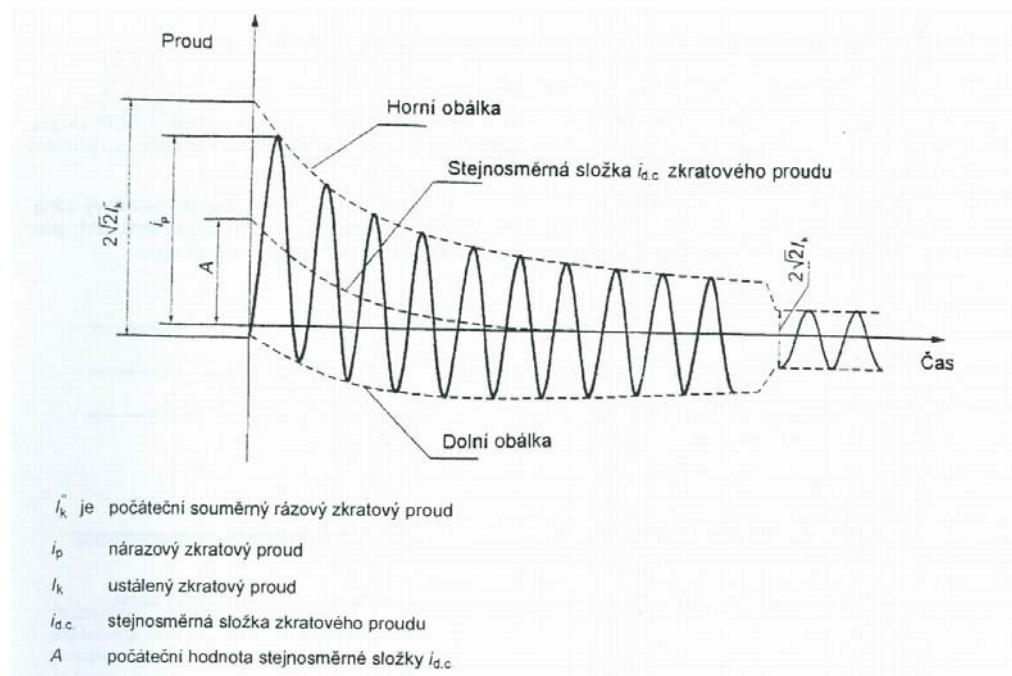
Max. instantaneous force on the conductor length unit

$$f = 2 \cdot k_1 \cdot k_2 \cdot 10^{-7} \frac{I_{\text{km}}^2}{a} \quad (\text{N/m})$$

$k_1$  – conductor shape coefficient

$k_2$  – conductors configuration and currents phase shift coefficient

$a$  – conductors distance



## Heat impacts

Key for dimensioning mainly at freely placed conductors.

They are given by heat accumulation influenced by time-changing current during short-circuit time  $t_k$  (adiabatic phenomenon).

Heat produced in conductors

$$Q = \int_0^{t_k} R(\vartheta) \cdot i_k^2(t) dt \quad (\text{J})$$

Thermal equivalent current – current RMS value which has the same heating effect in the short-circuit duration time as the real short-circuit current

$$I_{ke}^2 t_k = \int_0^{t_k} i_k^2(t) dt$$
$$I_{ke} = \sqrt{\frac{1}{t_k} \int_0^{t_k} i_k^2(t) dt} \quad (\text{A})$$

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Calculation according to  $k_e$  coefficient as  $I_k''$  multiple

$$I_{ke} = k_e I_k''$$