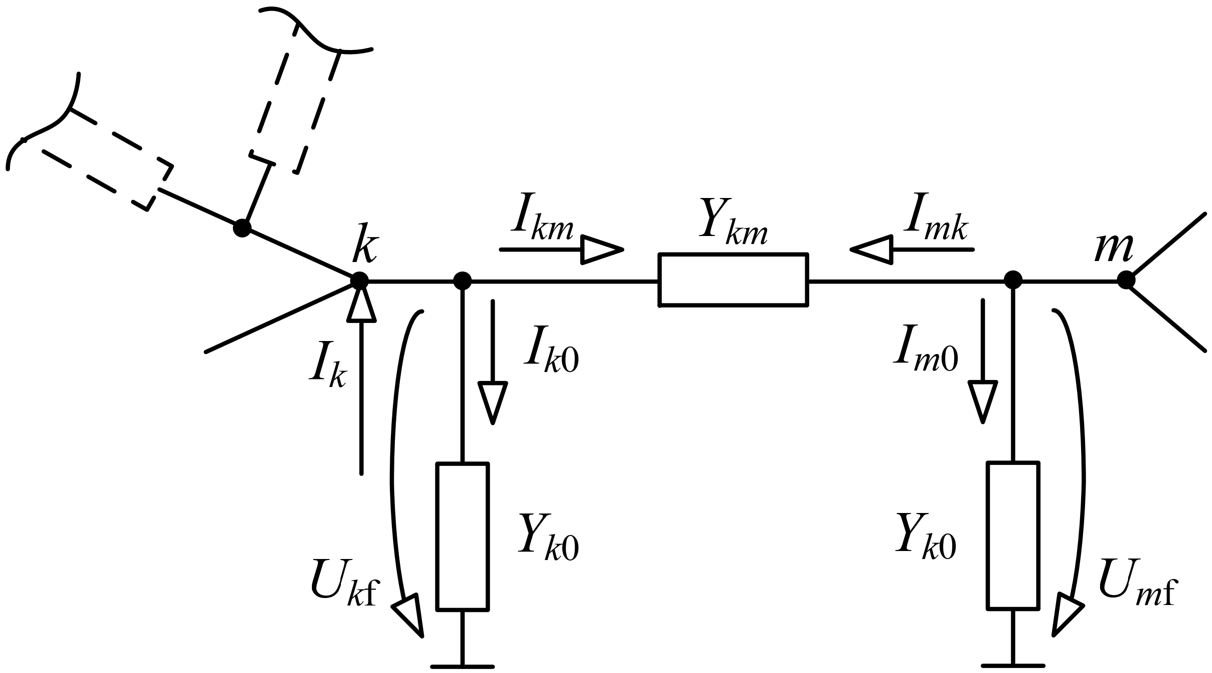
# LOAD-FLOW CALCULATIONS IN MESHED SYSTEMS

## Node voltage method

A system part with the node *k* and its neighbour




$$\hat{I}_k - \sum_{\substack{m=1 \\ m \neq k}}^{n} \hat{I}_{km} - \hat{I}_{k0} = 0$$

## Currents

$$\hat{I}_{km} = \left(\hat{U}_{fk} - \hat{U}_{fm}\right)\hat{Y}_{km}$$

$$\hat{I}_{k0} = \hat{U}_{fk}\hat{Y}_{k0}$$

$$\hat{I}_{k} = \sum_{\substack{m=1 \\ m\neq k}}^{n}\left(\hat{U}_{fk} - \hat{U}_{fm}\right)\hat{Y}_{km} + \hat{U}_{fk}\hat{Y}_{k0}$$

$$\hat{I}_{k} = \hat{U}_{fk}\left(\sum_{\substack{m=1 \\ m\neq k}}^{n}\hat{Y}_{km} + \hat{Y}_{k0}\right) - \sum_{\substack{m=1 \\ m\neq k}}^{n}\hat{U}_{fm}\hat{Y}_{km}$$

