

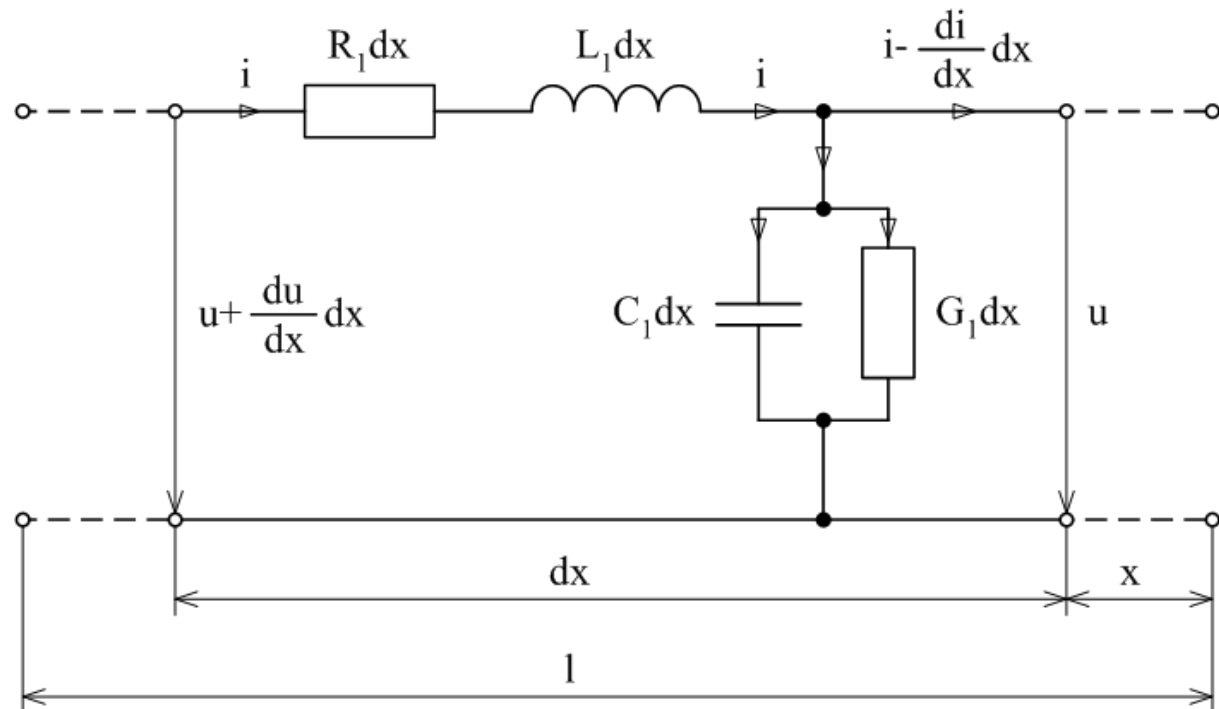
Three phase HV power lines

Transmission systems, international connections.

Aim: relations between sending and receiving ends conditions, losses, efficiency. RLGC

Line with equally distributed parameters

Homogeneous line - parameters R_1 , L_1 , G_1 , C_1 are equally distributed along the total line length.



2nd Kirchhoff's law

$$u + \frac{\partial u}{\partial x} dx - u - R_1 dx \cdot i - L_1 dx \frac{\partial i}{\partial t} = 0$$

$$\frac{\partial u}{\partial x} = R_1 i + L_1 \frac{\partial i}{\partial t}$$

1st Kirchhoff's law

$$i - \frac{\partial i}{\partial x} dx - i + G_1 dx \cdot u + C_1 dx \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial i}{\partial x} = G_1 u + C_1 \frac{\partial u}{\partial t}$$

For AC voltage (current)

$$u(t) = \text{Im} \left\{ \sqrt{2} \hat{U}_f e^{j\omega t} \right\}$$

$$\frac{\partial u(t)}{\partial x} = \sqrt{2} \text{Im} \left\{ \frac{d\hat{U}_f}{dx} e^{j\omega t} \right\}$$

$$\frac{\partial u(t)}{\partial t} = \sqrt{2} \text{Im} \left\{ j\omega \hat{U}_f e^{j\omega t} \right\}$$

After substitution

$$\frac{d\hat{U}_f}{dx} = (R_1 + j\omega L_1)\hat{I} = \hat{Z}_{l_1} \hat{I}$$

$$\frac{d\hat{I}}{dx} = (G_1 + j\omega C_1)\hat{U}_f = \hat{Y}_{q_1} \hat{U}_f$$

After derivation and substitution we get wave equations

$$\frac{d^2\hat{U}_f}{dx^2} = \hat{Z}_{l_1} \frac{d\hat{I}}{dx} = \hat{Z}_{l_1} \hat{Y}_{q_1} \hat{U}_f = \hat{\gamma}^2 \hat{U}_f$$

$$\frac{d^2\hat{I}}{dx^2} = \hat{Y}_{q_1} \frac{d\hat{U}_f}{dx} = \hat{Z}_{l_1} \hat{Y}_{q_1} \hat{I} = \hat{\gamma}^2 \hat{I}$$

Complex transmission constant

$$\hat{\gamma} = \sqrt{\hat{Z}_{l_1} \hat{Y}_{q_1}} \quad (\text{km}^{-1}; \Omega \cdot \text{km}^{-1}, \text{S} \cdot \text{km}^{-1})$$

Note: For time domain it is telegraph equation (mathematically it is wave equation for $R = G = 0$)

$$\frac{\partial^2 u(t, x)}{\partial x^2} = R_1 G_1 u(t, x) + (R_1 C_1 + L_1 G_1) \frac{\partial u(t, x)}{\partial t} + L_1 C_1 \frac{\partial^2 u(t, x)}{\partial t^2}$$

General wave equations solution, i.e. second-order linear ordinary differential equation (char. equation $\hat{\lambda}^2 - \hat{\gamma}^2 = 0$) – progressive and reflected wave

$$\hat{U}_f = \hat{K}_1 e^{\hat{\gamma}x} + \hat{K}_2 e^{-\hat{\gamma}x}$$

$$\hat{I} = \frac{d\hat{U}_f}{dx} \frac{1}{\hat{Z}_{l_1}} = \frac{\hat{\gamma}}{\hat{Z}_{l_1}} (\hat{K}_1 e^{\hat{\gamma}x} - \hat{K}_2 e^{-\hat{\gamma}x}) = \sqrt{\frac{\hat{Y}_{q_1}}{\hat{Z}_{l_1}}} (\hat{K}_1 e^{\hat{\gamma}x} - \hat{K}_2 e^{-\hat{\gamma}x})$$

$$\hat{I} = \frac{1}{\hat{Z}_v} (\hat{K}_1 e^{\hat{\gamma}x} - \hat{K}_2 e^{-\hat{\gamma}x})$$

Surge (wave, characteristic) impedance

$$\hat{Z}_v = \sqrt{\frac{\hat{Z}_{l_1}}{\hat{Y}_{q_1}}} \quad (\Omega; \Omega \cdot \text{km}^{-1}, \text{S} \cdot \text{km}^{-1})$$

Integration constants \hat{K}_1, \hat{K}_2 are calculated from boundary conditions. The line end ($x = 0$) index 2, the line beginning ($x = l$) index 1.

