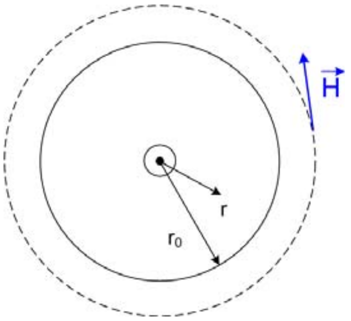
Calculation of inductance

1) internal inductance of a conductor



$$\oint_{l} \vec{H}.d\vec{l} = I$$

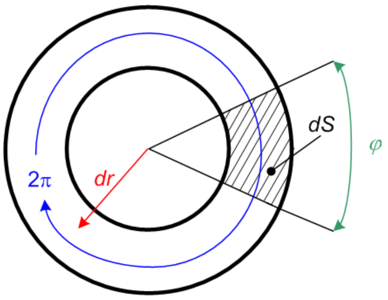
$$H.2\pi.r = I$$

$$H_i.2\pi.r = I\left(\frac{r}{r_0}\right)^2$$

$$H_i = I\frac{r}{2\pi.r_0^2}$$

- surface area calculation:

$$S = \pi.r^2$$
$$dS = \pi.2r.dr$$
$$dS = d\varphi.r.dr$$



- energy calculation:

$$w = \frac{1}{2}\cdot\vec{H}\cdot\vec{B} = \frac{1}{2}\mu.H^2 \quad (J/m^3)$$

$$W = w.V \quad (J)$$

$$W = \frac{1}{2}\cdot L\cdot I^2$$

- derivation of internal inductance:

$$\frac{1}{2}\cdot L_i\cdot I^2 = \int_0^l\int_0^{2\pi}\int_0^{r_0}\frac{1}{2}\mu\frac{I^2r^2}{4.\pi^2.r_0^4}r.dr.d\varphi.dl = \frac{\mu.I^2}{8.\pi^2.r_0^4}\cdot\frac{r_0^4}{4}\cdot 2\pi.l = \frac{\mu.I^2}{16.\pi}\cdot l$$

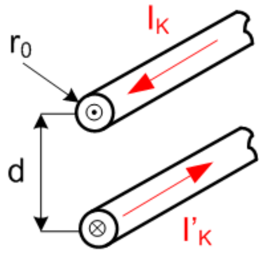
$$L_i = \frac{\mu}{8.\pi}\cdot l \quad (H)$$

$$L_i = \frac{\mu}{8.\pi} \quad (H/m)$$

2) external inductance of a conductor in a loop

$$\oint_l \vec{H}.d\vec{l} = I$$

$$H_e = \frac{I}{2\pi.r}$$



- derivation of external inductance:

$$\Phi = L \cdot I = \iint \vec{B} \cdot d\vec{S}$$

$$\Phi = \int_0^l \int_{r_0}^d \frac{\mu I}{2\pi r} dr.dl$$

$$\Phi = \frac{\mu I}{2\pi} \cdot \ln\frac{d}{r_0} \cdot l$$

$$L_e = \frac{\mu}{2.\pi} \cdot \ln\frac{d}{r_0} \cdot l \quad (H)$$

$$L_e = \frac{\mu}{2.\pi} \cdot \ln\frac{d}{r_0} \quad (H/m)$$

3) total inductance of one conductor in a loop

$$L_{kv} = L_i + L_e$$

$$= \frac{4\pi.10^{-7}}{8\pi} \cdot \alpha.\mu_r + \frac{4\pi.10^{-7}}{2\pi} \ln\frac{d}{r_0} =$$

$$= \frac{4\pi.10^{-7}}{8\pi} \cdot \alpha.\mu_r + \frac{4\pi.10^{-7}}{2\pi} \frac{\log\frac{d}{r_0}}{\log e} =$$

$$= \frac{1}{2} \cdot 10^{-7} \alpha.\mu_r + 2.10^{-7} \frac{1}{\log e} \log\frac{d}{r_0} \quad (H/m) =$$

