Reminder of basic formula for symmetric catenary

Catenary curve shape

$$y = c \cdot \cosh \frac{x}{c} \quad (\mathrm{m})$$

$$y' = \sinh \frac{x}{c}$$

Maximal sag

$$f_m = c\left(\cosh \frac{a}{2c} - 1\right)$$

Wire length

$$l_s = 2c \sinh \frac{a}{2c}$$

Catenary constant

$$\frac{\sigma_{\mathrm{H}}}{\gamma} = c$$

3. Stress in wire

Basic facts:

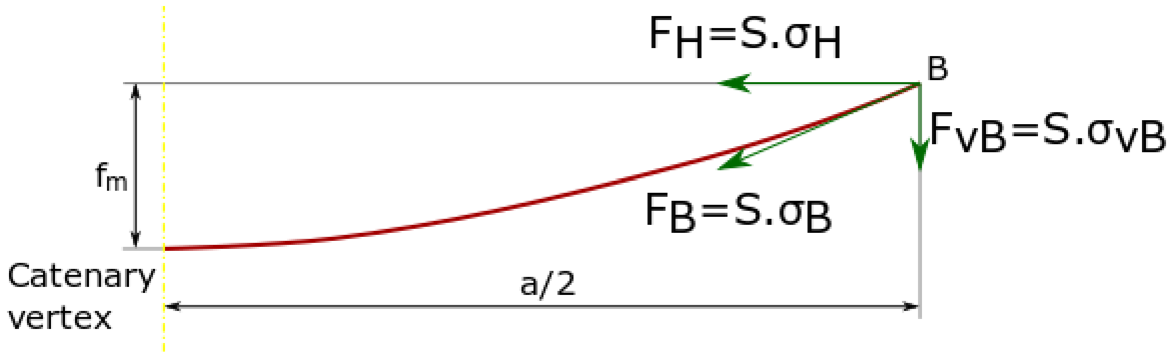
- Horizontal component of stress is constant along a catenary.

$$\sigma_H = const$$

- Vertical component of stress in a point at catenary is equal to the weight of weir between this point and catenary vertex.

$$\sigma_{vB} = \gamma \cdot \frac{l_s}{2}$$

- Stress in any point at catenary has always tangential direction to the catenary.



Force in wire can be expressed by several ways:

1) By vector sum of its vertical and horizontal component:

$$F_B = S \cdot \sqrt{{\sigma_H}^2 + {\sigma_{vB}}^2}$$

Using previous formulas for σ_{vB} and consequently for l_s and for catenary constant c, it can be rewritten to:

$$F_B = S \cdot \sigma_H \cdot \sqrt{1 + \left(\sinh \frac{a}{2c}\right)^2}$$

Which is equal to

$$F_B = S \cdot \sigma_H \cdot \cosh \frac{a}{2c}$$

2) The $\cosh \frac{a}{2c}$ in the previous formula can be exchanged for y_B/c (see the formula for catenary curve shape and take into account that x is x_B=a/2 for the point B)

$$F_B = S \cdot \sigma_H \cdot \frac{y_B}{c}$$

Using formula for catenary constant c:

$$F_B = S \cdot \gamma \cdot y_B$$

It means that the force in point B is the same as weight of wire of length y_B

3) The $\cosh \frac{a}{2c}$ in the formula above for F_B can be exchanged for $\frac{f_m}{c} + 1$ (See equation for maximal sag). Using formula for catenary constant c, following will be obtained:

$$F_B = S \cdot (\gamma \cdot f_m + \sigma_H)$$

It means, that the force in boint B is the same as sum of horizontal force and the weight of wire of the length of vertical distance between this point and the vertex (maximal sag).

4. Equation of state

An equation for horizontal stresses under different temperature and overloading will be derived.

- Change of wire’s length can due to a change of themperature reads

$$\Delta l_\vartheta = \alpha l_0(\vartheta_1 - \vartheta_0)$$

where
index 0 – initial state (known)
index 1 – new state (computed)
α - coefficient of thermal expansion($°C^{-1}$),
l_0 – initial wire’s length (m),
ϑ_0 - initial wire’s themperature ($°C^{-1}$),
ϑ_1 - new wire’s temperature ($°C^{-1}$).

- Change of wire’s length can due to a change of weight (overloading due to icing)

$$\Delta l_\sigma = \frac{l_0}{E}(\sigma_{H1} - \sigma_{H0})$$

where
E - Young's modulus (MPa),

σ_{H0} - horizontal stress component during initial state (MPa),

σ_{H1} - horizontal stress component during new state (MPa).

Overall change of wire's length as a sum of contributions:

$$\Delta l = l_1 - l_0 = \Delta l_\vartheta + \Delta l_\sigma = l_0 \left[\alpha \left(\vartheta_1 - \vartheta_0 \right) + \frac{1}{E} \left(\sigma_{H1} - \sigma_{H0} \right) \right]$$

Overall change of wire's length will now be expressed also from the catenary curve equation. Length of wire reads

$$l_s = 2c \sinh \frac{a}{2c}$$

Taking only first two elements of Thaylor series of this formula will give (it is also result of integration of parabolic approximation of catenary):

$$l_s = a + \frac{a^3 \gamma^2}{24\, \sigma_H{}^2}$$

$$l_k = a + \frac{a^3 \gamma_k^2}{24 \sigma_{Hk}^2}$$

where

a – span (m).

γ - specific weight per 1 m of wire ($\mathrm{MPa} \cdot \mathrm{m}^{-1}$).

Therefore the overall change of wire's length is

$$\Delta l = l_1 - l_0 = \frac{a^3}{24}\left(\frac{\gamma_1^2}{\sigma_{H1}^2} - \frac{\gamma_0^2}{\sigma_{H0}^2}\right)$$

Using the two equations for the overall change of wire's length will give the equation of state

$$l_0\left[\alpha(\vartheta_1 - \vartheta_0) + \frac{1}{E}(\sigma_{H1} - \sigma_{H0})\right] = \frac{a^3}{24}\left(\frac{\gamma_1^2}{\sigma_{H1}^2} - \frac{\gamma_0^2}{\sigma_{H0}^2}\right)$$

We can usually consider the aproximation

$$l_0 = a$$

and write

$$\alpha(\vartheta_1 - \vartheta_0) + \frac{1}{E}(\sigma_{H1} - \sigma_{H0}) = \frac{a^2}{24}\left(\frac{\gamma_1^2}{\sigma_{H1}^2} - \frac{\gamma_0^2}{\sigma_{H0}^2}\right)$$

After rearrangement that gives a cubic equation for the uknown σ_{H1}:

$$\sigma_{H1}^3 + \sigma_{H1}^2 \left[\frac{E\, a^2\, \gamma_0^2}{24 \sigma_{H0}^2} + \alpha E \left(\vartheta_1 - \vartheta_0 \right) - \sigma_{H0} \right] - \frac{a^2\, \gamma_1^2\, E}{24} = 0$$

It is also common to express the specific weights γ_0 a γ_1 using the specific weight of a pure conductor γ_v and an its overloading

$$\gamma_1 = \gamma_v\, z_1 \qquad \gamma_2 = \gamma_v\, z_2$$

The state equation than reads:

$$\sigma_{H1}^3 + \sigma_{H1}^2 \left[\frac{E\, \gamma_v^2}{24} \left(\frac{a\, z_0}{\sigma_{H0}} \right)^2 + \alpha E \left(\vartheta_1 - \vartheta_0 \right) - \sigma_{H0} \right] - \frac{E\, \gamma_v^2}{24} \left(a\, z_1 \right)^2 = 0$$