For $x = 0$:

$$\hat{U}_{f2} = \hat{K}_1 + \hat{K}_2$$
$$\hat{I}_2 = \frac{1}{\hat{Z}_v} (\hat{K}_1 - \hat{K}_2)$$

Hence

$$\hat{K}_1 = \frac{1}{2} (\hat{U}_{f2} + \hat{Z}_v \hat{I}_2)$$
$$\hat{K}_2 = \frac{1}{2} (\hat{U}_{f2} - \hat{Z}_v \hat{I}_2)$$

For $x = 1$:

$$\hat{U}_{f1} = \hat{U}_{f2} \frac{e^{\hat{\gamma}l} + e^{-\hat{\gamma}l}}{2} + \hat{Z}_v \hat{I}_2 \frac{e^{\hat{\gamma}l} - e^{-\hat{\gamma}l}}{2}$$

$$\hat{I}_1 = \frac{\hat{U}_{f2}}{\hat{Z}_v} \frac{e^{\hat{\gamma}l} - e^{-\hat{\gamma}l}}{2} + \hat{I}_2 \frac{e^{\hat{\gamma}l} + e^{-\hat{\gamma}l}}{2}$$

Definition of hyperbolic function

$$\hat{U}_{f1} = \hat{U}_{f2} \cosh \hat{\gamma}l + \hat{Z}_v \hat{I}_2 \sinh \hat{\gamma}l$$

$$\hat{I}_1 = \frac{\hat{U}_{f2}}{\hat{Z}_v} \sinh \hat{\gamma}l + \hat{I}_2 \cosh \hat{\gamma}l$$

Matrix form

$$\begin{pmatrix} \hat{U}_{f1} \\ \hat{I}_1 \end{pmatrix} = \begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix} \begin{pmatrix} \hat{U}_{f2} \\ \hat{I}_2 \end{pmatrix}$$

where $\hat{A}(-)$, $\hat{B}(\Omega)$, $\hat{C}(S)$, $\hat{D}(-)$ are Blondel's constants

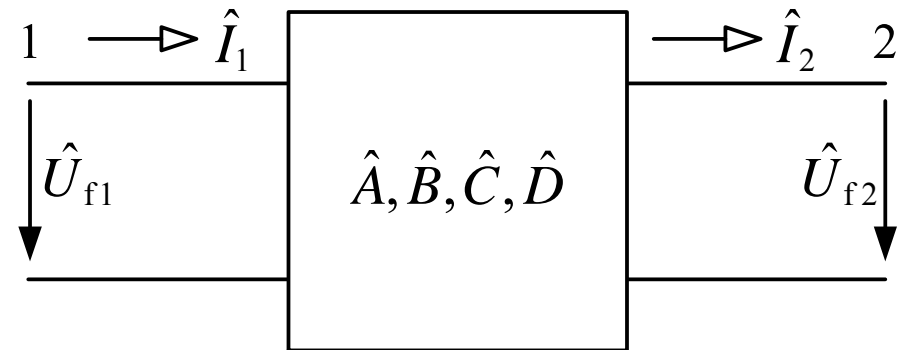
it is valid $\hat{A} = \hat{D}$, $\hat{A}\hat{D} - \hat{B}\hat{C} = 1$

$$(\cosh \hat{\gamma}l)^2 - (\sinh \hat{\gamma}l)^2 = 1$$

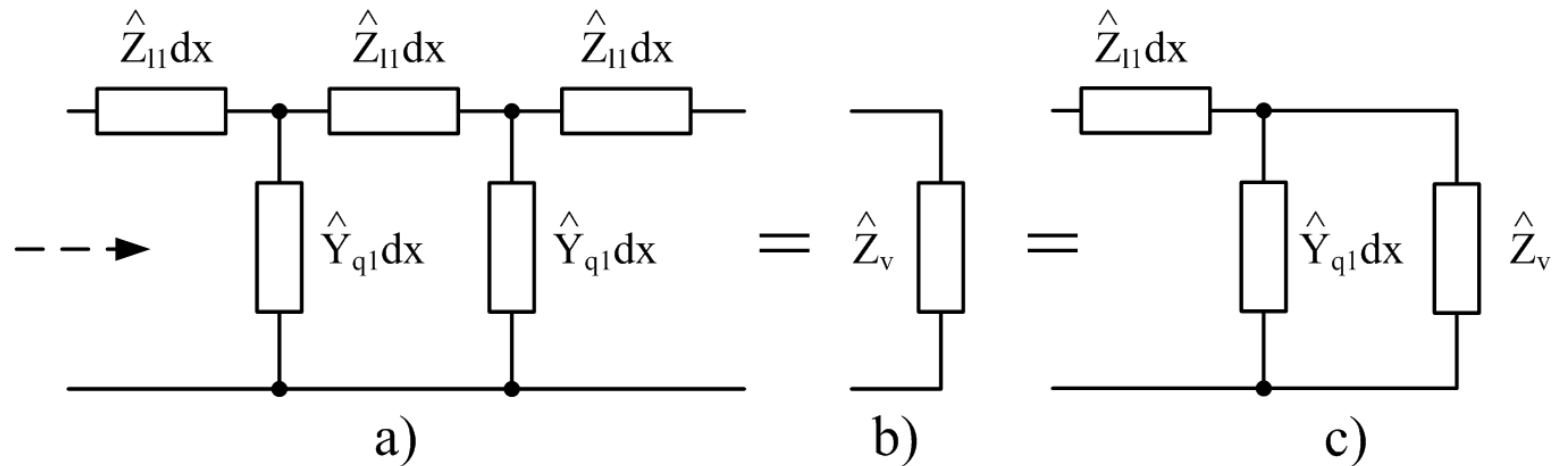
(symmetrical, passive two-port network)

Values at the line beginning are set

$$\begin{pmatrix} \hat{U}_{f2} \\ \hat{I}_2 \end{pmatrix} = \begin{pmatrix} \hat{D} & -\hat{B} \\ -\hat{C} & \hat{A} \end{pmatrix} \begin{pmatrix} \hat{U}_{f1} \\ \hat{I}_1 \end{pmatrix}$$



Surge impedance = impedance of the infinite long line



Input impedance

$$\hat{Z}_v = \hat{Z}_{l1} dx + \frac{\hat{Z}_v \cdot (\hat{Y}_{q1} dx)^{-1}}{\hat{Z}_v + (\hat{Y}_{q1} dx)^{-1}}$$

$$\hat{Z}_v^2 - \hat{Z}_{l1} dx \cdot \hat{Z}_v - \hat{Z}_{l1} dx \cdot (\hat{Y}_{q1} dx)^{-1} = 0$$

$$\hat{Z}_v = \frac{\hat{Z}_{l1} dx \pm \sqrt{(\hat{Z}_{l1} dx)^2 + 4\hat{Z}_{l1} dx \cdot (\hat{Y}_{q1} dx)^{-1}}}{2}$$

Continuous distribution of parameters for $dx \rightarrow 0$:

$$\hat{Z}_v = \sqrt{\frac{\hat{Z}_{l_1}}{\hat{Y}_{q_1}}}$$

Ideal line $-R = 0$ and $G = 0$. For higher voltage levels, the model does not allow to calculate losses.

$$\hat{\gamma} = \sqrt{(R_1 + jX_1)(G_1 + jB_1)} = j\sqrt{X_1 B_1} = j\beta$$

$$\hat{Z}_v = \sqrt{\frac{R_1 + jX_1}{G_1 + jB_1}} = \sqrt{\frac{L_1}{C_1}} = Z_v$$

Change to trigonometric functions

$$\cosh(j\beta l) = \frac{e^{j\beta l} + e^{-j\beta l}}{2} = \cos \beta l$$

$$\sinh(j\beta l) = \frac{e^{j\beta l} - e^{-j\beta l}}{2} = j \sin \beta l$$

Hence

$$\hat{U}_{f1} = \hat{U}_{f2} \cos \beta l + jZ_v \hat{I}_2 \sin \beta l$$

$$\hat{I}_1 = j \frac{\hat{U}_{f2}}{Z_v} \sin \beta l + \hat{I}_2 \cos \beta l$$

$$(\cos \beta l)^2 - (j \cdot \sin \beta l)^2 = 1$$

Surge impedance load (SIL) – the line end is loaded by the surge impedance (for lines transmission capacity comparison).

