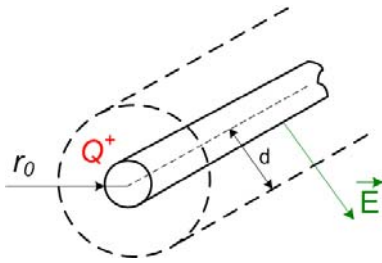


Calculation of capacities

- voltage from cylindrical conductor



$$\oiint_S \vec{D} \cdot d\vec{S} = Q$$

$$\int_0^l \int_0^{2\pi} \epsilon \cdot E \cdot r \cdot d\phi \cdot dl = Q$$

$$\epsilon \cdot E \cdot 2\pi \cdot l \cdot r = Q \quad (\text{the entire conductor length})$$

$$E = \frac{Q_1}{2\pi \cdot \epsilon \cdot r} \quad (\text{V/m})$$

$$\vec{E} = -\text{grad } \varphi$$

$$U^+ = -\int_{r_0}^d \vec{E} \cdot d\vec{l} = -\int_{r_0}^d \frac{Q_1}{2\pi \epsilon \cdot r} dr = -\frac{Q_1}{2\pi \epsilon} \cdot [\ln r]_{r_0}^d = \frac{Q_1}{2\pi \epsilon} \cdot \ln \frac{r_0}{d^+}$$

$$U^- = -\int_{r_0}^d \vec{E} \cdot d\vec{l} = -\int_{r_0}^d \frac{-Q_1}{2\pi \epsilon \cdot r} dr = \frac{Q_1}{2\pi \epsilon} \cdot [\ln r]_{r_0}^d = \frac{Q_1}{2\pi \epsilon} \cdot \ln \frac{d^-}{r_0}$$

$$U = U^+ + U^- = \frac{Q_1}{2\pi \epsilon} \cdot \ln \frac{d^-}{d^+} \quad (\text{V})$$

- cable capacity (cylindrical capacitor):

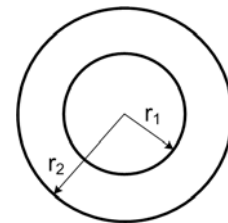
$$Q = C \cdot U$$

inside conductor Q^+ , voltage from r_1 to r_2 $U = -U^+$

$$U = \frac{Q_1}{2\pi \epsilon} \cdot \ln \frac{r_2}{r_1}$$

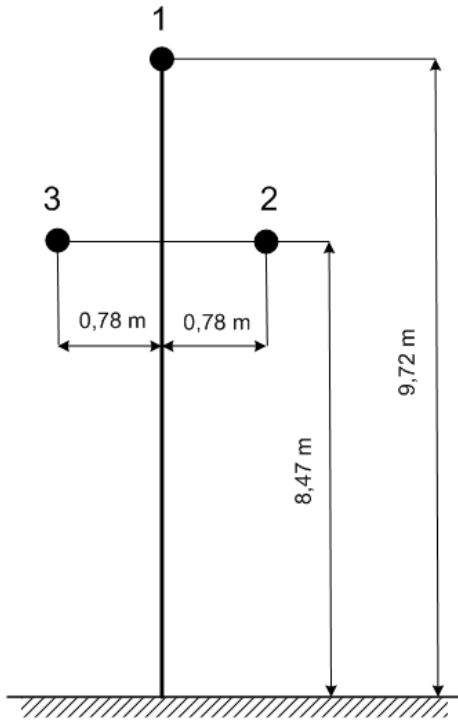
$$C = \frac{Q}{U} = \frac{2\pi \epsilon}{\ln \frac{r_2}{r_1}} = \frac{2\pi \cdot 8,854 \cdot 10^{-12} \cdot \epsilon_r}{\frac{\log \frac{r_2}{r_1}}{\log e}} = \frac{24,2 \cdot 10^{-12} \cdot \epsilon_r}{\log \frac{r_2}{r_1}} \quad (\text{F/m})$$

$$C = \frac{0,0242 \cdot \epsilon_r}{\log \frac{r_2}{r_1}} \quad (\mu\text{F/km})$$



Power line parameters – Capacities calculation

Ex. 1: Three-phase transposed power line is in the figure. Phase conductors are made of AlF70 mm², with radius r=5,8 mm. Calculate partial capacity to the ground of one conductor, partial mutual capacity and operational capacity (all capacities per 1 kilometre of overhead line).



