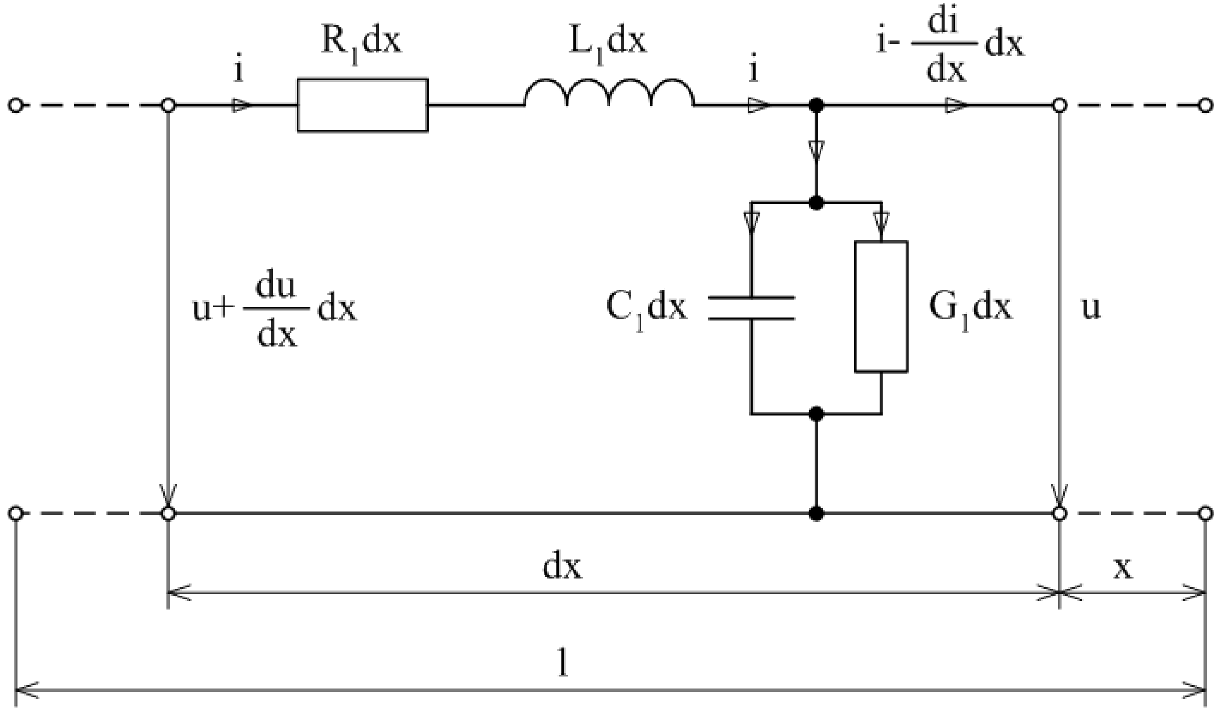
# Three phase HV systems

Transmission lines, international connections.
Goal: relations between input and output ends, losses, efficiency. RLGC

## Line with distributed parameters

Homogeneous line - parameters $R_1$, $L_1$, $G_1$, $C_1$ are equally distributed through all line length.



## 2nd Kirchhoff law

$$u + \frac{\partial u}{\partial x} dx - u - R_1 dx \cdot i - L_1 dx \frac{\partial i}{\partial t} = 0$$

$$\frac{\partial u}{\partial x} = R_1 i + L_1 \frac{\partial i}{\partial t}$$

## 1st Kirchhoff law

$$i - \frac{\partial i}{\partial x} dx - i + G_1 dx \cdot u + C_1 dx \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial i}{\partial x} = G_1 u + C_1 \frac{\partial u}{\partial t}$$

## For AC voltage (current):

$$u(t) = \mathrm{Im}\left\{\sqrt{2}\hat{U}_f e^{j\omega t}\right\}$$

$$\frac{\partial u(t)}{\partial x} = \sqrt{2}\,\mathrm{Im}\left\{\frac{d\hat{U}_f}{dx} e^{j\omega t}\right\} \qquad \frac{\partial u(t)}{\partial t} = \sqrt{2}\,\mathrm{Im}\left\{j\omega\hat{U}_f e^{j\omega t}\right\}$$

After substitution:

$$\frac{d\hat{U}_f}{dx} = \left(R_1 + j\omega L_1\right)\hat{I} = \hat{Z}_{l_1}\hat{I}$$

$$\frac{d\hat{I}}{dx} = \left(G_1 + j\omega C_1\right)\hat{U}_f = \hat{Y}_{q_1}\hat{U}_f$$

After derivation and substitution get wave equations:

$$\frac{d^2\hat{U}_f}{dx^2} = \hat{Z}_{l_1}\frac{d\hat{I}}{dx} = \hat{Z}_{l_1}\hat{Y}_{q_1}\hat{U}_f = \hat{\gamma}^2\hat{U}_f$$

$$\frac{d^2\hat{I}}{dx^2} = \hat{Y}_{q_1}\frac{d\hat{U}_f}{dx} = \hat{Z}_{l_1}\hat{Y}_{q_1}\hat{I} = \hat{\gamma}^2\hat{I}$$

Complex transmission constant:

$$\hat{\gamma} = \sqrt{\hat{Z}_{l_1}\hat{Y}_{q_1}} \quad \left(km^{-1}; \Omega \cdot km^{-1}, S \cdot km^{-1}\right)$$

Note: For time area get telegrapher's equations (mathematically is wave equation for R = G = 0)

$$\frac{\partial^2 u(t,x)}{\partial x^2} = R_1 G_1 u(t,x) + \left(R_1 C_1 + L_1 G_1\right)\frac{\partial u(t,x)}{\partial t} + L_1 C_1 \frac{\partial^2 u(t,x)}{\partial t^2}$$

Ordinary wave equation is second-order linear ordinary differential equation (char. equation $\hat{\lambda}^2 - \hat{\gamma}^2 = 0$) – forward-and-backward wave

$$\hat{U}_f = \hat{K}_1 e^{\hat{\gamma}x} + \hat{K}_2 e^{-\hat{\gamma}x}$$

$$\hat{I} = \frac{d\hat{U}_f}{dx}\frac{1}{\hat{Z}_{l_1}} = \frac{\hat{\gamma}}{\hat{Z}_{l_1}}\left(\hat{K}_1 e^{\hat{\gamma}x} - \hat{K}_2 e^{-\hat{\gamma}x}\right) = \sqrt{\frac{\hat{Y}_{q_1}}{\hat{Z}_{l_1}}}\left(\hat{K}_1 e^{\hat{\gamma}x} - \hat{K}_2 e^{-\hat{\gamma}x}\right)$$

$$\hat{I} = \frac{1}{\hat{Z}_v}\left(\hat{K}_1 e^{\hat{\gamma}x} - \hat{K}_2 e^{-\hat{\gamma}x}\right)$$

Characteristic impedance:

$$\hat{Z}_v = \sqrt{\frac{\hat{Z}_{l_1}}{\hat{Y}_{q_1}}} \quad \left(\Omega;\, \Omega \cdot \mathrm{km}^{-1}, \mathrm{S} \cdot \mathrm{km}^{-1}\right)$$

Integration constant $\hat{K}_1, \hat{K}_2$ are evaluated from boundary value. At the end of line (x = 0) index 2, beginning of the line (x = l) index 1.