## Let's define the node self-admittance

$$\hat{Y}_{kk} = \sum_{\substack{m=1 \\ m\neq k}}^{n}\hat{Y}_{km} + \hat{Y}_{k0} = \sum_{\substack{m=0 \\ m\neq k}}^{n}\hat{Y}_{km}$$

Hence for k$^{th}$ node current

$$\hat{I}_k = \hat{U}_{fk}\hat{Y}_{kk} - \sum_{\substack{m=1 \\ m\neq k}}^{n} \hat{U}_{fm}\hat{Y}_{km}$$

$$\hat{I}_k = \sum_{m=1}^{n} \hat{U}_{fm}\hat{Y}_{(k,m)}$$

Matrix expression

$$\left(\hat{I}\right) = \left(\hat{Y}\right)\left(\hat{U}_f\right) \qquad \sqrt{3}\left(\hat{I}\right) = \left(\hat{Y}\right)\left(\hat{U}\right)$$

Regular admittance matrix

$$\left(\hat{U}_f\right) = \left(\hat{Y}\right)^{-1}\left(\hat{I}\right) = \left(\hat{Z}\right)\left(\hat{I}\right)$$

Singular admittance matrix – node voltage x (1 ÷ n-1) defined

$$\begin{pmatrix} \left(\hat{I}_x\right) \\ \left(\hat{I}_y\right) \end{pmatrix} = \begin{pmatrix} \left(\hat{Y}_A\right) & \left(\hat{Y}_B\right) \\ \left(\hat{Y}_B\right)^T & \left(\hat{Y}_D\right) \end{pmatrix} \begin{pmatrix} \left(\hat{U}_{fx}\right) \\ \left(\hat{U}_{fy}\right) \end{pmatrix}$$

Hence

$$\left(\hat{I}_x\right) = \left(\hat{Y}_A\right)\left(\hat{U}_{fx}\right) + \left(\hat{Y}_B\right)\left(\hat{U}_{fy}\right)$$

$$\left(\hat{I}_y\right) = \left(\hat{Y}_B\right)^T\left(\hat{U}_{fx}\right) + \left(\hat{Y}_D\right)\left(\hat{U}_{fy}\right)$$

Let's calculate $\left(\hat{I}_x\right), \left(\hat{U}_{fy}\right)$

$$\left(\hat{U}_{fy}\right) = \left(\hat{Y}_D\right)^{-1}\left(\hat{I}_y\right) - \left(\hat{Y}_D\right)^{-1}\left(\hat{Y}_B\right)^T\left(\hat{U}_{fx}\right)$$

# Gauss-Seidel method

- iterative method for non-linear equations
- not always a good convergence

## Basic idea

$$f(x) \overset{!}{=} 0$$

Rewritten

$$x = g(x)$$

If $x^{(k)}$ is the estimation in $k^{th}$ step, the next iteration is

$$x^{(k+1)} = g(x^{(k)})$$

We continue until two following iterations difference is smaller than the prescribed precision ε

$$\left| x^{(k+1)} - x^{(k)} \right| \leq \varepsilon$$

Sometimes the convergence can be improved by the acceleration factor $\alpha$ ($\alpha < 1$ or $\alpha > 1$)

$$x^{(k+1)} = x^{(k)} + \alpha\left(g(x^{(k)}) - x^{(k)}\right)$$

## The system of n equations with n unknowns

$$f_1(x_1, x_2, \ldots, x_n) = c_1$$

$$f_2(x_1, x_2, \ldots, x_n) = c_2$$

.............

$$f_n(x_1, x_2, \ldots, x_n) = c_n$$

Each unknown is expressed from one equation

$$x_1 = c_1 + g_1(x_1, x_2, \dots, x_n)$$

$$x_2 = c_2 + g_2(x_1, x_2, \dots, x_n)$$

$$\dots\dots\dots\dots\dots$$

$$x_n = c_n + g_n(x_1, x_2, \dots, x_n)$$

Gauss: $k^{th}$ iteration from $(k\text{-}1)^{th}$ approximation

$$x_m^{(k)} = c_m + g_m(x_1^{(k-1)}, x_2^{(k-1)}, \dots, x_{m-1}^{(k-1)}, x_m^{(k-1)}, \dots, x_n^{(k-1)})$$

Gauss-Seidel: for $k^{th}$ iteration calculation also $k^{th}$ approximations from previous equations are used

$$x_m^{(k)} = c_m + g_m(x_1^{(k)}, x_2^{(k)}, \dots, x_{m-1}^{(k)}, x_m^{(k-1)}, \dots, x_n^{(k-1)})$$

Convergence is tested for each variable separately.

