$$opt\left\{ f\left(\overline{x}\right)\right\} / \overline{x} \in \mathcal{D} \subseteq \Re^n$$

nelineární funkce nebo podmínky.

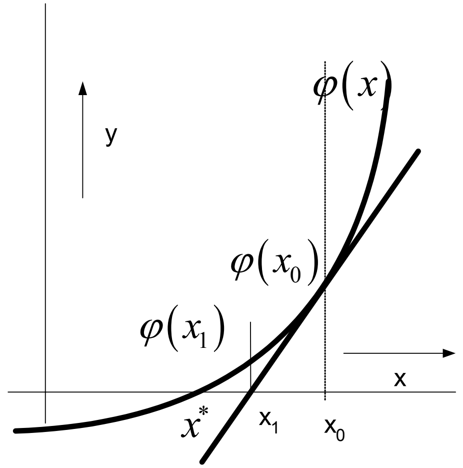
Obecný algoritmus optimalizace:

1. **iniciace startovacího bodu** $\overline{\xi}^0$ **; k=1**
2. **stanovení směru pohybu D**$^{(k)}$
3. **určení optimálního kroku** $\alpha^{(k)}$
4. **aktualizace** $\overline{\xi}^{k+1} = \overline{\xi}^k + \alpha^k \bullet \overline{D}^k$
5. **je li optimalita true pak konec jinak k:=k+1 a návrat na bod 2)**

Metody:

1	**Newton**	**9 (NR, Quasi-Newton)**
2	**Spádové metody**	**(maximální spád, konjugovaný gradient, DFP,BFG, redukovaný gradient)**
3	**Vnitřní trajektorie**	

Newtonovský přístup



$$\varphi\left(x_0\right) + \frac{\partial \varphi\left(x\right)_0}{\partial x}\underbrace{\left(x_1 - x_0\right)}_{\Delta x} = 0$$

$$\Delta x = -\frac{\partial \varphi\left(x\right)_0}{\partial x}^{-1} \varphi\left(x_0\right)$$

$$x_1 = x_0 + \Delta x$$

$$\overline{x} = \left[x_1, \ldots . x_{\partial x}\right]^T \qquad ,\left[J\right] = \left[\begin{matrix} \left[\frac{\partial \varphi_1}{\partial x_1}, \ldots, \frac{\partial \varphi_1}{\partial x_{\partial x}}\right] \\ \bullet\bullet\bullet \\ \left[\frac{\partial \varphi_{\partial y}}{\partial x_1}, \ldots, \frac{\partial \varphi_{\partial y}}{\partial x_{\partial x}}\right] \end{matrix}\right]$$

$$\overline{y} = \left[\varphi_1, \ldots, \varphi_{\partial y}\right]^T$$

$$\overline{\varphi}\left(\overline{x}_0\right) + \left[J_0\right]\Delta\overline{x} = \overline{0} \Rightarrow \Delta\overline{x} = -\left[J_0\right]^{-1}\overline{\varphi}\left(\overline{x}_0\right),$$

$$\overline{x}_1 = \overline{x}_0 + \Delta\overline{x}$$

Podmínky pro nulování:

BPF	Zadané bilance uzlů
OPF	Gradient Lagrangiánu

Postupné kvadratické programování:

$$H(\overline{\xi}^{(k)})\bullet\overline{D}^{(k)} = -\nabla f(\overline{\xi}^{(k)})$$

Základní algoritmus Newtonova přístupu:

1. **iniciace. k=0,** $\overline{\xi}=\overline{\xi}^{0}$
2. **výpočet** $\nabla L^{(k)}$**,** **stanovení množiny aktivních omezení**
3. **je li optimalita true pak konec jinak pokračuj**
4. **stanovení** $\left[J^{(k)}\right],\left[H^{(k)}\right],\left[W^{(k)}\right]$
5. **řešení rovnice** $\left[W^{(k)}\right]\bullet\Delta\overline{\xi}^{(k)}=-\nabla\overline{L}^{(k)}$ **;** **event. obsluha omezení**
6. **korekce** $\overline{\xi}^{(k+1)}=\vec{\xi}^{(k)}+\Delta\overline{\xi}^{(k)}$ **k=k+1, návrat na 2.**

Metoda největšího spádu .