Mean geometric height of phase conductors above the ground:

$$h = \sqrt[3]{h_1 \cdot h_2 \cdot h_3} = \sqrt[3]{9,72 \cdot 8,47 \cdot 8,47} = 8,86 \text{ m}$$

Mean geometric distance between phase conductors:

$$d = \sqrt[3]{d_{12} \cdot d_{13} \cdot d_{23}} = \sqrt[3]{\sqrt{0,78^2 + 1,25^2} \cdot \sqrt{0,78^2 + 1,25^2} \cdot (0,78 + 0,78)} = 1,5 \text{ m}$$

Potential coefficients:

$$\delta = \frac{1}{0,0242} \cdot \log \frac{2h}{r} = \frac{1}{0,0242} \cdot \log \frac{2 \cdot 8,86 \cdot 10^3}{5,8} = 144 \text{ km} / \mu\text{F}$$

$$\delta' = \frac{1}{0,0242} \cdot \log \frac{\sqrt{4 \cdot h^2 + d^2}}{d} = \frac{1}{0,0242} \cdot \log \frac{4 \cdot 8,86^2 + 1,5^2}{1,5} = 44,4 \text{ km} / \mu\text{F}$$

Partial capacity to the ground per 1 kilometre of the line length:

$$k_0 = \frac{1}{\delta + 2 \cdot \delta'} = \frac{1}{144 + 2 \cdot 44,4} = 0,00429 \mu\text{F} / \text{km}$$

Partial mutual capacity per 1 kilometre of the line length:

$$k' = \frac{\delta'}{(\delta + 2 \cdot \delta') \cdot (\delta - \delta')} = \frac{44,4}{(144 + 2 \cdot 44,4) \cdot (144 - 44,4)} = 0,00191 \mu\text{F} / \text{km}$$

Operational capacity of one conductor per 1 kilometre of the line length:

$$C = \frac{1}{\delta - \delta'} = \frac{1}{144 - 44,4} = 10,02 \cdot 10^{-3} \mu\text{F} / \text{km}$$

or $C = k_0 + 3 \cdot k' = 0,00429 + 3 \cdot 0,00191 = 10,02 \cdot 10^{-3} \mu\text{F} / \text{km}$

$$\begin{aligned}\hat{C}_A &= \frac{k_0 \cdot \hat{U}_A + k' \cdot (\hat{U}_A - \hat{U}_B) + k' \cdot (\hat{U}_A - \hat{U}_C)}{\hat{U}_A} = k_0 + k' \cdot (1 - \hat{a}^2) + k' \cdot (1 - \hat{a}) = \\ &= k_0 + k' \cdot (2 - \hat{a}^2 - \hat{a}) = k_0 + 3k'\end{aligned}$$

$$(\delta_{km}) = \begin{pmatrix} \delta & \delta' & \delta' \\ \delta' & \delta & \delta' \\ \delta' & \delta' & \delta \end{pmatrix}$$

$$(c_{km}) = (\delta_{km})^{-1} = \begin{pmatrix} c & c' & c' \\ c' & c & c' \\ c' & c' & c \end{pmatrix}$$

$$c = \frac{\delta + \delta'}{\delta^2 + \delta \cdot \delta' - 2\delta'^2} = \frac{\delta + \delta'}{(\delta - \delta')(\delta + 2\delta')}$$