# Newton-Raphson method

- the most often method for non-linear equations
- it uses Taylor polynomial
- it converts non-linear equations solution to linear equations solution, gradually higher precision of the estimation

## Basic idea

$$f(x) = c$$

If $x^{(0)}$ is the initial estimation and $\Delta x^{(0)}$ is the difference from the right solution, then

$$f(x^{(0)} + \Delta x^{(0)}) = c$$

Expansion to the Taylor series

$$f(x^{(0)}) + \left(\frac{df}{dx}\right)^{(0)} \Delta x^{(0)} + \frac{1}{2!}\left(\frac{d^2 f}{dx^2}\right)^{(0)} (\Delta x^{(0)})^2 + \ldots = c$$

Higher orders neglecting (linearization)

$$\Delta c^{(0)} \approx \left(\frac{df}{dx}\right)^{(0)} \Delta x^{(0)}$$

where

$$\Delta c^{(0)} = c - f(x^{(0)})$$

is called "defect".

Adding $\Delta x^{(0)}$ to the initial estimation gives the second approximation

$$x^{(1)} = x^{(0)} + \frac{\Delta c^{(0)}}{\left(\frac{df}{dx}\right)^{(0)}}$$

(Note: impossible if the derivative equals zero)

The same relation in the next steps give the method algorithm:

$$\Delta c^{(k)} = c - f(x^{(k)})$$

$$\Delta x^{(k)} = \frac{\Delta c^{(k)}}{\left(\frac{df}{dx}\right)^{(k)}}$$

$$x^{(k+1)} = x^{(k)} + \Delta x^{(k)}$$

$$\Delta c^{(k+1)} = c - f(x^{(k+1)})$$

# The system of n equations with n unknowns

$$f_1(x_1, x_2, ..., x_n) = c_1$$

$$f_2(x_1, x_2, ..., x_n) = c_2$$

.............

$$f_n(x_1, x_2, ..., x_n) = c_n$$

Expansion to the Taylor series

$$(f_1)^{(0)} + \left(\frac{\partial f_1}{\partial x_1}\right)^{(0)} \Delta x_1^{(0)} + \left(\frac{\partial f_1}{\partial x_2}\right)^{(0)} \Delta x_2^{(0)} + \ldots + \left(\frac{\partial f_1}{\partial x_n}\right)^{(0)} \Delta x_n^{(0)} = c_1$$

$$(f_2)^{(0)} + \left(\frac{\partial f_2}{\partial x_1}\right)^{(0)} \Delta x_1^{(0)} + \left(\frac{\partial f_2}{\partial x_2}\right)^{(0)} \Delta x_2^{(0)} + \ldots + \left(\frac{\partial f_2}{\partial x_n}\right)^{(0)} \Delta x_n^{(0)} = c_2$$

................

$$(f_n)^{(0)} + \left(\frac{\partial f_n}{\partial x_1}\right)^{(0)} \Delta x_1^{(0)} + \left(\frac{\partial f_n}{\partial x_2}\right)^{(0)} \Delta x_2^{(0)} + \ldots + \left(\frac{\partial f_n}{\partial x_n}\right)^{(0)} \Delta x_n^{(0)} = c_n$$