$F(\overline{v}),\ \overline{v}=[v_1,...,v_{\partial v}]^T$,..*funkce*, *vektor proměnných*

$\nabla^T F(\overline{v})=[g_1,...,g_{\partial v}]^T=\overline{g}(\overline{v})$.....*gradient funkce*

$\overline{D}=[d_1,...,d_{\partial v}]^T$,..*normovaný směrový vektor* $\left(\sum_{\forall i} d_i^2=1\right)$

Deriavace ve směru $\overline{D}$

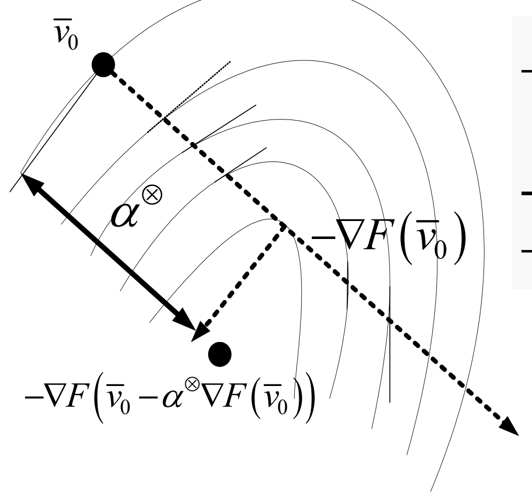
$v_i=v_{0i}+t.d_i \Rightarrow dv_i/dt=d_i$

$$\frac{dF(\overline{v}_0)}{d\overline{D}}=\lim_{t\to 0}\frac{F(\overline{v}_0+t.\overline{D})-F(\overline{v}_0)}{t}=\sum_{i=1}^{\partial v}\overbrace{\frac{\partial F(\overline{v})_0}{\partial v_i}}^{g_{i0}}\overbrace{\frac{dv_i}{dt}}^{d_i}=\nabla F(\overline{v}_0).\overline{D}$$

α...vzdálenost od $\overline{v}_0$, $\overline{D}=-\nabla^T F(\overline{v}_0)$...směr gradientní metody

$$F\left(\overbrace{\overline{v}_0+\alpha\overline{D}}^{\overline{v}}\right)=\Phi(\alpha),\quad \alpha^{\otimes}=\arg\min\Phi(\alpha)\text{..optimální krok}$$

$$\frac{d\Phi(\alpha)}{d\alpha}=0;\Rightarrow \nabla^T F\left(\overline{v}_0-\alpha^{\otimes}\nabla^T F(\overline{v}_0)\right).\nabla F(\overline{v}_0)=0\ \textit{součin}\perp\textit{směrů}$$



$$-\nabla F\left(\overline{v}_0-\alpha^{\otimes}\nabla F(\overline{v}_0)\right)$$
$$\perp$$
$$-\nabla F(\overline{v}_0)$$

$$\overline{v}_{k+1} = \overline{v}_k - \alpha_k \overline{g}_k,$$

$$\alpha_k = \arg \min_{\alpha > 0} F\left(v_k - \alpha g_k\right) \Rightarrow \frac{\partial F\left(\overline{v}_k - \alpha \overline{g}_k\right)}{\partial \alpha} = 0$$

kvadratická aproximace:

$$F\left(\overline{v}\right) = F\left(v^*\right) + \frac{1}{2}\left(\overline{v} - \overline{v}^*\right)^T [H]\left(\overline{v} - \overline{v}^*\right)$$

$$\left(\frac{\partial F\left(\overline{v}\right)}{\partial \overline{v}}\right)^T = [H]\left(\overline{v} - \overline{v}^*\right) = \overline{g}\left(v\right)$$

$$\left(\frac{\partial^2 F\left(\overline{v}\right)}{\partial \overline{v}^2}\right) = [H]$$

$$F\left(\overline{v}_{k+1}\right) = F\left(v^*\right) + \frac{1}{2}\left(\overline{v}_k - \alpha_k \overline{g}_k - \overline{v}^*\right)^T [H]\left(\overline{v}_k - \alpha_k \overline{g}_k - \overline{v}^*\right) = \Phi\left(\alpha_k\right)$$

$$\frac{\partial \Phi\left(\alpha_k\right)}{\partial \alpha_k} = \alpha_k . \overline{g}_k^T [H] \overline{g}_k - \overline{g}_k^T [H]\left(\overline{v}_k - \overline{v}^*\right) = 0$$

$$\alpha_k = \frac{\overline{g}_k^T [H]\left(\overline{v}_k - \overline{v}^*\right)}{\overline{g}_k^T [H] \overline{g}_k} = \frac{\overline{g}_k^T \overline{g}_k}{\overline{g}_k^T [H] \overline{g}_k}$$