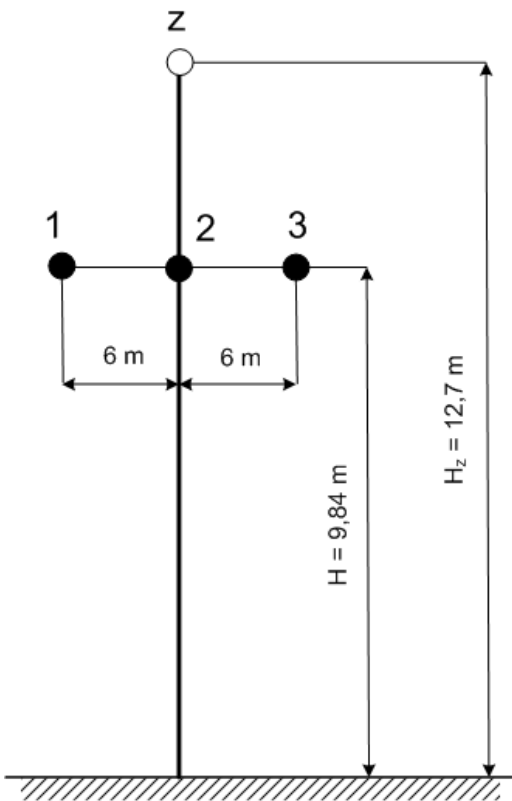
$$c' = \frac{-\delta'}{\delta^2 + \delta \cdot \delta' - 2\delta'^2} = \frac{-\delta'}{(\delta - \delta')(\delta + 2\delta')}$$

$$k_0 = c + 2c' = \frac{\delta + \delta' - 2\delta'}{(\delta - \delta')(\delta + 2\delta')} = \frac{1}{\delta + 2\delta'}$$

$$k' = -c' = \frac{\delta'}{(\delta - \delta')(\delta + 2\delta')}$$

$$C = k_0 + 3k' = \frac{1}{\delta + 2\delta'} + \frac{3\delta'}{(\delta - \delta')(\delta + 2\delta')} = \frac{1}{\delta - \delta'}$$

Ex. 2: Three-phase transposed line is in the figure. There are marked suspension heights of phase conductors and of ground wire.



Phase conductors (unbundle):

AlFe 150 mm^2 , $2 \cdot r = 24 \text{ mm}$

sag: $p = 1,2 \text{ m}$

Ground wire:

Fe 70 mm^2 , $2 \cdot r_z = 11 \text{ mm}$

sag: $p_z = 1 \text{ m}$

Calculate partial capacity to the ground of one conductor, partial mutual capacity and operational capacity (all capacities per 1 kilometre of overhead line).

Calculation height (phase conductors):

$$h_1 = h_2 = h_3 = H - 0,7 \cdot p = 9,84 - 0,7 \cdot 1,2 = 9 \text{ m}$$

Calculation height (ground wire):

$$h_z = H_z - 0,7 \cdot p_z = 12,7 - 0,7 \cdot 1 = 12 \text{ m}$$

Mean geometric height of phase conductors above the ground:

$$h = \sqrt[3]{h_1 \cdot h_2 \cdot h_3} = \sqrt[3]{9^3} = 9 \text{ m}$$

Mean geometric distance between phase conductors:

$$d = \sqrt[3]{d_{12} \cdot d_{13} \cdot d_{23}} = \sqrt[3]{6 \cdot 12 \cdot 6} = 7,56 \text{ m}$$

Mean geometric distance between ground wire and phase conductors:

$$d_{vz} = \sqrt[3]{d_{1z} \cdot d_{2z} \cdot d_{3z}} = \sqrt[3]{\sqrt{6^2 + 3^2} \cdot 3 \cdot \sqrt{6^2 + 3^2}} = 5,13 \text{ m}$$

Potential coefficients (phase conductors only):

$$\delta = \frac{1}{0,0242} \cdot \log \frac{2 \cdot h}{r} = \frac{1}{0,0242} \cdot \log \frac{2 \cdot 9 \cdot 10^3}{12} = 131,24 \text{ km} / \mu\text{F}$$

$$\delta' = \frac{1}{0,0242} \cdot \log \frac{\sqrt{4 \cdot h^2 + d^2}}{d} = \frac{1}{0,0242} \cdot \log \frac{\sqrt{4 \cdot 9^2 + 7,56^2}}{7,56} = 17,02 \text{ km} / \mu\text{F}$$