For x = 0:

$$\hat{U}_{f2} = \hat{K}_1 + \hat{K}_2$$

$$\hat{I}_2 = \frac{1}{\hat{Z}_v}\left(\hat{K}_1 - \hat{K}_2\right)$$

Hence:

$$\hat{K}_1 = \frac{1}{2}\left(\hat{U}_{f2} + \hat{Z}_v \hat{I}_2\right)$$

$$\hat{K}_2 = \frac{1}{2}\left(\hat{U}_{f2} - \hat{Z}_v \hat{I}_2\right)$$

For x = l:

$$\hat{U}_{f1} = \hat{U}_{f2}\frac{e^{\hat{\gamma}l} + e^{-\hat{\gamma}l}}{2} + \hat{Z}_v\hat{I}_2\frac{e^{\hat{\gamma}l} - e^{-\hat{\gamma}l}}{2}$$

$$\hat{I}_1 = \frac{\hat{U}_{f2}}{\hat{Z}_v}\frac{e^{\hat{\gamma}l} - e^{-\hat{\gamma}l}}{2} + \hat{I}_2\frac{e^{\hat{\gamma}l} + e^{-\hat{\gamma}l}}{2}$$

Definition of hyperbolic function:

$$\underline{\hat{U}_{f1} = \hat{U}_{f2}\cosh\hat{\gamma}l + \hat{Z}_v\hat{I}_2\sinh\hat{\gamma}l}$$

$$\underline{\hat{I}_1 = \frac{\hat{U}_{f2}}{\hat{Z}_v}\sinh\hat{\gamma}l + \hat{I}_2\cosh\hat{\gamma}l}$$

At matrix form:

$$\begin{pmatrix} \hat{U}_{f1} \\ \hat{I}_1 \end{pmatrix} = \begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix} \begin{pmatrix} \hat{U}_{f2} \\ \hat{I}_2 \end{pmatrix}$$

where $\hat{A}\,(-),\ \hat{B}\,(\Omega),\ \hat{C}(S),\ \hat{D}(-)$ are Blondel constant

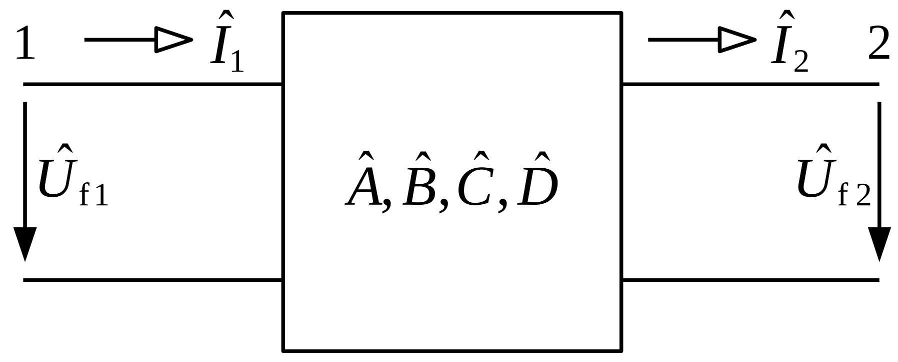
valid $\hat{A} = \hat{D},\ \hat{A}D - \hat{B}\hat{C} = 1$

$$(\cosh \hat{\gamma} l)^2 - (\sinh \hat{\gamma} l)^2 = 1$$

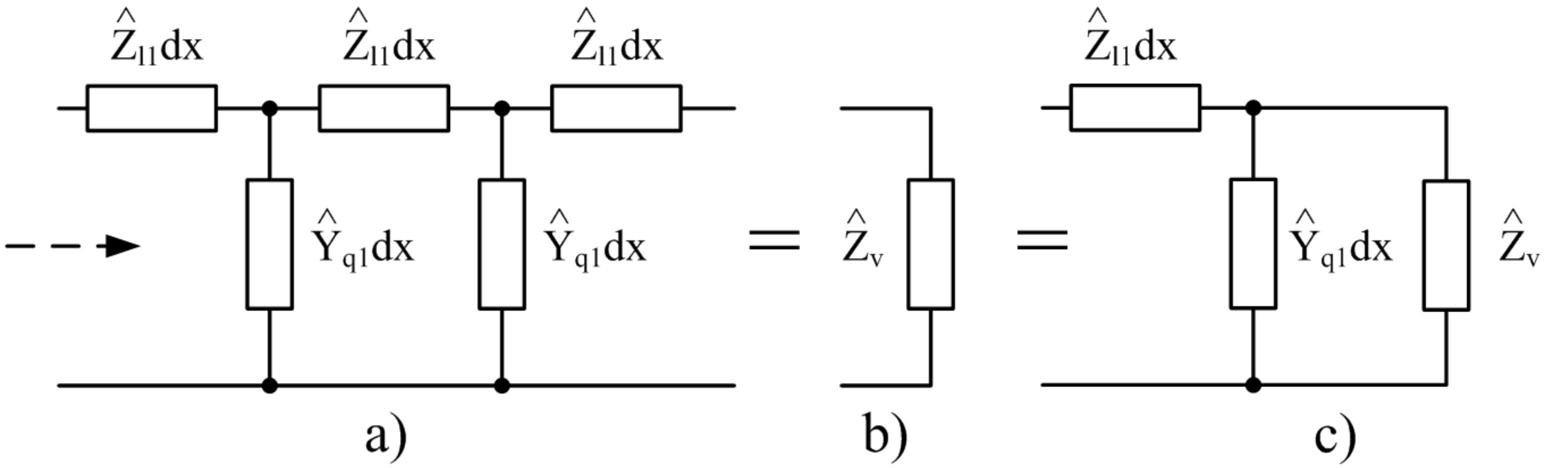
(symmetrical, passive two-port connection)

Set values at the beginning of the line:

$$\begin{pmatrix} \hat{U}_{f2} \\ \hat{I}_2 \end{pmatrix} = \begin{pmatrix} \hat{D} & -\hat{B} \\ -\hat{C} & \hat{A} \end{pmatrix} \begin{pmatrix} \hat{U}_{f1} \\ \hat{I}_1 \end{pmatrix}$$



Characteristic impedance = impedance of the infinite long line.




Input impedance:

$$\hat{Z}_v = \hat{Z}_{l_1} dx + \frac{\hat{Z}_v \cdot \left(\hat{Y}_{q_1} dx\right)^{-1}}{\hat{Z}_v + \left(\hat{Y}_{q_1} dx\right)^{-1}}$$

$$\hat{Z}_v^2 - \hat{Z}_{l_1} dx \cdot \hat{Z}_v - \hat{Z}_{l_1} dx \cdot \left(\hat{Y}_{q_1} dx\right)^{-1} = 0$$

$$\hat{Z}_v = \frac{\hat{Z}_{l_1} dx \pm \sqrt{\left(\hat{Z}_{l_1} dx\right)^2 + 4\hat{Z}_{l_1} dx \cdot \left(\hat{Y}_{q_1} dx\right)^{-1}}}{2}$$

Continues distribution of parameters for dx $\rightarrow$ 0:

$$\hat{Z}_v = \sqrt{\frac{\hat{Z}_{l_1}}{\hat{Y}_{q_1}}}$$

<u>Ideal line</u> –R = 0 and G = 0. For higher voltage levels model does not allow to calculate losses.

$$\hat{\gamma} = \sqrt{(R_1 + jX_1)(G_1 + jB_1)} = j\sqrt{X_1 B_1} = j\beta$$

$$\hat{Z}_v = \sqrt{\frac{R_1 + jX_1}{G_1 + jB_1}} = \sqrt{\frac{L_1}{C_1}} = Z_v$$