Metoda konjugovaných směrů.

$$\overline{s}_i [Q] \overline{s}_j = 0 \qquad \forall i, j, \qquad i \neq j$$

$\overline{s}$ ....*směry konjugované k* $[Q]$

$[Q]$...*symetrická poz.def. matice*

$$F\left(\overline{v}\right) = \overline{a}^T \overline{v} + \frac{1}{2} \overline{v}^T [Q] \overline{v}$$

$$\overline{g}_k\left(\overline{v}\right) = \overline{a} + [Q]\overline{v}, \Rightarrow 0 = \overline{a} + [Q]\overline{v}^*$$

předpoklad: známe $\overline{s}_k$, $k = 0 \ldots n-1$

$$\overline{v}_{k+1} = \overline{v}_k + \alpha_k \overline{s}_k, \quad \alpha_k = \arg \min_{\alpha > 0} F\left(\overline{v}_k + \alpha \overline{s}_k\right)$$

$$\overline{v}_k - \overline{v}_0 = \sum_{j=0}^{k-1} \alpha_j \overline{s}_j$$

$$\overline{s}_k^T [Q]\left(\overline{v}_k - \overline{v}_0\right) = 0 \Rightarrow \overline{s}_k^T [Q] \overline{v}_k = \overline{s}_k^T [Q] \overline{v}_0$$

$$\overline{v}_n - \overline{v}_0 = \sum_{j=0}^{n-1} \alpha_j \overline{s}_j$$

$$\overline{s}_k^T [Q]\left(\overline{v}_n - \overline{v}_0\right) = \sum_{j=0}^{n-1} \alpha_j \overline{s}_j = \alpha_k \overline{s}_k^T [Q] \overline{s}_k$$

$$\alpha_k = \frac{\overline{s}_k^T [Q]\left(\overline{v}_n - \overline{v}_0\right)}{\overline{s}_k^T [Q] \overline{s}_k} = \frac{\overline{s}_k^T \left(\overbrace{[Q]\overline{v}^*}^{-\overline{a}} - \overbrace{[Q]\overline{v}_k}^{\overline{g}_k - a} \right)}{\overline{s}_k^T [Q] \overline{s}_k} = -\frac{\overline{s}_k^T \overline{g}_k}{\overline{s}_k^T [Q] \overline{s}_k}$$

Redukovaný gradient $\nabla_{\overline{\rho}} F$

cílová funkce: $F\left(\underbrace{\overline{\rho},\, \overline{\sigma}(\overline{\rho})}_{\overline{x}}\right),$ $\overline{\rho},\left(\overline{\sigma}=\overline{\sigma}(\overline{\rho})\right)$ vektor nezávisle, (závisle) proměnných

$\overline{x}=\begin{bmatrix}\overline{\rho}\\ \overline{\sigma}\end{bmatrix}$...... $\partial\overline{x}$ – *rozm.* složený vektor proměnných

$\overline{\mathcal{R}}(\overline{\rho},\overline{\sigma})=$ $\partial\overline{\mathcal{R}}$ – *rozm.* vektor vazeb.podmínek:

$$0=\mathrm{d}\overline{\mathcal{R}}(\overline{\rho},\overline{\sigma})=\left(\frac{\partial\overline{\mathcal{R}}}{\partial\overline{\rho}}\right)\mathrm{d}\overline{\rho}+\left(\frac{\partial\overline{\mathcal{R}}}{\partial\overline{\sigma}}\right)\mathrm{d}\overline{\sigma}\Rightarrow$$