As an imaginary infinitely long line → power is transmitted only by progressive wave, reflected one is zero.

$$\hat{I}_2 = \frac{\hat{U}_{f2}}{\hat{Z}_v}$$

$$\hat{U}_{odr} = \frac{1}{2} (\hat{U}_{f2} - \hat{Z}_v \hat{I}_2) e^{-\hat{\gamma} l} = \hat{K}_2 e^{-\hat{\gamma} l} = 0$$

$$\hat{S}_{p2} = 3\hat{U}_{f2}\hat{I}_2^* = 3\hat{U}_{f2}\left(\frac{\hat{U}_{f2}}{\hat{Z}_v}\right)^* = \frac{U_2^2}{\hat{Z}_v^*}$$

(Active part is much bigger than imaginary one → often in MW.)

$$\hat{U}_{f1} = \hat{U}_{f2}(\cosh \hat{\gamma}l + \sinh \hat{\gamma}l)$$

$$\hat{I}_1 = \hat{I}_2(\sinh \hat{\gamma}l + \cosh \hat{\gamma}l)$$

Phase between voltage and current does not change → reactive powers at L and C are equal.

$$\frac{\hat{U}_{f(x)}}{\hat{I}_{(x)}} = \frac{\hat{U}_{f2}}{\hat{I}_2} = \hat{Z}_v$$

Voltage and current magnitudes (and active power) decrease along the line.

$$\hat{U}_{f(x)} = \hat{U}_{f2}(\cosh \hat{\gamma}x + \sinh \hat{\gamma}x) = \hat{U}_{f2} \cdot e^{\hat{\gamma}x} = \hat{U}_{f2} \cdot e^{\alpha x} \cdot e^{j\beta x}$$

Overhead line

$$Z_v = (250 \div 400) \Omega \quad \text{pro} \quad (400 \div 22) \text{ kV}$$

$$S_p = (1 \div 580) \text{ MW} \quad \text{pro} \quad (22 \div 400) \text{ kV}$$

Lower for cable lines $Z_v = (50 \div 70) \Omega \rightarrow$ higher S_p

Voltage and current magnitudes don't decrease for ideal line:

$$\hat{U}_{f1} = \hat{U}_{f2} \cos \beta l + j \hat{U}_{f2} \sin \beta l = \hat{U}_{f2} e^{j\beta l}$$

$$U_{f1} = U_{f2}$$

$$\hat{I}_1 = \hat{I}_2 e^{j\beta l}$$

Open-circuit

$$\hat{I}_2 = 0$$

$$\hat{U}_{f10} = \hat{U}_{f2} \cosh \hat{\gamma}l$$

$$\hat{I}_{10} = \frac{\hat{U}_{f2}}{\hat{Z}_v} \sinh \hat{\gamma}l$$

For ideal line

$$\hat{U}_{f10} = \hat{U}_{f2} \cos \beta l$$

$$\hat{I}_{10} = j \frac{\hat{U}_{f2}}{Z_v} \sin \beta l$$

It is valid $U_{f10} \leq U_{f2} \rightarrow$ Ferranti effect
Line character is like capacity.

Short-circuit

$$\hat{U}_{f2} = 0$$

$$\hat{U}_{f1} = \hat{Z}_v \hat{I}_2 \sinh \hat{\gamma}l$$

$$\hat{I}_1 = \hat{I}_2 \cosh \hat{\gamma}l$$

For ideal line

$$\hat{U}_{f1} = jZ_v \hat{I}_2 \sin \beta l$$

$$\hat{I}_1 = \hat{I}_2 \cos \beta l$$

Voltage decreases from the beginning to the end.

Line character is like inductance.

Example:

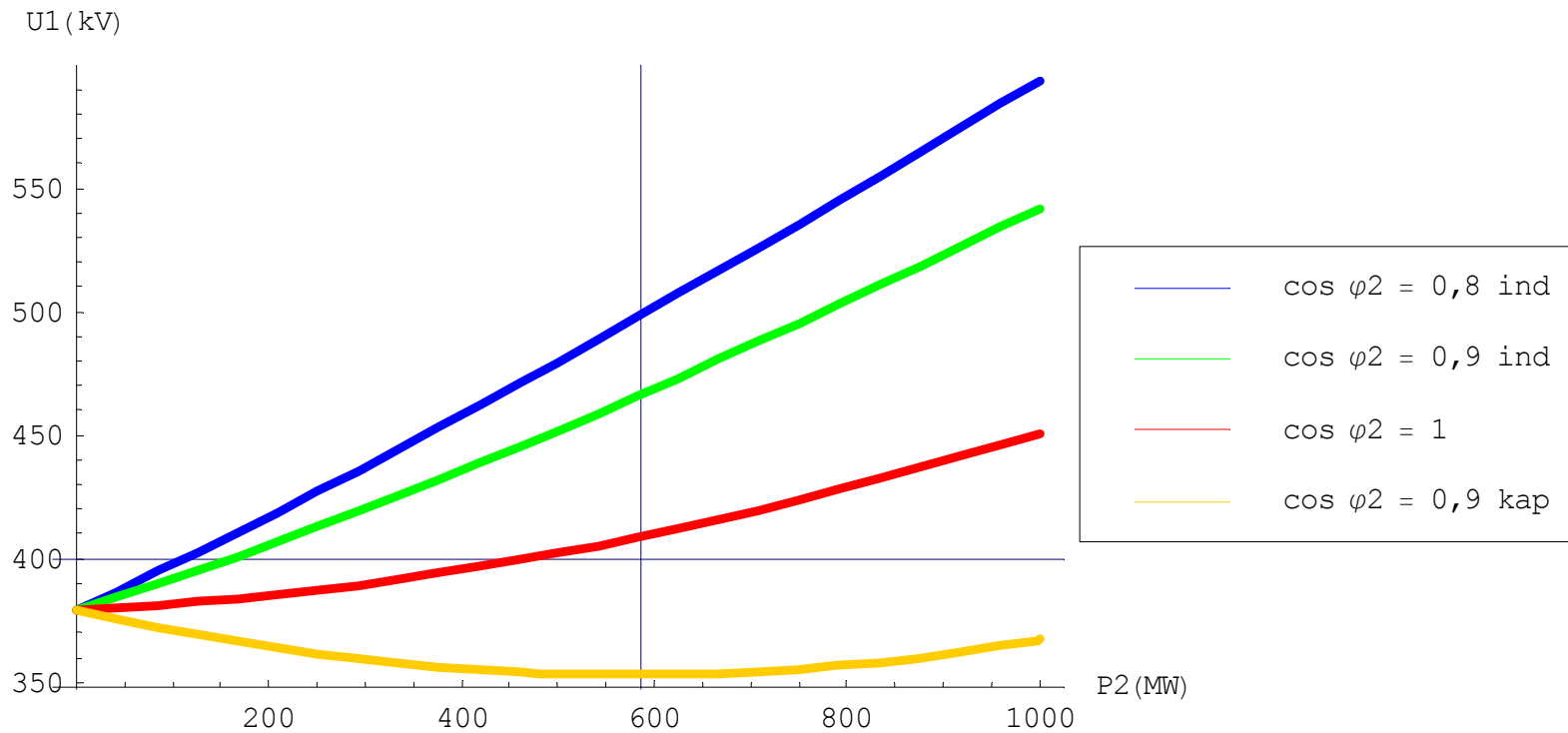
line 1 x 400 kV with two ground wires

phase conductor: 3xACSR 450/52, ground wire: ACSR 185/31, $l = 300$ km

$R_1 = 0,021 \Omega/\text{km}$; $X_1 = 0,293 \Omega/\text{km}$; $G_1 = 2 \cdot 10^{-8} \text{ S}/\text{km}$; $B_1 = 3,9 \cdot 10^{-6} \text{ S}/\text{km}$