Change to trigonometric functions:

$$\cosh(j\beta l) = \frac{e^{j\beta l} + e^{-j\beta l}}{2} = \cos\beta l$$

$$\sinh(j\beta l) = \frac{e^{j\beta l} - e^{-j\beta l}}{2} = j\sin\beta l$$

Than

$$\hat{U}_{f1} = \hat{U}_{f2} \cos\beta l + jZ_v \hat{I}_2 \sin\beta l$$

$$\hat{I}_1 = j\frac{\hat{U}_{f2}}{Z_v} \sin\beta l + \hat{I}_2 \cos\beta l$$

$$(\cos\beta l)^2 - (j \cdot \sin\beta l)^2 = 1$$

<u>Matched load</u> – line at the end has load equal to characteristic impedance (for line's transmission capacity evaluation).
For imaginary infinite long line → power is transited just by forward wave, reflected is zero.

$$\hat{I}_2 = \frac{\hat{U}_{f2}}{\hat{Z}_v}$$

$$\hat{U}_{odr} = \frac{1}{2}\left(\hat{U}_{f2} - \hat{Z}_v \hat{I}_2\right)e^{-\hat{\gamma}l} = \hat{K}_2 e^{-\hat{\gamma}l} = 0$$

$$\hat{S}_{p2} = 3\hat{U}_{f2}\hat{I}_2^* = 3\hat{U}_{f2}\left(\frac{\hat{U}_{f2}}{\hat{Z}_v}\right)^* = \frac{U_2^2}{\hat{Z}_v^*}$$

(Active part is much bigger than imaginary → often in MW.)

$$\hat{U}_{f1} = \hat{U}_{f2}\left(\cosh\hat{\gamma}l + \sinh\hat{\gamma}l\right)$$

$$\hat{I}_1 = \hat{I}_2\left(\sinh\hat{\gamma}l + \cosh\hat{\gamma}l\right)$$

Phase between voltage and current does not change → reactive power at L and C are equal.

$$\frac{\hat{U}_{f(x)}}{\hat{I}_{(x)}} = \frac{\hat{U}_{f2}}{\hat{I}_2} = \hat{Z}_v$$

Along the line amplitude of voltage and current is declined (and active power).

$$\hat{U}_{f(x)} = \hat{U}_{f2}\left(\cosh\hat{\gamma}x + \sinh\hat{\gamma}x\right) = \hat{U}_{f2}\cdot e^{\hat{\gamma}x} = \hat{U}_{f2}\cdot e^{\alpha x}\cdot e^{j\beta x}$$

Overhead line:

$$Z_v = (250 \div 400)\,\Omega \quad \text{pro} \quad (400 \div 22)\,\text{kV}$$

$$S_p = (1 \div 580)\,\text{MW} \quad \text{pro} \quad (22 \div 400)\,\text{kV}$$

For cable line is lower $Z_v = (50 \div 70)\,\Omega$ $\rightarrow$ higher $S_p$

At ideal line voltage and current does not decline:

$$\hat{U}_{f1} = \hat{U}_{f2} \cos \beta l + j \hat{U}_{f2} \sin \beta l = \hat{U}_{f2} e^{j\beta l}$$

$$U_{f1} = U_{f2}$$

$$\hat{I}_1 = \hat{I}_2 e^{j\beta l}$$

Open-circuit:

$$\hat{I}_2 = 0$$

$$\hat{U}_{f10} = \hat{U}_{f2} \cosh \hat{\gamma} l$$

$$\hat{I}_{10} = \frac{\hat{U}_{f2}}{\hat{Z}_v} \sinh \hat{\gamma} l$$

For ideal line:

$$\hat{U}_{f10} = \hat{U}_{f2} \cos \beta l$$

$$\hat{I}_{10} = j \frac{\hat{U}_{f2}}{Z_v} \sin \beta l$$

Valid $U_{f10} \leq U_{f2}$ $\rightarrow$ Ferranti effect

Line is equal to capacity.

Short-circuit:

$$\hat{U}_{f2} = 0$$

$$\hat{U}_{f1} = \hat{Z}_v \hat{I}_2 \sinh \hat{\gamma} l$$

$$\hat{I}_1 = \hat{I}_2 \cosh \hat{\gamma} l$$

For ideal line:

$$\hat{U}_{f1} = jZ_v \hat{I}_2 \sin \beta l$$

$$\hat{I}_1 = \hat{I}_2 \cos \beta l$$

Voltage is declining from beginning to end.

Line is equal to induction.

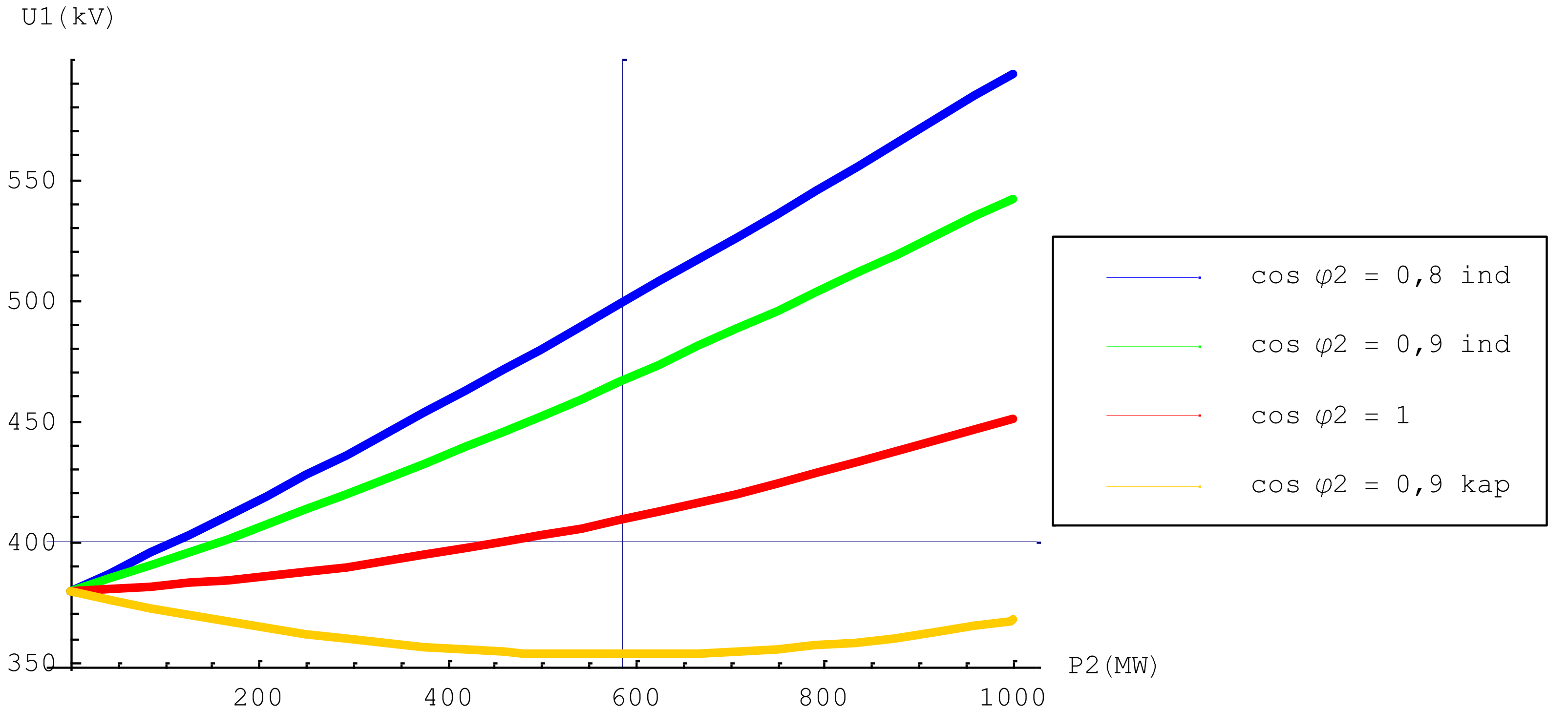
Example:

Line 1 x 400kV with two ground wires

Phase conductor: 3x ACSR 450/52, ground wire: ACSR 185/31, l = 300 km

$$R_1 = 0{,}021\,\Omega/\mathrm{km};\ X_1 = 0{,}293\,\Omega/\mathrm{km};\ G_1 = 2\cdot 10^{-8}\ \mathrm{S/km};\ B_1 = 3{,}9\cdot 10^{-6}\ \mathrm{S/km}$$

# Voltage level ($U_2$ = 400 kV)

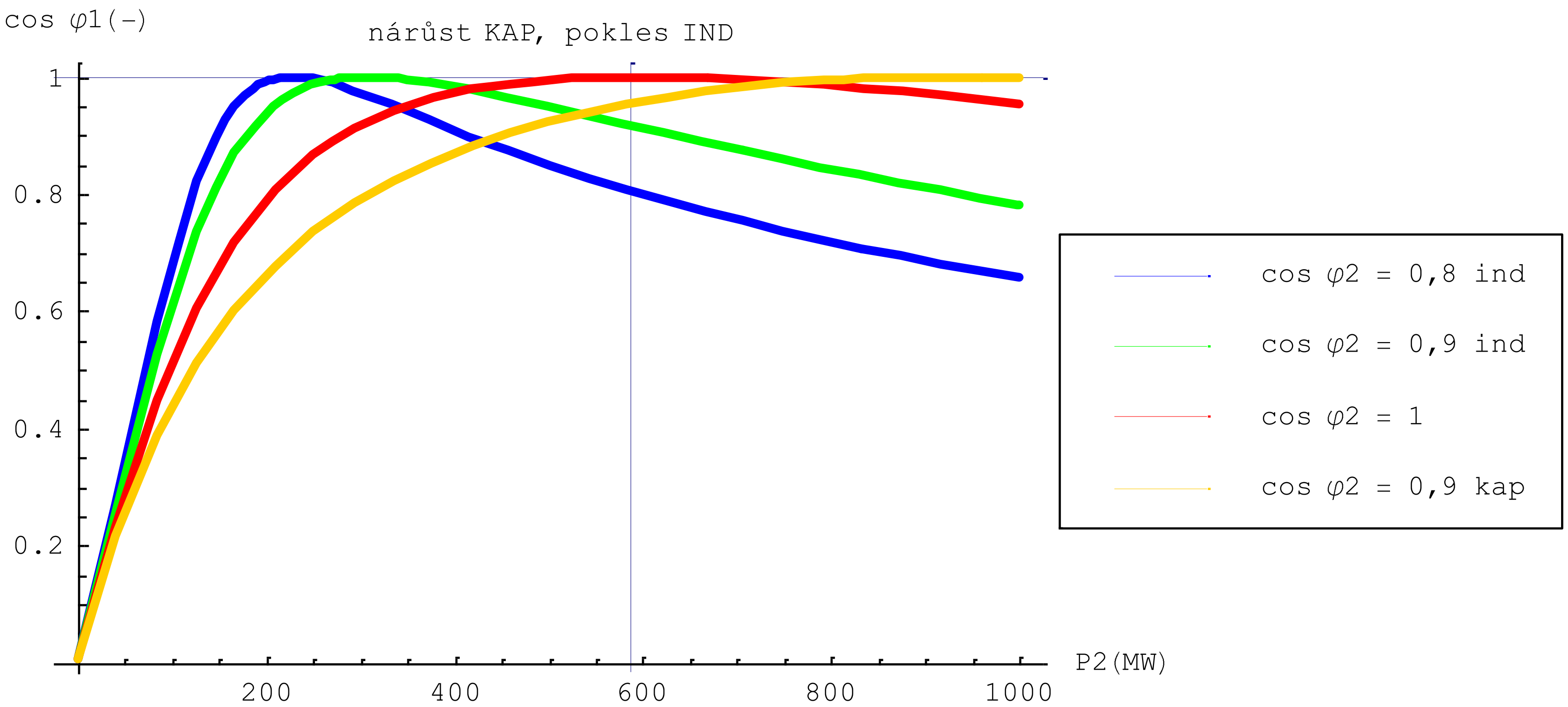



$U_1 < U_n$: Ferranti effect