$$\Rightarrow\frac{\mathrm{d}\overline{\mathcal{R}}(\overline{\rho},\overline{\sigma})}{\mathrm{d}\overline{\rho}}=\left(\frac{\partial\overline{\mathcal{R}}}{\partial\overline{\rho}}\right)+\left(\frac{\partial\overline{\mathcal{R}}}{\partial\overline{\sigma}}\right)\cdot\left(\frac{\mathrm{d}\overline{\sigma}}{\mathrm{d}\overline{\rho}}\right)\Rightarrow$$

$$\Rightarrow\frac{\mathrm{d}\overline{\sigma}}{\mathrm{d}\overline{\rho}}=-\left[\frac{\partial\overline{\mathcal{R}}}{\partial\overline{\sigma}}\right]^{-1}\cdot\frac{\partial\overline{\mathcal{R}}}{\partial\overline{\rho}}=\begin{bmatrix}\frac{\mathrm{d}\sigma_1}{\partial\rho_1} & \cdot & \frac{\partial\sigma_1}{\partial\rho_{\partial x}}\\ \cdot & & \cdot\\ \frac{\partial\sigma_{\partial\mathcal{R}}}{\partial\rho_1} & \cdot & \frac{\partial\sigma_{\partial\mathcal{R}}}{\partial\rho_{\partial x}}\end{bmatrix}$$

$$\nabla F_{\rho}=\frac{\mathrm{d}F}{\mathrm{d}\overline{\rho}}=\begin{bmatrix}\frac{\mathrm{d}F}{\mathrm{d}\rho_1}\\ \cdot\\ \frac{\mathrm{d}F}{\mathrm{d}\rho_{\partial x}}\end{bmatrix}=\begin{bmatrix}\frac{\partial F}{\partial\rho_1}+\sum_{j=1}^{\mathrm{M}}\frac{\partial F}{\partial\sigma_j}\cdot\frac{\partial\sigma_j}{\partial\rho_1}\\ \cdot\\ \frac{\partial F}{\partial\rho_{\partial x}}+\sum_{j=1}^{\mathrm{M}}\frac{\partial F}{\partial\sigma_j}\cdot\frac{\partial\sigma_j}{\partial\rho_{\partial x}}\end{bmatrix}$$

$$\frac{\mathrm{d}F}{\mathrm{d}\overline{\rho}}=\nabla F_{\rho}+\left[\frac{\partial\overline{\sigma}}{\partial\overline{\rho}}\right]^{\mathrm{T}}\nabla F_{\sigma}=\nabla F_{\rho}+\left(\nabla F_{\sigma}^{\mathrm{T}}\left[\frac{\partial\overline{\sigma}}{\partial\overline{\rho}}\right]\right)^{\mathrm{T}}$$

$$\frac{\mathrm{d}F}{\mathrm{d}\overline{\rho}}=\nabla F_{\rho}-\left(\nabla F_{\sigma}^{\mathrm{T}}\cdot\left[\frac{\partial\overline{\mathcal{R}}}{\partial\overline{\sigma}}\right]^{-1}\cdot\frac{\partial\overline{\mathcal{R}}}{\partial\overline{\rho}}\right)^{\mathrm{T}}$$

formulace optimálního režimu:

$$L(\overline{x})=F(\overline{x})+\overline{\lambda}_R^T\bullet\underbrace{\overline{\mathcal{R}}(\overline{x})}_{\substack{podmínky\\ chodu}}+\underbrace{\Psi_p(\overline{x})}_{penalizace}$$

podmínky 1.řádu:

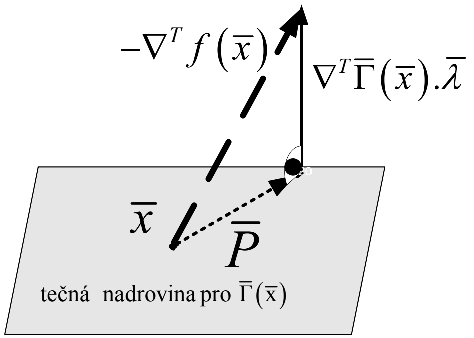
$$\frac{\partial L}{\partial\overline{\sigma}}=\frac{\partial F}{\partial\overline{\sigma}}+\frac{\partial\overline{\mathcal{R}}(\overline{x})}{\partial\overline{\sigma}}.\overline{\lambda}=0\Rightarrow\overline{\lambda}=-\left[\frac{\partial\overline{\mathcal{R}}^T}{\partial\overline{\sigma}}\right]\left(\frac{\partial F}{\partial\overline{\sigma}}\right),$$