## Matrix expression

$$\begin{pmatrix} c_1 - (f_1^{(0)}) \\ c_1 - (f_2^{(0)}) \\ \vdots \\ c_n - (f_n^{(0)}) \end{pmatrix} = \begin{pmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^{(0)} & \left(\frac{\partial f_1}{\partial x_2}\right)^{(0)} & \ldots & \left(\frac{\partial f_1}{\partial x_n}\right)^{(0)} \\ \left(\frac{\partial f_2}{\partial x_1}\right)^{(0)} & \left(\frac{\partial f_2}{\partial x_2}\right)^{(0)} & \ldots. & \left(\frac{\partial f_2}{\partial x_n}\right)^{(0)} \\ \vdots & \vdots & \vdots & \vdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^{(0)} & \left(\frac{\partial f_n}{\partial x_2}\right)^{(0)} & \ldots & \left(\frac{\partial f_n}{\partial x_n}\right)^{(0)} \end{pmatrix} \cdot \begin{pmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \\ \vdots \\ \Delta x_n^{(0)} \end{pmatrix}$$

## in short

$$\left(\Delta C^{(0)}\right) = \left(J^{(0)}\right) \cdot \left(\Delta X^{(0)}\right)$$

Hence

$$\left(\Delta X^{(0)}\right)=\left(J^{(0)}\right)^{-1}\cdot\left(\Delta C^{(0)}\right)$$

The method algorithm:

$$\left(\Delta C^{(k)}\right)=\begin{pmatrix} c_1-(f_1^{(k)}) \\ c_1-(f_2^{(k)}) \\ \vdots \\ c_n-(f_n^{(k)}) \end{pmatrix}$$

$$\left(\Delta X^{(k)}\right)=\left(J^{(k)}\right)\cdot\left(\Delta C^{(k)}\right)$$

$$\left(X^{(k+1)}\right)=\left(X^{(k)}\right)+\left(\Delta X^{(k)}\right)$$

$$\left(\Delta C^{(k+1)}\right)=\begin{pmatrix} c_1-(f_1^{(k+1)}) \\ c_1-(f_2^{(k+1)}) \\ \vdots \\ c_n-(f_n^{(k+1)}) \end{pmatrix}$$

where

$$\left(\Delta X^{(k)}\right)=\begin{pmatrix} \Delta x_1^{(k)} \\ \Delta x_2^{(k)} \\ \vdots \\ \Delta x_n^{(k)} \end{pmatrix}$$

$$\left(J^{(k)}\right)=\begin{pmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_1}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^{(k)} \\ \left(\frac{\partial f_2}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_2}{\partial x_2}\right)^{(k)} & \cdots\cdot & \left(\frac{\partial f_2}{\partial x_n}\right)^{(k)} \\ \vdots & \vdots & \vdots & \vdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_n}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^{(k)} \end{pmatrix}$$

$(J^{(k)})$ – Jakobi matrix, regularity assumption

# Load Flow solution

U-I equations system can be extended to voltage-power dependence

$$\hat{I}_k = \sum_{m=1}^{n} \hat{U}_{fm} \hat{Y}_{km}$$

$$\hat{S}_k = 3\hat{S}_{fk} = 3\hat{U}_{fk}\hat{I}_k^* = 3\hat{U}_{fk} \sum_{m=1}^{n} \hat{U}_{fm}^* \hat{Y}_{km}^*$$

$$\hat{S}_k = \hat{U}_k \sum_{m=1}^{n} \hat{U}_m^* \hat{Y}_{km}^*$$

$$\left(\hat{S}\right) = \left(\hat{U}_{diag}\right)\left(\hat{Y}^*\right)\left(\hat{U}^*\right)$$

$$\begin{pmatrix} \hat{S}_1 \\ \dots \\ \hat{S}_k \\ \dots \\ \hat{S}_n \end{pmatrix} = \begin{pmatrix} \hat{U}_1 & 0 & 0 & 0 & 0 \\ 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & \hat{U}_k & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \hat{U}_n \end{pmatrix} \cdot \begin{pmatrix} \hat{Y}^*_{11} & \dots & \dots & \dots & \hat{Y}^*_{1n} \\ \dots & \dots & \dots & \dots & \dots \\ \hat{Y}^*_{k1} & \dots & \hat{Y}^*_{kk} & \dots & \hat{Y}^*_{kn} \\ \dots & \dots & \dots & \dots & \dots \\ \hat{Y}^*_{n1} & \dots & \dots & \dots & \hat{Y}^*_{nn} \end{pmatrix} \cdot \begin{pmatrix} \hat{U}^*_1 \\ \dots \\ \hat{U}^*_k \\ \dots \\ \hat{U}^*_n \end{pmatrix}$$