Potential coefficients (ground wire only):

$$\delta_{zz} = \frac{1}{0,0242} \cdot \log \frac{2 \cdot h_z}{r_z} = \frac{1}{0,0242} \cdot \log \frac{2 \cdot 12 \cdot 10^3}{5,5} = 150,41 \text{ km} / \mu\text{F}$$

Potential coefficients (phase conductors + ground wire):

$$\delta_{vz} = \frac{1}{0,0242} \cdot \log \frac{\sqrt{4 \cdot h \cdot h_z + d_{vz}^2}}{d_{vz}} = \frac{1}{0,0242} \cdot \log \frac{\sqrt{4 \cdot 9 \cdot 12 + 5,13^2}}{5,13} = 25,64 \text{ km} / \mu\text{F}$$

Correction potential coefficient:

$$\delta_k = \frac{\delta_{vz}^2}{\delta_{zz}} = \frac{25,64^2}{150,41} = 4,37 \text{ km} / \mu\text{F}$$

Factors:

$$N = \delta - \delta_k = 131,24 - 4,37 = 126,87 \text{ km} / \mu\text{F}$$

$$N' = \delta' - \delta_k = 17,02 - 4,37 = 12,65 \text{ km} / \mu\text{F}$$

Partial capacity of one conductor to the ground:

$$k_0 = \frac{1}{N + 2 \cdot N'} = \frac{1}{126,87 + 2 \cdot 12,65} = 6,57 \cdot 10^{-3} \mu\text{F} / \text{km}$$

Partial mutual capacity:

$$k' = \frac{N'}{(N + 2 \cdot N') \cdot (N - N')} = \frac{12,65}{(126,87 + 2 \cdot 12,65) \cdot (126,87 - 12,65)} = 0,728 \cdot 10^{-3} \mu\text{F} / \text{km}$$

Operational capacity of one conductor per 1 kilometre of the line length:

$$C = \frac{1}{N - N'} = \frac{1}{126,87 - 12,65} = 8,755 \cdot 10^{-3} \mu\text{F} / \text{km}$$

$$\text{or } C = k_0 + 3 \cdot k' = (6,57 + 3 \cdot 0,728) \cdot 10^{-3} = 8,755 \cdot 10^{-3} \mu\text{F} / \text{km}$$

$$(\delta_{km}) = \begin{pmatrix} \delta & \delta' & \delta' & \delta_{vz} \\ \delta' & \delta & \delta' & \delta_{vz} \\ \delta' & \delta' & \delta & \delta_{vz} \\ \delta_{vz} & \delta_{vz} & \delta_{vz} & \delta_{zz} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$(\delta_{km})_{mod} = A - B \cdot D^{-1} \cdot C$$

$$(\delta_{kor}) = B \cdot D^{-1} \cdot C = \begin{pmatrix} \delta_{vz} \\ \delta_{vz} \\ \delta_{vz} \end{pmatrix} (\delta_{zz}^{-1}) \begin{pmatrix} \delta_{vz} & \delta_{vz} & \delta_{vz} \end{pmatrix} = \begin{pmatrix} \frac{\delta_{vz}^2}{\delta_{zz}} & \frac{\delta_{vz}^2}{\delta_{zz}} & \frac{\delta_{vz}^2}{\delta_{zz}} \\ \frac{\delta_{vz}^2}{\delta_{zz}} & \frac{\delta_{vz}^2}{\delta_{zz}} & \frac{\delta_{vz}^2}{\delta_{zz}} \\ \frac{\delta_{vz}^2}{\delta_{zz}} & \frac{\delta_{vz}^2}{\delta_{zz}} & \frac{\delta_{vz}^2}{\delta_{zz}} \end{pmatrix} = \begin{pmatrix} \delta_{kor} & \delta_{kor} & \delta_{kor} \\ \delta_{kor} & \delta_{kor} & \delta_{kor} \\ \delta_{kor} & \delta_{kor} & \delta_{kor} \end{pmatrix}$$