Voltage level ($U_2 = 400 \text{ kV}$)

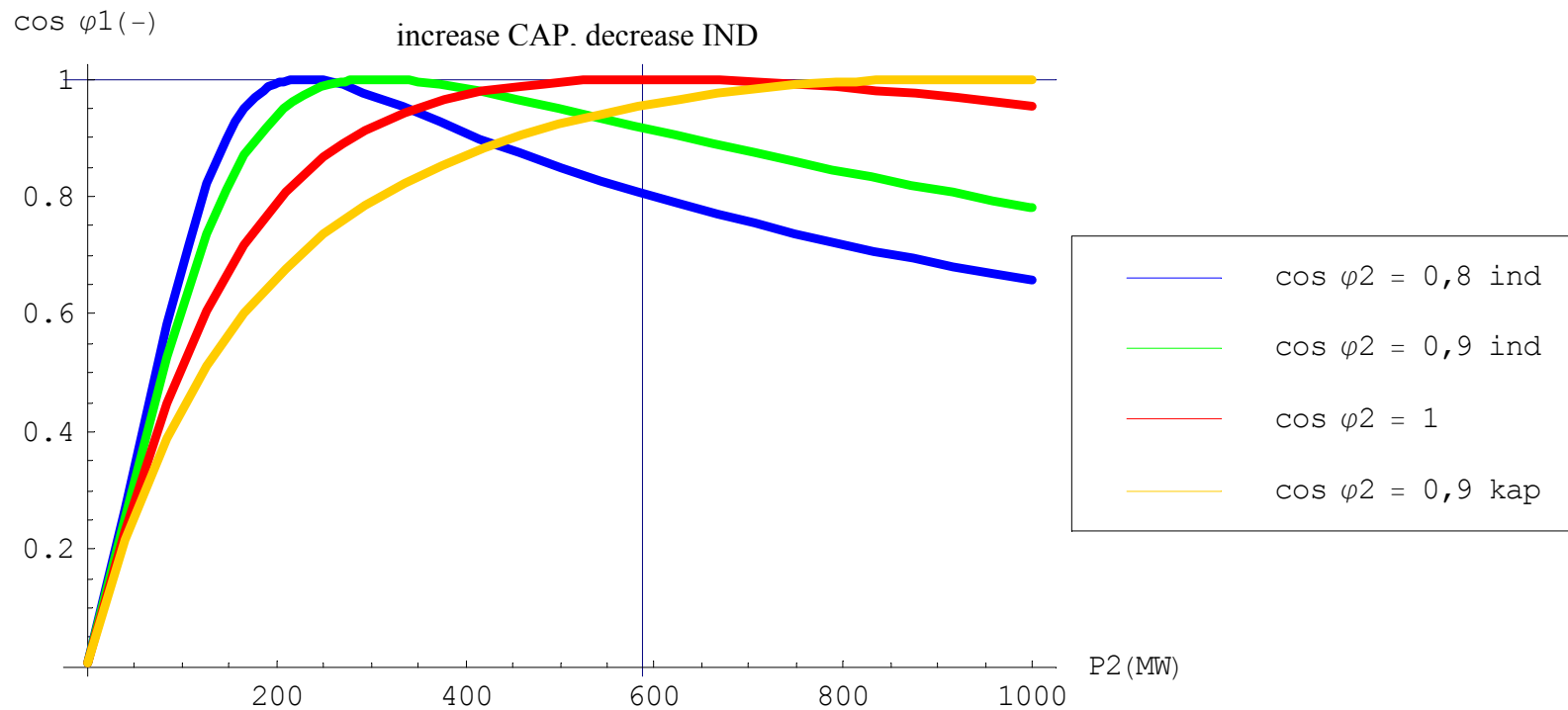


$U_1 < U_n$: Ferranti effect

$U_1 \sim U_n$ for S_p area and $\cos \varphi = 1$

Transmission power factor

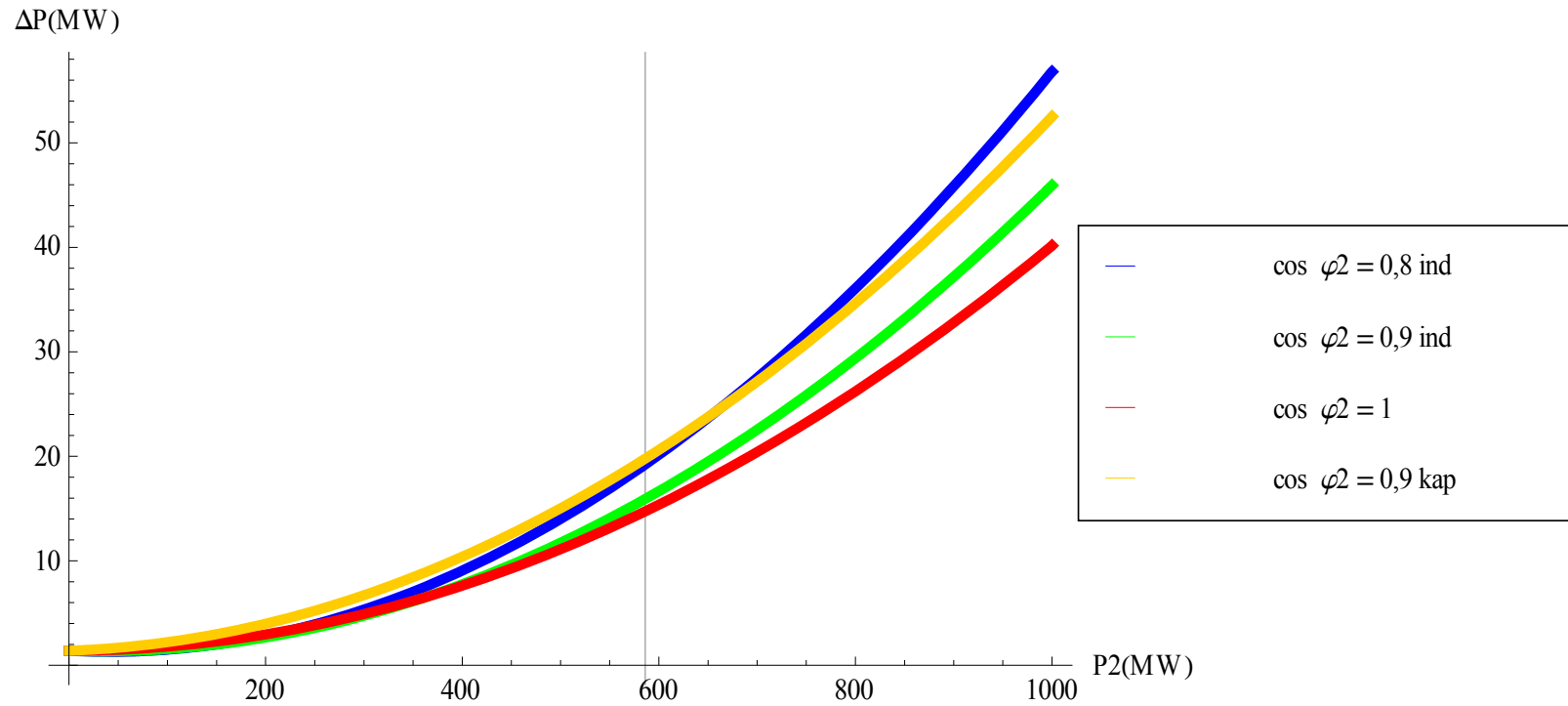
$$\cos \varphi_1 = \frac{P_1}{S_1}$$