- powers defined → nonlinearity

Aim: to calculate **P, Q, U, δ** in nodes and branches

# Node types

| Node power | | Node voltage phasor components | |
|---|---|---|---|
| defined | to be calculated | defined | to be calculated |
| – | $P$, $Q$ | $U$, $\vartheta$ | – |
| $P$, $Q$ | – | – | $U$, $\vartheta$ |
| $P$ | $Q$ | $U$ | $\vartheta$ |
| $Q$ | $P$ | $\vartheta$ | $U$ |

slack – „balance node“, balance P, Q for losses, as a huge system, large generation

PQ – loads

PU – generators, controlled voltage

Quantities

- fixed – requirements (P,Q for loads; P for generators)
- state – independent variables (U,δ for loads; δ for generators)
- control – here no changes (U for slack and generators), they change in optimization procedures

Node current (single phase)

$$\hat{I}_i = \hat{U}_i \sum_{\substack{j=0 \\ j \neq i}}^{n} \hat{Y}_{ij} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \hat{Y}_{ij} \hat{U}_j$$

Node power

$$P_i + jQ_i = \hat{U}_i \hat{I}_i^*$$

$$\hat{I}_i = \frac{P_i - jQ_i}{\hat{U}_i^*}$$

hence

$$\frac{P_i - jQ_i}{\hat{U}_i^*} = \hat{U}_i \sum_{\substack{j=0 \\ j \neq i}}^{n} \hat{Y}_{ij} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \hat{Y}_{ij} \hat{U}_j$$

# Gauss-Seidel Power Flow Solution

Solution for U, δ:

$$\hat{U}_i^{(k+1)} = \frac{\dfrac{P_i - jQ_i}{\hat{U}_i^{*(k)}} + \sum_{\substack{j=1 \\ j\neq i}}^{n} \hat{Y}_{ij}\hat{U}_j^{(k)}}{\sum_{\substack{j=0 \\ j\neq i}}^{n} \hat{Y}_{ij}}$$

(note: for loads P, Q < 0)

Solution for P:

$$P_i^{(k+1)} = \mathrm{Re}\left\{ \hat{U}_i^{*(k)} \left[ \hat{U}_i^{(k)} \sum_{\substack{j=0 \\ j\neq i}}^{n} \hat{Y}_{ij} - \sum_{\substack{j=1 \\ j\neq i}}^{n} \hat{Y}_{ij}\hat{U}_j^{(k)} \right] \right\}$$

## Solution for Q:

$$Q_i^{(k+1)} = -\mathrm{Im}\left\{ \hat{U}_i^{*(k)} \left[ \hat{U}_i^{(k)} \sum_{\substack{j=0 \\ j\neq i}}^{n} \hat{Y}_{ij} - \sum_{\substack{j=1 \\ j\neq i}}^{n} \hat{Y}_{ij} \hat{U}_j^{(k)} \right] \right\}$$

## Admittance matrix diagonal elements

$$\sum_{\substack{j=0 \\ j\neq i}}^{n} \hat{Y}_{ij} = \hat{Y}_{ii}$$

## Slack voltage known → 2(n-1) equations

PQ: $\hat{U}_i^{(k+1)} = f(P_i, Q_i, \hat{U}_j^{(k)})$

PU: $Q_i^{(k+1)} = f(\hat{U}_i^{(k)}, \hat{U}_j^{(k)})$

$\hat{U}_i^{(k+1)} = f(P_i, Q_i^{(k+1)}, \hat{U}_j^{(k)})$

imaginary part taken, real part to be calculated

$$\left(e_i^{(k+1)}\right)^2 + \left(f_i^{(k+1)}\right)^2 = \left|\hat{U}_i\right|^2$$