$$\frac{\partial L}{\partial\overline{\rho}}=\frac{\partial F}{\partial\overline{\rho}}+\frac{\partial\overline{\mathcal{R}}(\overline{x})}{\partial\overline{\rho}}.\overline{\lambda}=0,\qquad\frac{\partial L}{\partial\overline{\lambda}}=\overline{\mathcal{R}}(\overline{x})=0$$

$$\nabla_{\rho}L=\frac{\partial F}{\partial\overline{\rho}}-\frac{\partial\overline{\mathcal{R}}(\overline{x})}{\partial\overline{\rho}}.\underbrace{\left[\frac{\partial\overline{\mathcal{R}}^T}{\partial\overline{\sigma}}\right]\left(\frac{\partial F}{\partial\overline{\sigma}}\right)}_{-\overline{\lambda}}$$

Projektovaný gradient.

$$L = F\left(\bar{x}\right) + \bar{\lambda}^T.\bar{\Gamma}\left(\bar{x}\right) \quad \text{....}\textit{Lagrangeova funkce}$$



$$\textit{KKT podmínky pro } \nabla^T F\left(\bar{x}^*\right) \uparrow\uparrow \nabla^T\bar{\Gamma}\left(\bar{x}^*\right)$$

$$\nabla^T F\left(\bar{x}^*\right) + \left[\nabla^T\bar{\Gamma}\left(\bar{x}^*\right)\right].\bar{\lambda} = \bar{0} \quad \textit{v optimu}$$

$$\bar{P} = -\nabla^T F - \left[\nabla^T\bar{\Gamma}\right].\bar{\lambda} \text{..}\textit{hledaný směr projekce gradientu}$$

$$\left[\nabla\bar{\Gamma}\right].\bar{P} = 0, \qquad \left[\nabla^T\bar{\Gamma}\right]\bar{\lambda} \textit{ a } \bar{P} \textit{ jsou navzájem kolmé}$$

$$0 = -\left[\nabla\bar{\Gamma}\right]\nabla^T F - \left[\nabla\bar{\Gamma}\right].\left[\nabla^T\bar{\Gamma}\right].\bar{\lambda} \Rightarrow$$

$$\bar{\lambda} = -\left(\nabla\bar{\Gamma}.\nabla^T\bar{\Gamma}\right)^{-1}\nabla\bar{\Gamma}.\nabla^T F$$

$$\bar{P} = -\underbrace{\left\{\left[diag\,❶\right] - \nabla^T\bar{\Gamma}.\left(\nabla\bar{\Gamma}.\nabla^T\bar{\Gamma}\right)^{-1}\nabla\bar{\Gamma}\right\}}_{\text{Projekční matice}}.\nabla^T F$$

Spádové metody (redukovaný gradient):

$$\bar{D}^{(k)} = -\nabla^T L(\bar{\xi}^{(k)})$$

Výpočetní postup:

1	$\bar{\rho} = \bar{\rho}_0$, **k=1**	**Počáteční iniciace**
2	$\bar{\mathcal{R}}\left(\bar{x}\right) = \bar{0}$	**Řešení ustáleného stavu**
3	$\bar{\lambda} = -\left[\frac{\partial\bar{\mathcal{R}}^T}{\partial\bar{\sigma}}\right]\left(\frac{\partial F}{\partial\bar{\sigma}}\right)$	**Výpočet** $\bar{\lambda}$
4	$\nabla_{\bar{\rho}}F = \frac{\partial F}{\partial\bar{\rho}} + \frac{\partial\bar{\mathcal{R}}\left(\bar{x}\right)}{\partial\bar{\rho}}.\bar{\lambda}$	**Výpočet** $\nabla_{\bar{\rho}}F$
5	$\nabla'_{\rho}F$	**projekce** $\nabla_{\bar{\rho}}F$
6	$if \left\|\nabla'_u F\right\| \leq \varepsilon\ pak\ END\ else\ 7$	**kontrola optimality**
7	**α=opt(•)**	**Určení kroku α**
8	$\bar{\rho} = \bar{\rho} - \alpha.\nabla_{\rho}F'_u$	**korekce**
9	**k=k+1; Goto 2**	**návrat**