$U_1 \sim U_n$ for $S_p$ area and cos φ = 1

# Transmission power factor

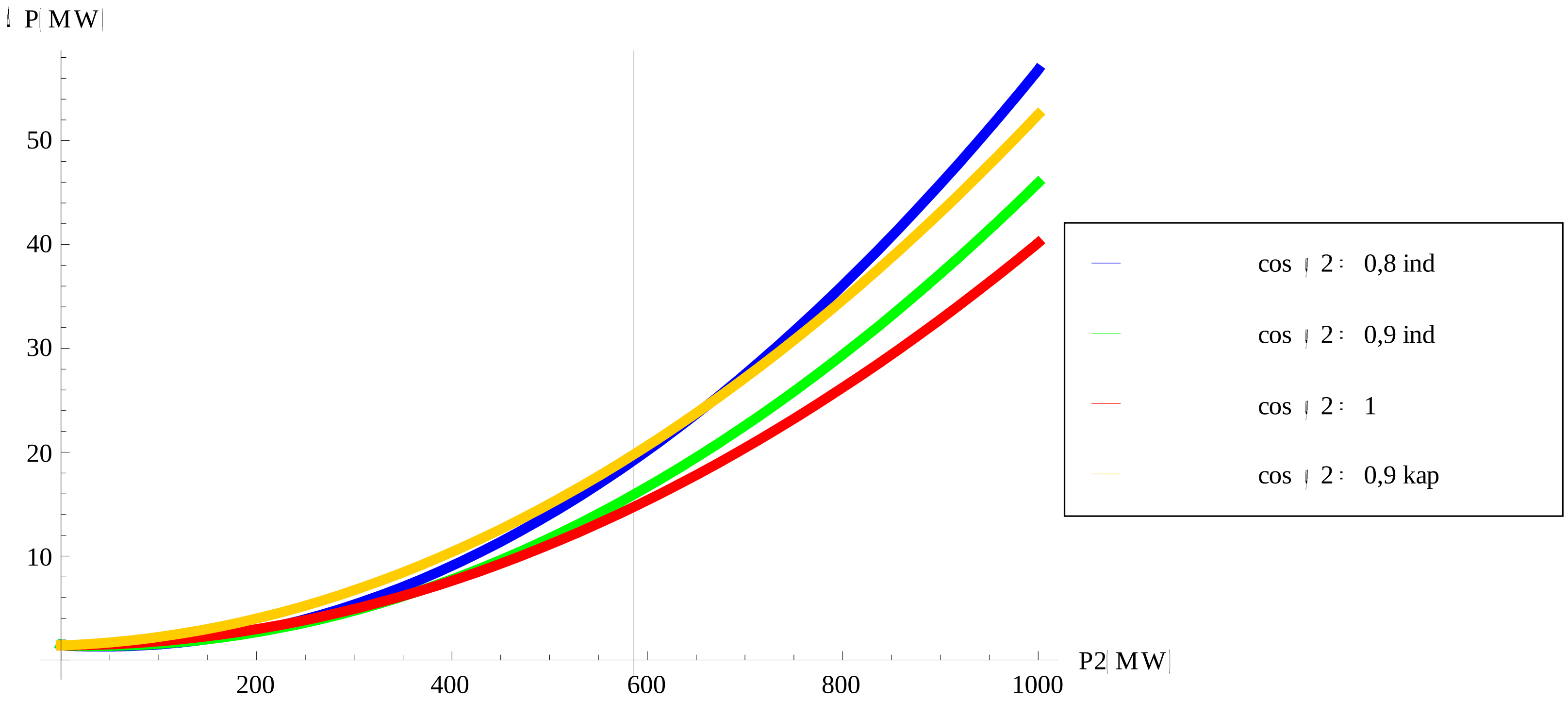
$$\cos \varphi_1 = \frac{P_1}{S_1}$$

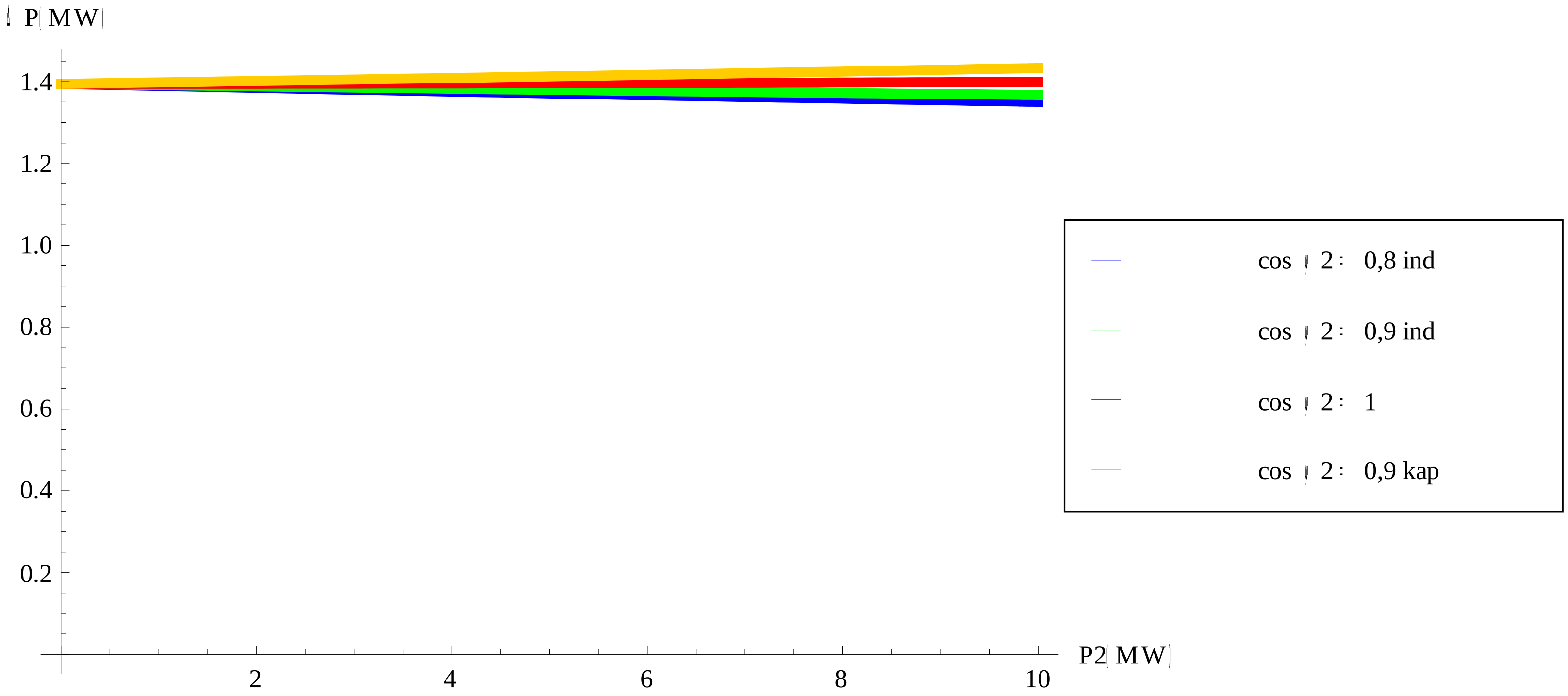



Open-circuit → line is equal to capacity load

Higher power → „Self-compensating“ line

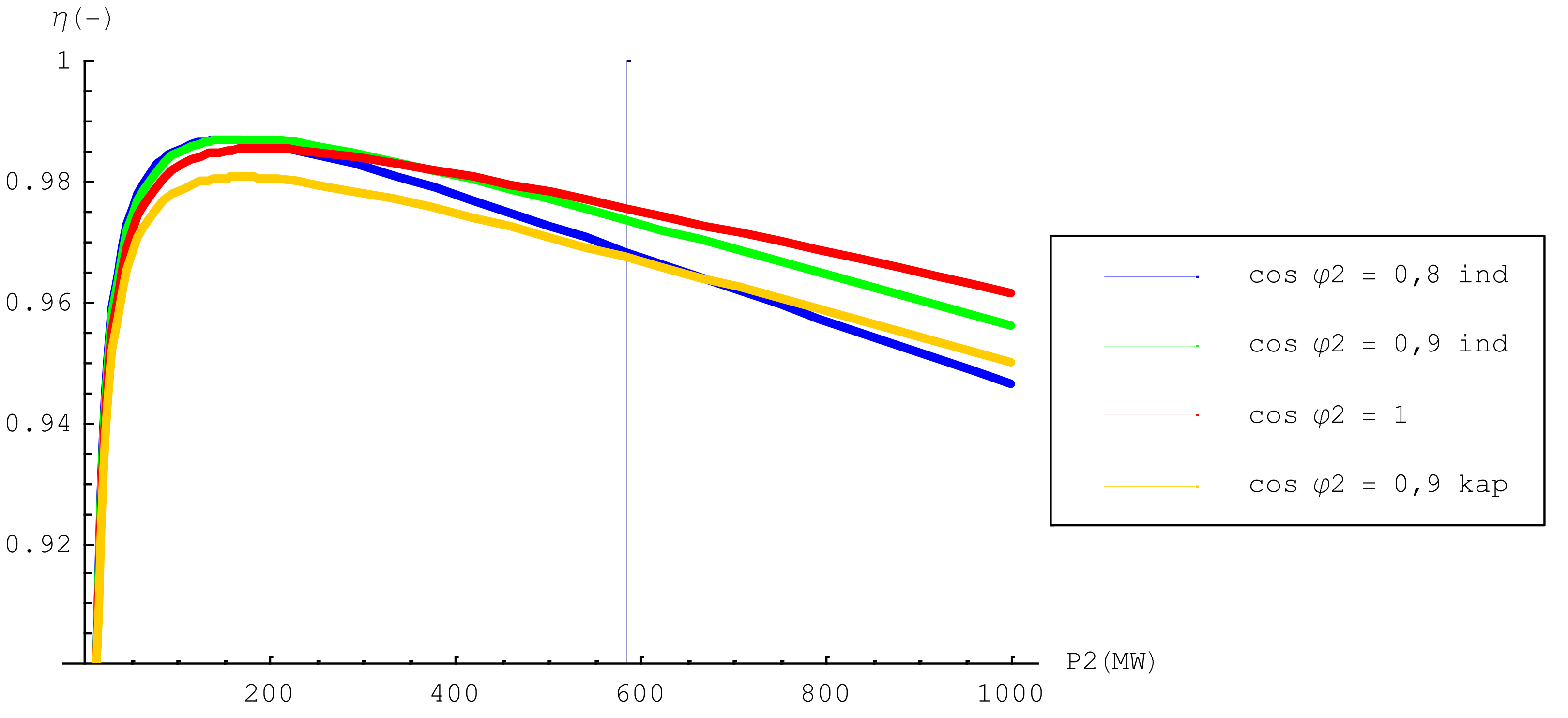
# Line losses
## = open-circuit ~ $U^2$ + load ~ $I^2$