$$e_i^{(k+1)} = \sqrt{\left|\hat{U}_i\right|^2 - \left(f_i^{(k+1)}\right)^2}$$

## Newton-Raphson Power Flow Solution

$$\hat{S}_k = \hat{U}_k \sum_{m=1}^{n} \hat{U}_m^* \hat{Y}_{km}^* = U_k^2 \hat{Y}_{kk}^* + \hat{U}_k \sum_{\substack{m=1 \\ m \neq k}}^{n} \hat{U}_m^* \hat{Y}_{km}^*$$

$$\hat{S}_k = f_k\left(\left(\hat{U}\right)\right)$$

### Exponential form

$$\hat{S}_k = P_k + jQ_k \qquad \hat{U}_k = U_k e^{j\vartheta_k} \qquad \hat{Y}_{km} = Y_{km} e^{j\theta_{km}}$$

$$\hat{S}_k = U_k e^{j\vartheta_k} \sum_{m=1}^{n} U_m Y_{km} e^{-j(\vartheta_m + \theta_{km})}$$

Power separated into the real and imaginary part

$$P_k = \sum_{m=1}^{n} U_k U_m Y_{km} \cos(\vartheta_k - \vartheta_m - \theta_{km})$$

$$Q_k = \sum_{m=1}^{n} U_k U_m Y_{km} \sin(\vartheta_k - \vartheta_m - \theta_{km})$$

The power changes are expressed

$$\Delta \hat{S}_k = \sum_{m=1}^{n} \left( \frac{\partial \hat{S}_k}{\partial \vartheta_m} \Delta \vartheta_m + \frac{\partial \hat{S}_k}{\partial U_m} \Delta U_m \right)$$

$$\Delta P_k = \sum_{m=1}^{n} \left( \frac{\partial P_k}{\partial \vartheta_m} \Delta \vartheta_m + \frac{\partial P_k}{\partial U_m} \Delta U_m \right)$$

$$\Delta Q_k = \sum_{m=1}^{n} \left( \frac{\partial Q_k}{\partial \vartheta_m} \Delta \vartheta_m + \frac{\partial Q_k}{\partial U_m} \Delta U_m \right)$$

# Complete equation description

$$
\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta P_{n-1} \\ \Delta P_n \\ \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_{n-1} \\ \Delta Q_n \end{bmatrix} =
$$