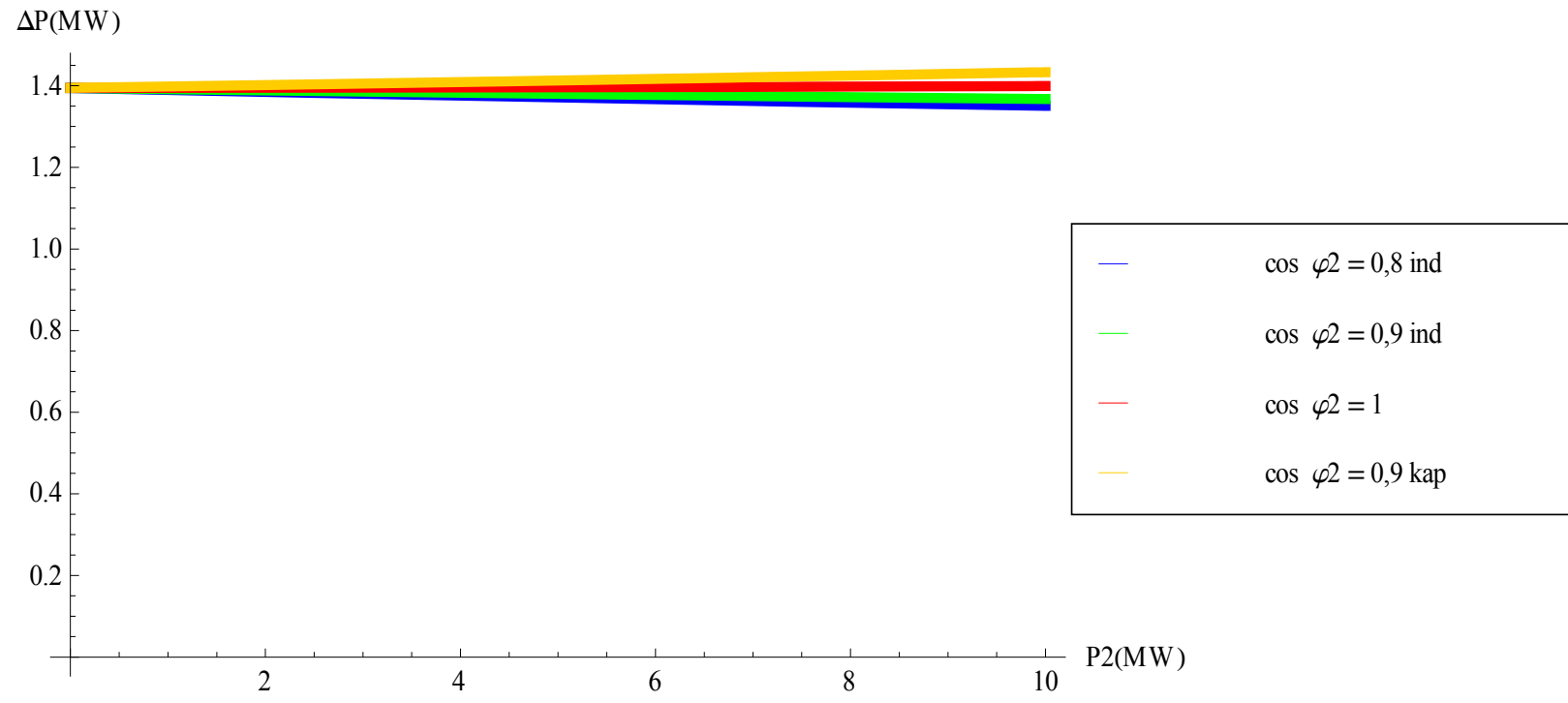


open-circuit \rightarrow line is like capacitive load
higher power \rightarrow line „self-compensation“

Line losses

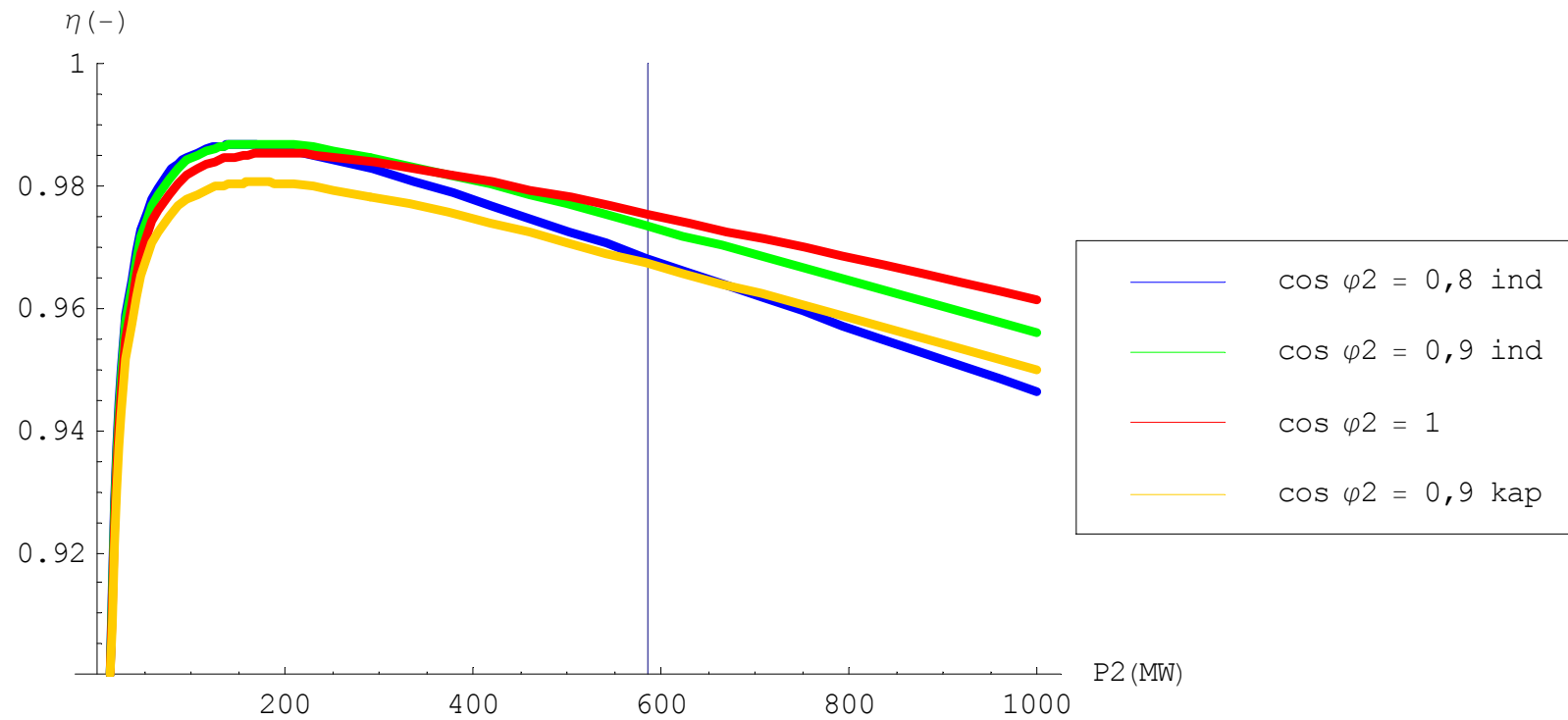
= open-circuit $\sim U^2$ + load $\sim I^2$