P[MW]
1.4
1.2
1.0
0.8
0.6
0.4
0.2
2
4
6
8
10
P2[MW]
cos 2 : 0,8 ind
cos 2 : 0,9 ind
cos 2 : 1
cos 2 : 0,9 kap

# Transmission power factor

$$\eta = \frac{P_2}{P_1}$$




maximum for low powers
for higher power curve is flat.

Hyperbolic function series
(Taylor series at x = 0)

$$f(x)_{x_0} = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k = f(x_0) + f'(x_0)\cdot(x - x_0) + \frac{f''(x_0)}{2}\cdot(x - x_0)^2 + \ldots$$

$$\cosh\hat{\gamma}l = 1 + \frac{(\hat{\gamma}l)^2}{2} + \frac{(\hat{\gamma}l)^4}{24} + \ldots = 1 + \frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{2} l^2 + \ldots$$

$$\sinh\hat{\gamma}l = \hat{\gamma}l + \frac{(\hat{\gamma}l)^3}{6} + \ldots = \sqrt{\hat{Z}_{l_1}\hat{Y}_{q_1}}\, l + \frac{\left(\hat{Z}_{l_1}\hat{Y}_{q_1}\right)^{3/2}}{6} l^3 + \ldots$$

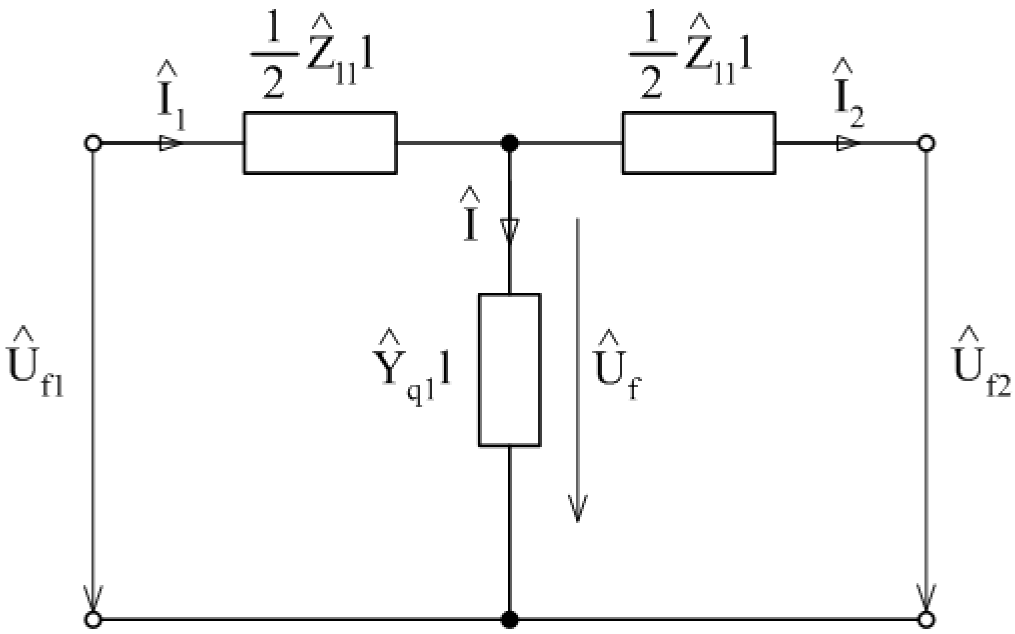
$$\hat{Z}_v \sinh\hat{\gamma}l = \sqrt{\frac{\hat{Z}_{l_1}}{\hat{Y}_{q_1}}}\sinh\hat{\gamma}l = \hat{Z}_{l_1} l + \hat{Z}_{l_1} l \frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{6} l^2 + \ldots$$

$$\frac{1}{\hat{Z}_v}\sinh\hat{\gamma}l = \sqrt{\frac{\hat{Y}_{q_1}}{\hat{Z}_{l_1}}}\sinh\hat{\gamma}l = \hat{Y}_{q_1} l + \hat{Y}_{q_1} l \frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{6} l^2 + \ldots$$

# Line with concentrated parameters

For ordinary calculation (nodal grids) is possible to replace by cell with good accuracy (by line length).

T-cell – short line, transformers; adding another node (replacement for overhead lines up to 200 km, cable up to 80 km)



Voltage and current at the beginning of the line:

$$\hat{U}_{f1} = \hat{U}_{f2} + \frac{1}{2}\hat{Z}_{l_1} l \cdot \hat{I}_2 + \frac{1}{2}\hat{Z}_{l_1} l \cdot \hat{I}_1 \qquad \hat{I}_1 = \hat{I}_2 + \hat{I}$$

Voltage and current at shunt branch:

$$\hat{U}_f = \hat{U}_{f2} + \frac{1}{2}\hat{Z}_{l_1} l \cdot \hat{I}_2$$

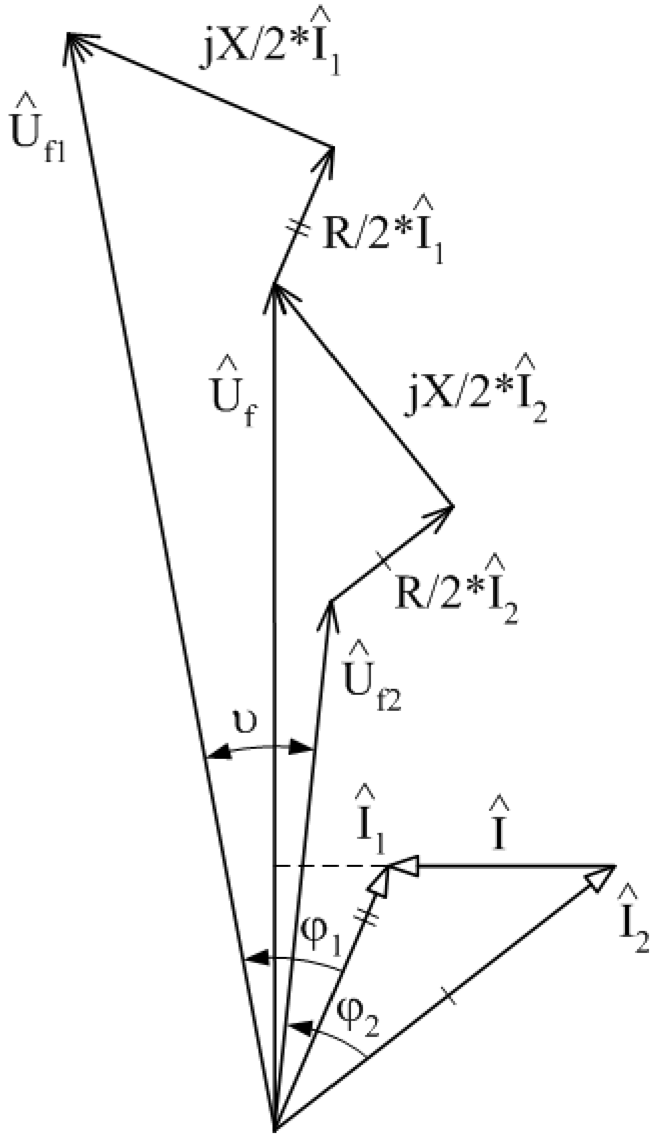
$$\hat{I} = \hat{Y}_{q_1} l \cdot \hat{U}_f = \hat{U}_{f2}\hat{Y}_{q_1} l + \hat{I}_2 \frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{2} l^2$$