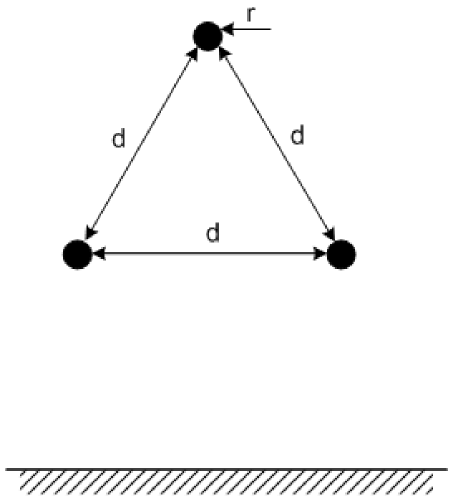
$$= 0{,}05\alpha.\mu_r + 0{,}2 \cdot 2{,}3 \cdot \log\frac{d}{r_0} \quad (mH/km)$$

$$\left(0{,}05\alpha.\mu_r = 0{,}46.\log\frac{1}{\xi} \quad \rightarrow \quad \xi = 10^{-\frac{0{,}05.\mu_r.\alpha}{0{,}46}} \right)$$

$$= 0{,}46.\log\frac{d}{\xi.r_0} \quad (mH/km)$$

Electrical parameters of a power line – calculation of inductances

Ex. 1: Three-phase power line is arranged symmetrically on the pole as shown. Conductors have a radius r = 4 mm with coefficient (of current density inequality in cross section) $\xi = 0{,}809$. The distance between phase conductors is d = 3 m. Calculate operational inductance and inductive reactance of this power line.



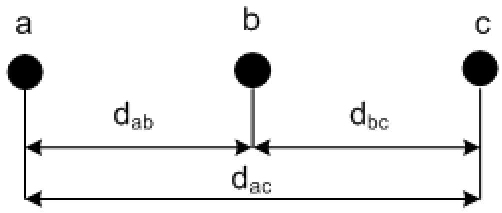
Operational inductance:

$$L = 0{,}46 \cdot \log \frac{d}{\xi \cdot r} = 0{,}46 \cdot \log \frac{3 \cdot 10^3}{0{,}809 \cdot 4} = 1{,}365 \;\; \mathrm{mH/km}$$

Inductive reactance (f = 50 Hz):

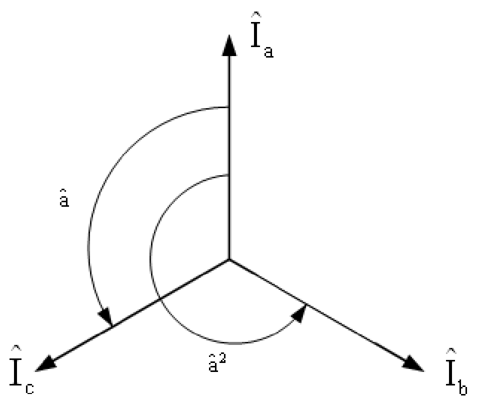
$$X = \omega \cdot L = 2 \cdot \pi \cdot f \cdot L = 2 \cdot 3{,}14 \cdot 50 \cdot 1{,}365 \cdot 10^{-3} = 0{,}4288 \;\Omega/\mathrm{km}$$

Ex. 2: Three-phase non-transposed power line is arranged asymmetrically on the pole as shown. The distances between phase conductors are $d_{ab} = d_{bc} = 9$ m and $d_{ac} = 2 \cdot d_{ab} = 18$ m. Phase conductors have a radius r = 27,7 mm with coefficient (of current density inequality in cross section) $\xi = 0{,}81$. Calculate operational inductance of this power line.



The power line is asymmetrical and non-transposed so operational inductances of each phase are different and moreover complex.

Assuming symmetrical load:



$$\hat{I}_a = I_a$$
$$\hat{I}_b = \hat{a}^2 \cdot I_a$$
$$\hat{I}_c = \hat{a} \cdot I_a$$

$$\hat{a} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

$$\hat{a}^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

Operational inductances of phase a:

$$\hat{L}_a = \frac{M_{aa} \cdot \hat{I}_a + M_{ab} \cdot \hat{I}_b + M_{ac} \cdot \hat{I}_c}{\hat{I}_a} = \frac{M_{aa} \cdot I_a + M_{ab} \cdot \hat{a}^2 \cdot I_a + M_{ac} \cdot \hat{a} \cdot I_a}{I_a} =$$

$$= \frac{I_a \cdot \left(M_{aa} + M_{ab} \cdot \hat{a}^2 + M_{ac} \cdot \hat{a}\right)}{I_a} = M_{aa} + M_{ab} \cdot \hat{a}^2 + M_{ac} \cdot \hat{a} =$$

$$= M_{aa} + M_{ab} \cdot \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + M_{ac} \cdot \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) =$$

$$= M_{aa} - \frac{1}{2} \cdot M_{ab} - \frac{1}{2} \cdot M_{ac} + j \cdot \left(-\frac{\sqrt{3}}{2} \cdot M_{ab} + \frac{\sqrt{3}}{2} \cdot M_{ac}\right)$$