Transmission efficiency

$$\eta = \frac{P_2}{P_1}$$



maximum for low powers
for higher powers a flat curve

Hyperbolic functions series (Taylor series at $x = 0$)

$$f(x)_{x_0} = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!} (x - x_0)^k = f(x_0) + f'(x_0) \cdot (x - x_0) + \frac{f''(x_0)}{2} \cdot (x - x_0)^2 + \dots$$

$$\cosh \hat{y}l = 1 + \frac{(\hat{y}l)^2}{2} + \frac{(\hat{y}l)^4}{24} + \dots = 1 + \frac{\hat{Z}_{l_1} \hat{Y}_{q_1}}{2} l^2 + \dots$$

$$\sinh \hat{y}l = \hat{y}l + \frac{(\hat{y}l)^3}{6} + \dots = \sqrt{\hat{Z}_{l_1} \hat{Y}_{q_1}} l + \frac{(\hat{Z}_{l_1} \hat{Y}_{q_1})^{3/2}}{6} l^3 + \dots$$

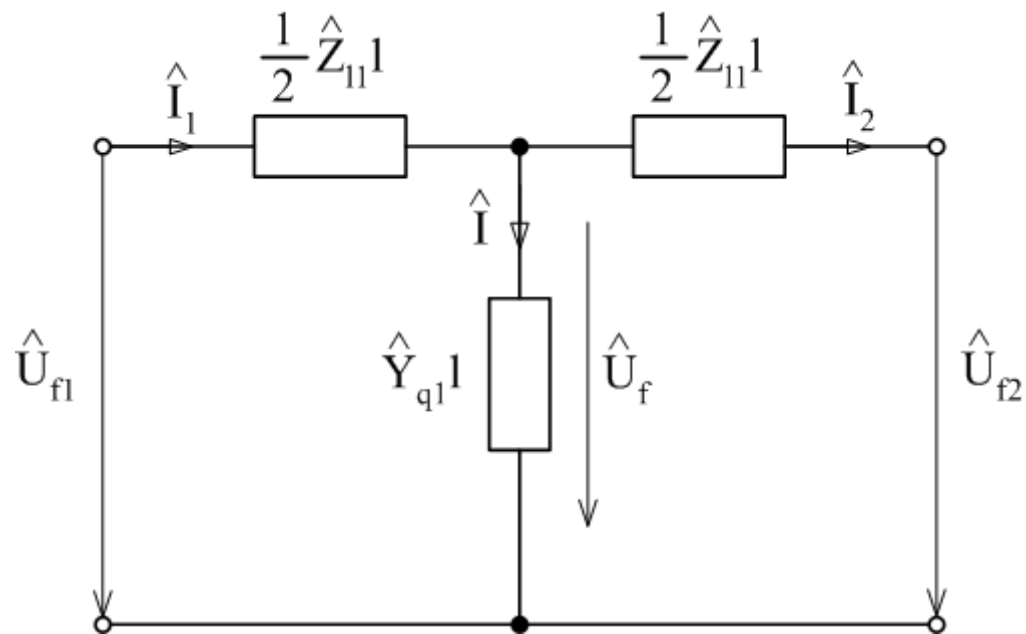
$$\hat{Z}_v \sinh \hat{y}l = \sqrt{\frac{\hat{Z}_{l_1}}{\hat{Y}_{q_1}}} \sinh \hat{y}l = \hat{Z}_{l_1} l + \hat{Z}_{l_1} l \frac{\hat{Z}_{l_1} \hat{Y}_{q_1}}{6} l^2 + \dots$$

$$\frac{1}{\hat{Z}_v} \sinh \hat{y}l = \sqrt{\frac{\hat{Y}_{q_1}}{\hat{Z}_{l_1}}} \sinh \hat{y}l = \hat{Y}_{q_1} l + \hat{Y}_{q_1} l \frac{\hat{Z}_{l_1} \hat{Y}_{q_1}}{6} l^2 + \dots$$

Line with lumped parameters

For ordinary calculations (meshed grids) it is possible to use substitution networks with a good accuracy (according to line length).

T-network – short lines, transformers; it adds another node (substitution diagram for overhead lines up to 200 km, cable lines up to 80 km)



Voltage and current at the line beginning

$$\hat{U}_{f1} = \hat{U}_{f2} + \frac{1}{2} \hat{Z}_{l_1} \mathbf{1} \cdot \hat{I}_2 + \frac{1}{2} \hat{Z}_{l_1} \mathbf{1} \cdot \hat{I}_1 \quad \hat{I}_1 = \hat{I}_2 + \hat{I}$$

Voltage and current for the cross branch

$$\hat{U}_f = \hat{U}_{f2} + \frac{1}{2} \hat{Z}_{l_1} \mathbf{1} \cdot \hat{I}_2$$

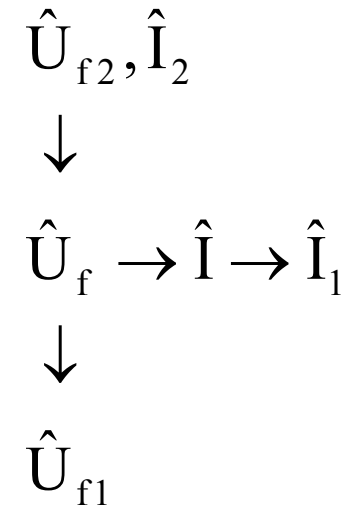
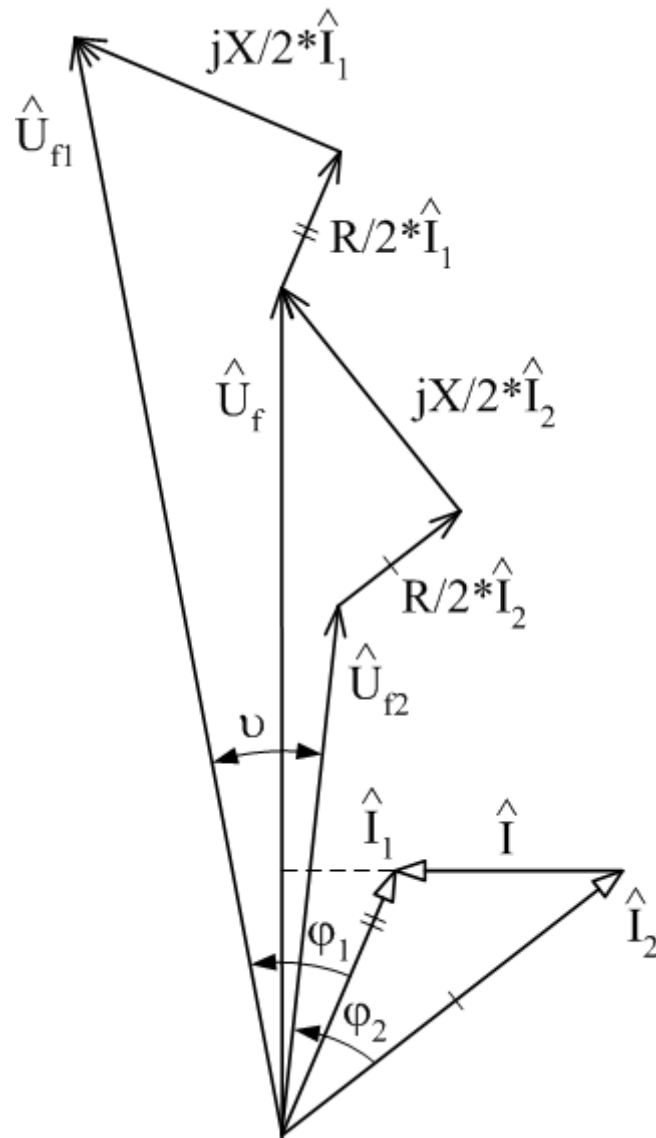
$$\hat{I} = \hat{Y}_{q_1} \mathbf{1} \cdot \hat{U}_f = \hat{U}_{f2} \hat{Y}_{q_1} \mathbf{1} + \hat{I}_2 \frac{\hat{Z}_{l_1} \hat{Y}_{q_1} \mathbf{1}^2}{2}$$