Hence (using Blondel constants)

$$\hat{U}_{f1} = \hat{U}_{f2}\left(1 + \frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{2} l^2\right) + \hat{I}_2 \hat{Z}_{l_1} l \left(1 + \frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{4} l^2\right)$$

$$\hat{I}_1 = \hat{U}_{f2}\hat{Y}_{q_1} l + \hat{I}_2\left(1 + \frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{2} l^2\right)$$

# Phasor diagram (G = 0)




$$\hat{U}_{f2}, \hat{I}_2$$

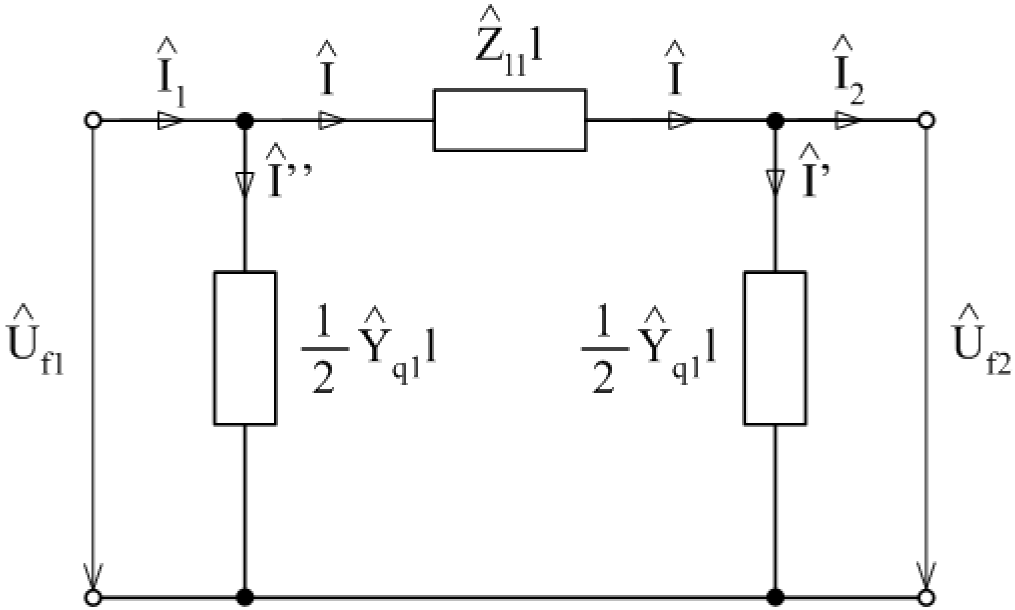
$$\downarrow$$

$$\hat{U}_f \rightarrow \hat{I} \rightarrow \hat{I}_1$$

$$\downarrow$$

$$\hat{U}_{f1}$$

π-cell – longer line, more accurate (replacement for overhead lines up to 250 km, cable up to 100 km)




Voltage and current at the beginning of the line:

$$\hat{U}_{f1} = \hat{U}_{f2} + \hat{Z}_{l_1} l \cdot \hat{I} = \hat{U}_{f2} + \hat{Z}_{l_1} l \cdot \left(\hat{I}_2 + \hat{I}'\right)$$

$$\hat{I}_1 = \hat{I}_2 + \hat{I}' + \hat{I}''$$

Current at series branch:

$$\hat{I}' = \frac{1}{2}\hat{Y}_{q_1} l \hat{U}_{f2}$$

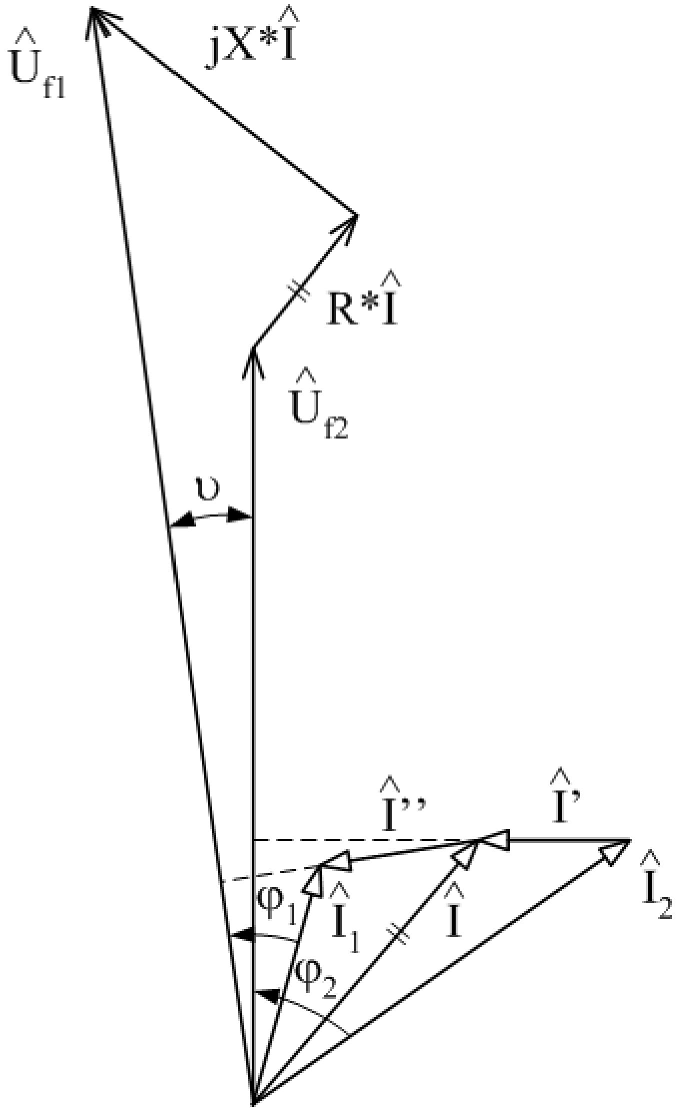
$$\hat{I}'' = \frac{1}{2}\hat{Y}_{q_1} l \hat{U}_{f1}$$

After modification (using Blondel constants)

$$\hat{U}_{f1} = \hat{U}_{f2}\left(1 + \frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{2} l^2\right) + \hat{I}_2 \hat{Z}_{l_1} l$$