All phase conductors have the same radius and the same design, hence:

$$M_{aa} = M_{bb} = M_{cc} = 0{,}46 \cdot \log \frac{D_g}{\xi \cdot r}$$

Because of different phase conductors distances:

$$M_{ab} = 0{,}46 \cdot \log \frac{D_g}{d_{ab}} \quad \neq M_{ac} \neq M_{bc}$$

After the substitution to the derived equation:

$$\hat{L}_a = 0{,}46 \cdot \log \frac{\sqrt{d_{ab} \cdot d_{ac}}}{\xi \cdot r} + j \cdot 0{,}46 \cdot \sqrt{3} \cdot \log \sqrt{\frac{d_{ab}}{d_{ac}}} = 0{,}46 \cdot \log \frac{\sqrt{9 \cdot 18 \cdot 10^{-3}}}{0{,}81 \cdot 21{,}7} + j \cdot 0{,}46 \cdot \sqrt{3} \cdot \log \sqrt{\frac{9}{18}} =$$

$$= (1{,}3155 - j \cdot 0{,}12) \ \text{mH/km}$$

Operational inductance of phase b as well as c is computed similarly as phase a:

$$\hat{L}_b = 0{,}46 \cdot \log \frac{\sqrt{d_{ab} \cdot d_{bc}}}{\xi \cdot r} + j \cdot 0{,}46 \cdot \sqrt{3} \cdot \log \sqrt{\frac{d_{bc}}{d_{ab}}} = 1{,}246 \ \text{mH/km}$$

$$\hat{L}_c = 0{,}46 \cdot \log \frac{\sqrt{d_{ac} \cdot d_{bc}}}{\xi \cdot r} + j \cdot 0{,}46 \cdot \sqrt{3} \cdot \log \sqrt{\frac{d_{ac}}{d_{bc}}} = (1{,}3155 + j \cdot 0{,}12) \ \text{mH/km}$$

Ex. 3: Three-phase power line from ex. 2 is transposed. Calculate operational inductance and inductive reactance of this power line.

a) Operational inductance using the calculated results from ex. 2 (substituting into the formula for transposed power line):

$$L = \frac{1}{3} \cdot \left(\hat{L}_a + \hat{L}_b + \hat{L}_c\right) = \frac{1}{3} \cdot \left(1{,}3155 - j \cdot 0{,}12 + 1{,}246 + 1{,}3155 + j \cdot 0{,}12\right) = 1{,}292 \ \text{mH/km}$$

b) Operational inductance according to the derived formula for the operational inductance of transposed power line:

$$L = L_a = L_b = L_c = 0{,}46 \cdot \log\frac{d}{\xi \cdot r} = 0{,}46 \cdot \log\frac{11{,}34 \cdot 10^3}{0{,}81 \cdot 21{,}7} = 1{,}292 \ \text{mH/km}$$

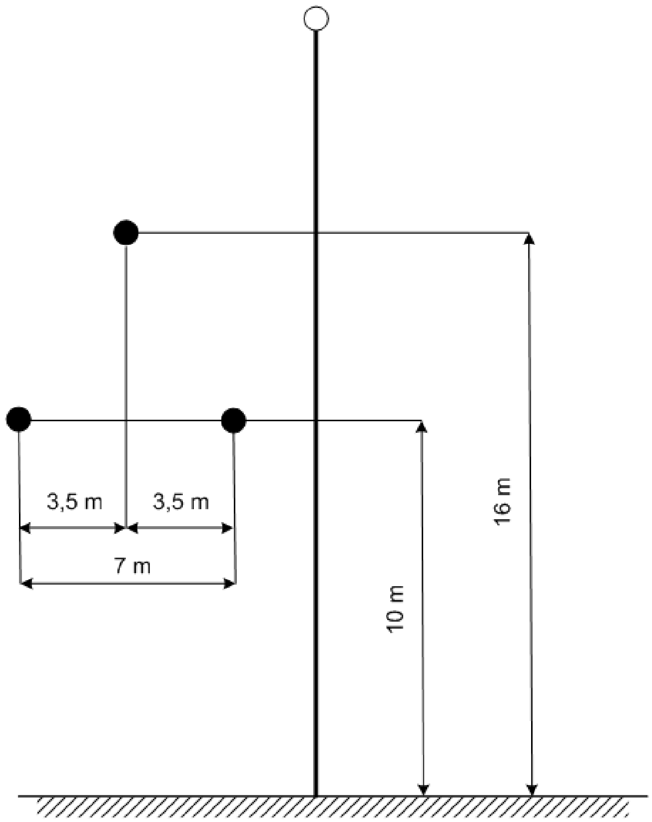
Mean geometrical distance between phase conductors:

$$d = \sqrt[3]{d_{ab} \cdot d_{ac} \cdot d_{bc}} = \sqrt[3]{9 \cdot 18 \cdot 9} = 9 \cdot \sqrt[3]{2} = 11{,}34 \ \text{m}$$

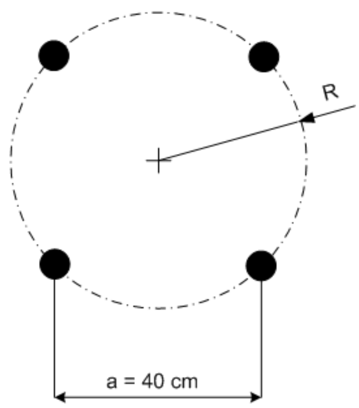
Inductive reactance (f = 50 Hz):

$$X = \omega \cdot L = 2 \cdot \pi \cdot f \cdot L = 2 \cdot 3{,}14 \cdot 50 \cdot 1{,}292 \cdot 10^{-3} = 0{,}406 \ \Omega/\text{km}$$

Ex. 4: Three-phase transposed power line with one ground wire is on the pole as shown. Phase conductors are made as bundle conductors as shown. One partial conductor is made with the conductor $6\,\text{AlFe}\,240\ \text{mm}^2$ with diameter $2 \cdot r = 21{,}7$ mm with coefficient (of current density inequality in cross section) $\xi = 0{,}826$. Resistance per kilometre of a partial conductor in a bundle is $R_{dl} = 0{,}122\ \Omega$/km. Calculate longitudinal impedance of this power line.



Bundle conductor



The radius of the circumscribed circle of a bundle conductor (half diagonal of a square):

$$R = a \cdot \frac{\sqrt{2}}{2} = 400 \cdot \frac{\sqrt{2}}{2} = 282{,}84 \text{ mm}$$

Equivalent bundle radius:

$$r_e = R \cdot \sqrt[n]{r \cdot \frac{n}{R}} = 282{,}84 \sqrt[4]{10{,}85 \cdot \frac{4}{282{,}84}} = 177{,}02 \text{ mm}$$

Equivalent coefficient:

$$\xi_e = \sqrt[n]{\xi} = \sqrt[4]{0{,}826} = 0{,}953$$

Mean geometrical distance of phase conductors:

$$d = \sqrt[3]{d_{12} \cdot d_{23} \cdot d_{13}} = \sqrt[3]{\sqrt{6^2 + 3{,}5^2} \cdot \sqrt{6^2 + 3{,}5^2} \cdot (3{,}5 + 3{,}5)} = 6{,}97 \text{ m}$$

Operational inductance of a bundle conductor per kilometre:

$$L_1 = 0{,}46 \cdot \log \frac{d}{\xi_e \cdot r_e} = 0{,}46 \cdot \log \frac{6{,}97 \cdot 10^3}{0{,}953 \cdot 177{,}02} = 0{,}74 \text{ mH/km}$$

Inductive reactance of a bundle conductor per kilometre (f = 50 Hz):

$$X_1 = \omega \cdot L_1 = 2 \cdot \pi \cdot f \cdot L_1 = 2 \cdot 3{,}14 \cdot 50 \cdot 0{,}74 \cdot 10^{-3} = 0{,}233 \ \Omega/\text{km}$$

Resistance of one bundle conductor per kilometre:

$$R_1 = \frac{R_{d1}}{n} = \frac{0{,}122}{4} = 0{,}0305 \ \Omega/\text{km}$$

Longitudinal impedance:

$$\hat{Z}_{11} = R_1 + j \cdot X_1 = 0{,}0305 + j \cdot 0{,}233 \ \Omega/\text{km}$$