Hence (Blondel's constants relations)

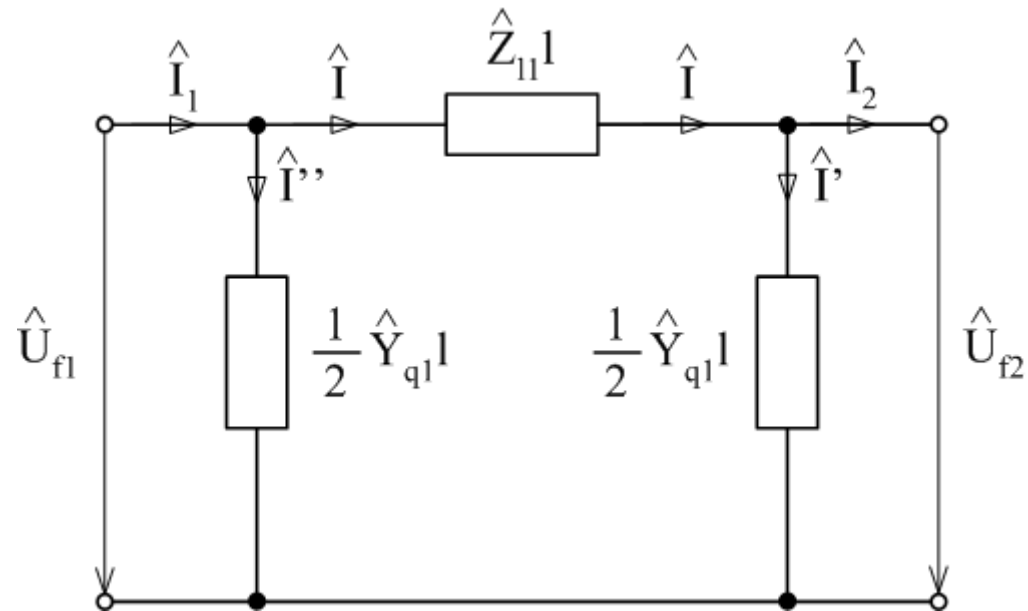
$$\hat{U}_{f1} = \hat{U}_{f2} \left(1 + \frac{\hat{Z}_{l_1} \hat{Y}_{q_1} \mathbf{1}^2}{2} \right) + \hat{I}_2 \hat{Z}_{l_1} \mathbf{1} \left(1 + \frac{\hat{Z}_{l_1} \hat{Y}_{q_1} \mathbf{1}^2}{4} \right)$$

$$\hat{I}_1 = \hat{U}_{f2} \hat{Y}_{q_1} \mathbf{1} + \hat{I}_2 \left(1 + \frac{\hat{Z}_{l_1} \hat{Y}_{q_1} \mathbf{1}^2}{2} \right)$$

Phasor diagram ($G = 0$)



π -network – longer lines, more accurate (substitution diagram for overhead lines up to 250 km, cable lines up to 100 km)



Voltage and current at the line beginning

$$\hat{U}_{f1} = \hat{U}_{f2} + \hat{Z}_{11} \cdot \hat{I} = \hat{U}_{f2} + \hat{Z}_{11} \cdot (\hat{I}_2 + \hat{I}')$$

$$\hat{I}_1 = \hat{I}_2 + \hat{I}' + \hat{I}''$$

Currents for cross branches

$$\hat{I}' = \frac{1}{2} \hat{Y}_{q_1} \mathbf{1} \hat{U}_{f_2}$$

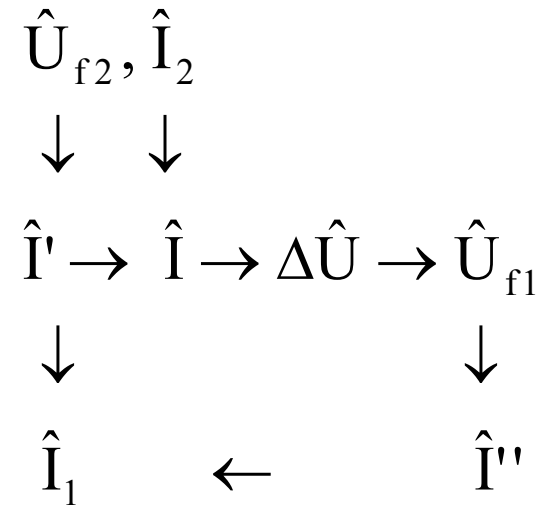
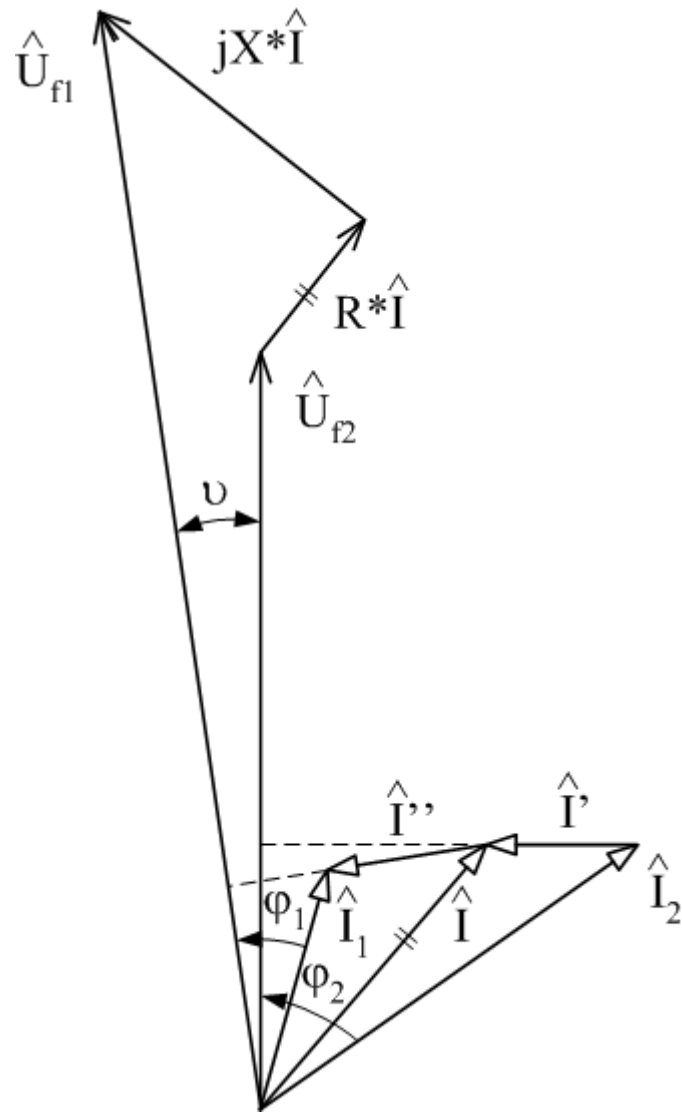
$$\hat{I}'' = \frac{1}{2} \hat{Y}_{q_1} \mathbf{1} \hat{U}_{f_1}$$