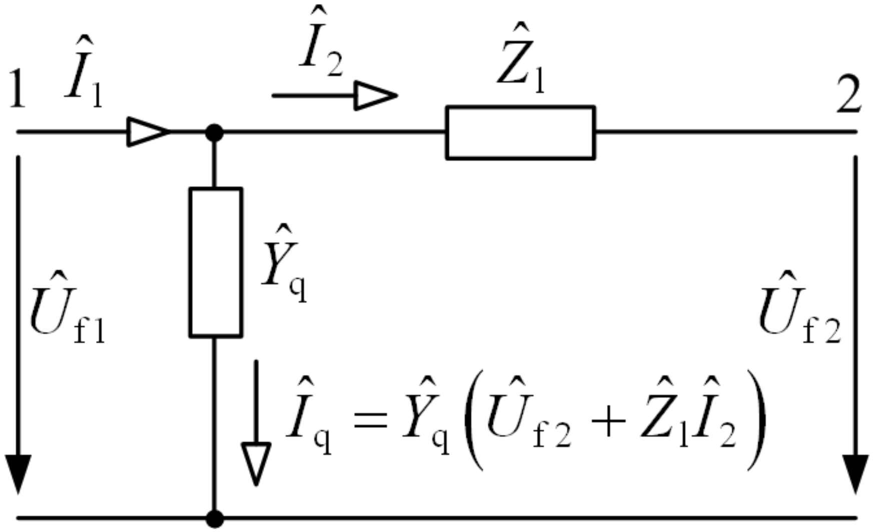
$$\hat{I}_1 = \hat{U}_{f2}\hat{Y}_{q_1} l\left(1 + \frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{4} l^2\right) + \hat{I}_2\left(1 + \frac{\hat{Z}_{l_1}\hat{Y}_{q_1}}{2} l^2\right)$$

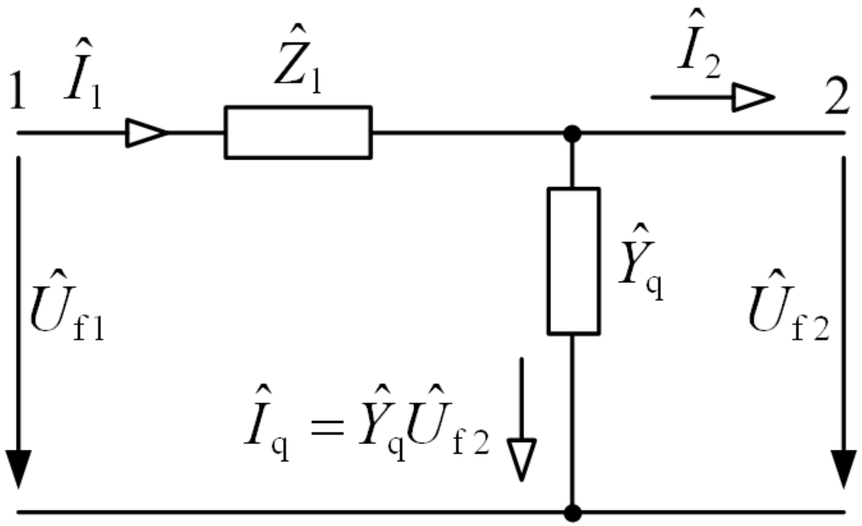
# Phasor diagram (G = 0)




$$
\begin{array}{ccccccc}
\hat{U}_{f2}, & \hat{I}_2 & & & & & \\
\downarrow & \downarrow & & & & & \\
\hat{I}' & \rightarrow & \hat{I} & \rightarrow \Delta\hat{U} & \rightarrow & \hat{U}_{f1} \\
\downarrow & & & & & \downarrow \\
\hat{I}_1 & & \leftarrow & & & \hat{I}''
\end{array}
$$

Γ-cell (gamma) – used rare, for shorter lines (overhead up to 80 km, cable up to 25 km), transformers.



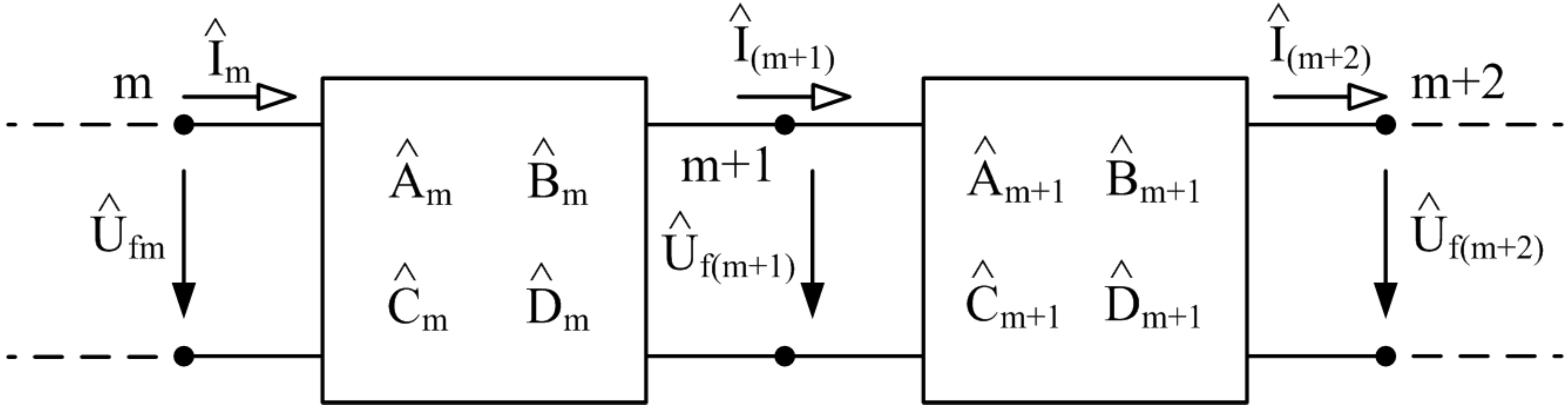




$$\hat{U}_{f1} = \hat{U}_{f2} + \hat{I}_2\hat{Z}_{l_1} l \qquad \hat{U}_{f1} = \hat{U}_{f2}\left(1 + \hat{Z}_{l_1}\hat{Y}_{q_1} l^2\right) + \hat{I}_2\hat{Z}_{l_1} l$$

$$\hat{I}_1 = \hat{U}_{f2}\hat{Y}_{q_1} l + \hat{I}_2\left(1 + \hat{Z}_{l_1}\hat{Y}_{q_1} l^2\right) \qquad \hat{I}_1 = \hat{U}_{f2}\hat{Y}_{q_1} l + \hat{I}_2$$

valid $\hat{A} \neq \hat{D},\ \hat{A}\hat{D} - \hat{B}\hat{C} = 1$ (unsymmetrical, passive two-port)

Longer line → cascade connection of cells for shorter sections (additional nodes)




$$\begin{pmatrix} \hat{U}_{fm} \\ \hat{I}_m \end{pmatrix} = \begin{pmatrix} \hat{A}_m & \hat{B}_m \\ \hat{C}_m & \hat{D}_m \end{pmatrix} \begin{pmatrix} \hat{U}_{f(m+1)} \\ \hat{I}_{(m+1)} \end{pmatrix}$$

$$\begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix} = \prod_{m=1}^{n} \begin{pmatrix} \hat{A}_m & \hat{B}_m \\ \hat{C}_m & \hat{D}_m \end{pmatrix}$$