$$
= \begin{bmatrix}
\frac{\partial P_1}{\partial \vartheta_1} & \frac{\partial P_1}{\partial \vartheta_1} & \frac{\partial P_1}{\partial \vartheta_{n-1}} & \frac{\partial P_1}{\partial \vartheta_n} & \frac{\partial P_1}{\partial U_1} & \frac{\partial P_1}{\partial U_2} & \frac{\partial P_1}{\partial U_{n-1}} & \frac{\partial P_1}{\partial U_n} \\
\frac{\partial P_2}{\partial \vartheta_1} & \frac{\partial P_2}{\partial \vartheta_2} & \frac{\partial P_2}{\partial \vartheta_{n-1}} & \frac{\partial P_2}{\partial \vartheta_n} & \frac{\partial P_2}{\partial U_1} & \frac{\partial P_2}{\partial U_2} & \frac{\partial P_2}{\partial U_{n-1}} & \frac{\partial P_2}{\partial U_n} \\
\frac{\partial P_{n-1}}{\partial \vartheta_1} & \frac{\partial P_{n-1}}{\partial \vartheta_2} & \frac{\partial P_{n-1}}{\partial \vartheta_{n-1}} & \frac{\partial P_{n-1}}{\partial \vartheta_n} & \frac{\partial P_{n-1}}{\partial U_1} & \frac{\partial P_{n-1}}{\partial U_2} & \frac{\partial P_{n-1}}{\partial U_{n-1}} & \frac{\partial P_{n-1}}{\partial U_n} \\
\frac{\partial P_n}{\partial \vartheta_1} & \frac{\partial P_n}{\partial \vartheta_2} & \frac{\partial P_n}{\partial \vartheta_{n-1}} & \frac{\partial P_n}{\partial \vartheta_n} & \frac{\partial P_n}{\partial U_1} & \frac{\partial P_n}{\partial U_2} & \frac{\partial P_n}{\partial U_{n-1}} & \frac{\partial P_n}{\partial U_n} \\
\frac{\partial Q_1}{\partial \vartheta_1} & \frac{\partial Q_1}{\partial \vartheta_2} & \frac{\partial Q_1}{\partial \vartheta_{n-1}} & \frac{\partial Q_1}{\partial \vartheta_n} & \frac{\partial Q_1}{\partial U_1} & \frac{\partial Q_1}{\partial U_2} & \frac{\partial Q_1}{\partial U_{n-1}} & \frac{\partial Q_1}{\partial U_n} \\
\frac{\partial Q_2}{\partial \vartheta_1} & \frac{\partial Q_2}{\partial \vartheta_2} & \frac{\partial Q_2}{\partial \vartheta_{n-1}} & \frac{\partial Q_2}{\partial \vartheta_n} & \frac{\partial Q_2}{\partial U_1} & \frac{\partial Q_2}{\partial U_2} & \frac{\partial Q_2}{\partial U_{n-1}} & \frac{\partial Q_2}{\partial U_n} \\
\frac{\partial Q_{n-1}}{\partial \vartheta_1} & \frac{\partial Q_{n-1}}{\partial \vartheta_2} & \frac{\partial Q_{n-1}}{\partial \vartheta_{n-1}} & \frac{\partial Q_{n-1}}{\partial \vartheta_n} & \frac{\partial Q_{n-1}}{\partial \vartheta_1} & \frac{\partial Q_{n-1}}{\partial \vartheta_2} & \frac{\partial Q_{n-1}}{\partial \vartheta_{n-1}} & \frac{\partial Q_{n-1}}{\partial \vartheta_n} \\
\frac{\partial Q_n}{\partial \vartheta_1} & \frac{\partial Q_n}{\partial \vartheta_2} & \frac{\partial Q_n}{\partial \vartheta_{n-1}} & \frac{\partial Q_n}{\partial \vartheta_n} & \frac{\partial Q_n}{\partial U_1} & \frac{\partial Q_n}{\partial U_2} & \frac{\partial Q_n}{\partial U_{n-1}} & \frac{\partial Q_n}{\partial U_n}
\end{bmatrix}
\begin{bmatrix} \Delta \vartheta_1 \\ \Delta \vartheta_2 \\ \Delta \vartheta_{n-1} \\ \Delta \vartheta_n \\ \Delta U_1 \\ \Delta U_2 \\ \Delta U_{n-1} \\ \Delta U_n \end{bmatrix}
$$

## More compact equations form

$$\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} = \begin{pmatrix} \frac{\partial P}{\partial \vartheta} & \frac{\partial P}{\partial U} \\ \frac{\partial Q}{\partial \vartheta} & \frac{\partial Q}{\partial U} \end{pmatrix} \begin{pmatrix} \Delta \vartheta \\ \Delta U \end{pmatrix}$$

$$(J) = \begin{pmatrix} \frac{\partial P}{\partial \vartheta} & \frac{\partial P}{\partial U} \\ \frac{\partial Q}{\partial \vartheta} & \frac{\partial Q}{\partial U} \end{pmatrix} = \begin{pmatrix} H & K \\ L & M \end{pmatrix}$$

# Iterative solution idea

$$\begin{pmatrix} \vartheta \\ U \end{pmatrix}_k$$

defect $\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix}$

$$\begin{pmatrix} \Delta\vartheta \\ \Delta U \end{pmatrix} = (J)^{-1} \begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix}$$

$$\begin{pmatrix} \vartheta \\ U \end{pmatrix}_{k+1} = \begin{pmatrix} \vartheta \\ U \end{pmatrix}_k + \begin{pmatrix} \Delta\vartheta \\ \Delta U \end{pmatrix}$$