After modification (Blondel's constants relations)

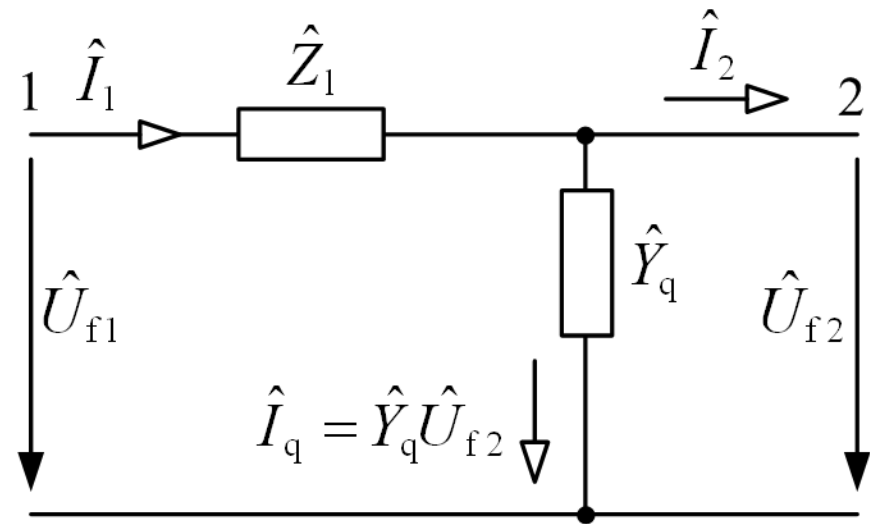
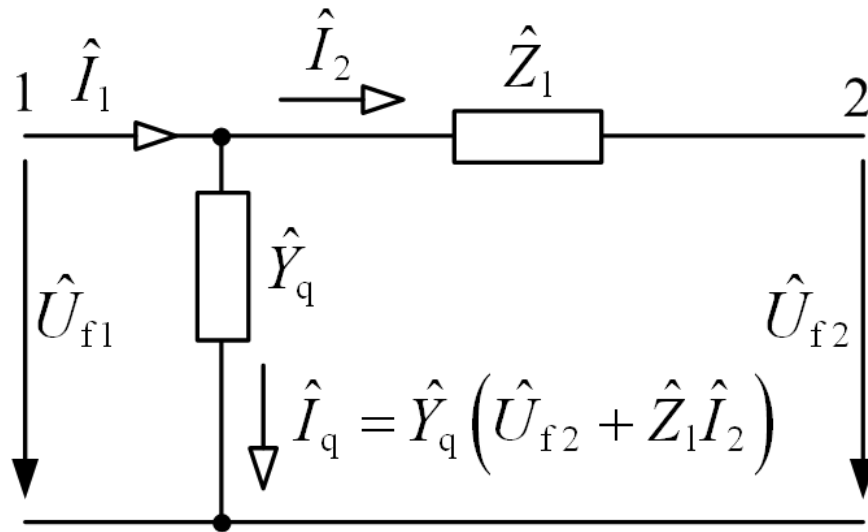
$$\hat{U}_{f_1} = \hat{U}_{f_2} \left(1 + \frac{\hat{Z}_{l_1} \hat{Y}_{q_1}}{2} \mathbf{1}^2 \right) + \hat{I}_2 \hat{Z}_{l_1} \mathbf{1}$$

$$\hat{I}_1 = \hat{U}_{f_2} \hat{Y}_{q_1} \mathbf{1} \left(1 + \frac{\hat{Z}_{l_1} \hat{Y}_{q_1}}{4} \mathbf{1}^2 \right) + \hat{I}_2 \left(1 + \frac{\hat{Z}_{l_1} \hat{Y}_{q_1}}{2} \mathbf{1}^2 \right)$$

Phasor diagram (G = 0)



Γ -network (gamma) – relatively rare utilization, for shorter lines (overhead up to 80 km, cable up to 25 km), transformers



$$\hat{U}_{f1} = \hat{U}_{f2} + \hat{I}_2 \hat{Z}_{l_1} 1$$

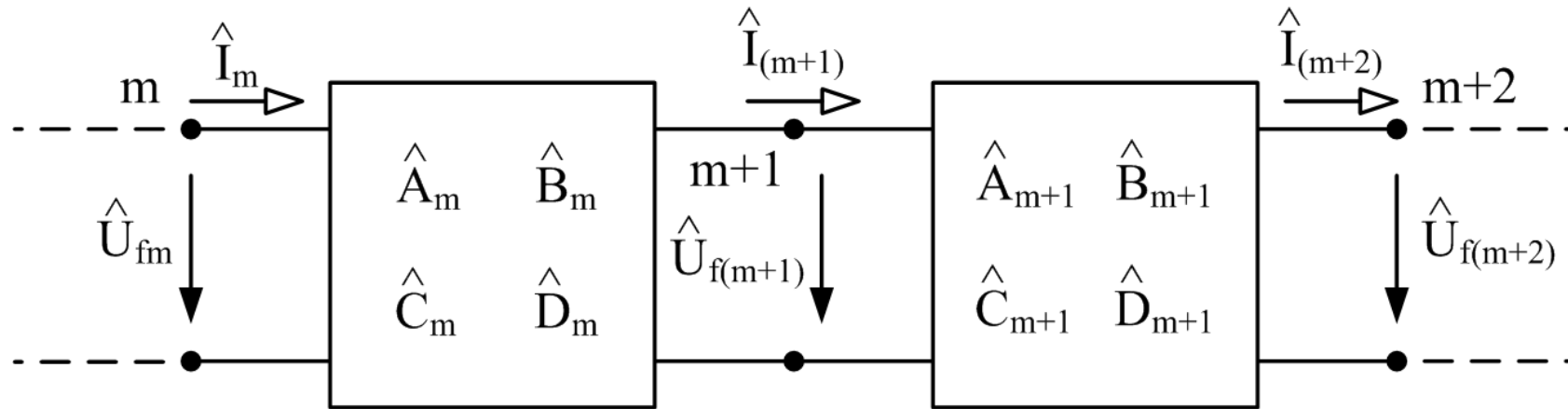
$$\hat{I}_1 = \hat{U}_{f2} \hat{Y}_{q_1} 1 + \hat{I}_2 (1 + \hat{Z}_{l_1} \hat{Y}_{q_1} 1^2)$$

$$\hat{U}_{f1} = \hat{U}_{f2} (1 + \hat{Z}_{l_1} \hat{Y}_{q_1} 1^2) + \hat{I}_2 \hat{Z}_{l_1} 1$$

$$\hat{I}_1 = \hat{U}_{f2} \hat{Y}_{q_1} 1 + \hat{I}_2$$

it is valid $\hat{A} \neq \hat{D}$, $\hat{A}\hat{D} - \hat{B}\hat{C} = 1$ (unbalanced, passive two-port network)

Longer lines \rightarrow cascade connection of networks for shorter sections
(additional nodes)



$$\begin{pmatrix} \hat{U}_{fm} \\ \hat{I}_m \end{pmatrix} = \begin{pmatrix} \hat{A}_m & \hat{B}_m \\ \hat{C}_m & \hat{D}_m \end{pmatrix} \begin{pmatrix} \hat{U}_{f(m+1)} \\ \hat{I}_{(m+1)} \end{pmatrix}$$

$$\begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix} = \prod_{m=1}^n \begin{pmatrix} \hat{A}_m & \hat{B}_m \\ \hat{C}_m & \hat{D}_m \end{pmatrix}$$