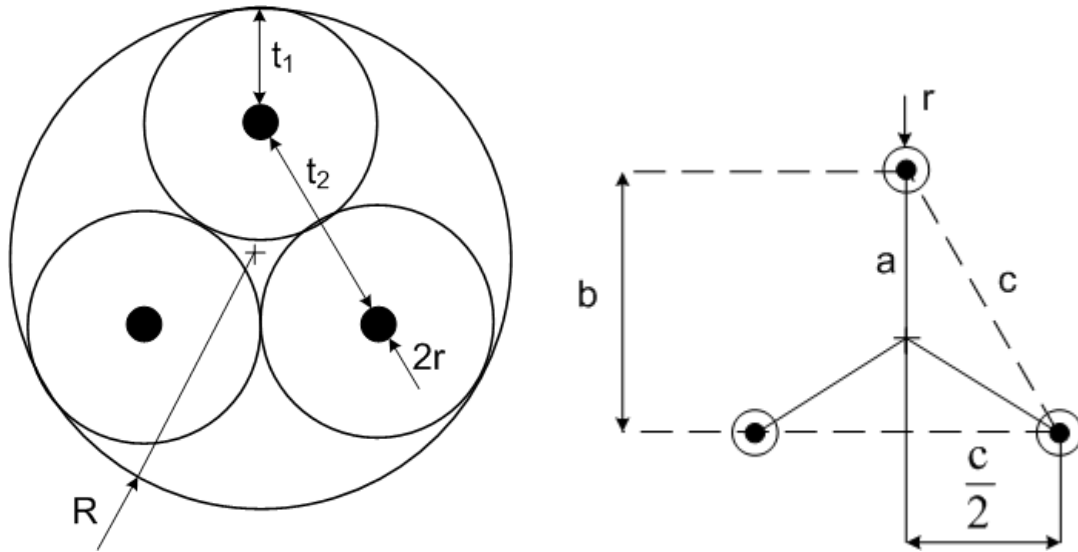
$$(\delta_{km})_{mod} = \begin{pmatrix} \delta - \delta_{kor} & \delta' - \delta_{kor} & \delta' - \delta_{kor} \\ \delta' - \delta_{kor} & \delta - \delta_{kor} & \delta' - \delta_{kor} \\ \delta' - \delta_{kor} & \delta' - \delta_{kor} & \delta - \delta_{kor} \end{pmatrix} = \begin{pmatrix} N & N' & N' \\ N' & N & N' \\ N' & N' & N \end{pmatrix}$$

$$k_0 = \frac{1}{N + N'} = \frac{1}{\delta - \delta_{kor} + 2(\delta' - \delta_{kor})} = \frac{1}{\delta + 2\delta' - 3\delta_{kor}} \quad \text{higher}$$

$$k' = \frac{N'}{(N - N')(N + 2N')} = \frac{\delta' - \delta_{kor}}{(\delta - \delta')(\delta + 2\delta' - 3\delta_{kor})} \quad \text{lower}$$

$$C = k_0 + 3k' = \frac{1}{N - N'} = \frac{1}{\delta - \delta'} \quad \text{equal}$$

Ex. 3: Calculate capacities for three-core cable with Aluminium screen. Conductors: Al 300 mm² ($r = 11,25$ mm). Conductor insulation thickness is $t_1 = 3,75$ mm, insulation thickness between conductors is $t_2 = 7,5$ mm and insulation permittivity is $\epsilon_r = 4,2$.



From figure:

$$c = t_2 + 2 \cdot r = 7,5 + 2 \cdot 11,25 = 30 \text{ mm}$$

$$b = \sqrt{c^2 - \left(\frac{c}{2}\right)^2} = \frac{c}{2} \cdot \sqrt{3} = \frac{30}{2} \cdot \sqrt{3} = 26 \text{ mm}$$

$$a = \frac{2}{3} \cdot b = \frac{2}{3} \cdot 26 = 17,32 \text{ mm}$$

$$R = a + r + t_1 = 17,32 + 11,25 + 3,75 = 32,32 \text{ mm}$$

Potential coefficients:

$$\delta = \frac{1}{0,0242 \cdot \epsilon_r} \cdot \log \frac{R^2 - a^2}{R \cdot r} = \frac{1}{0,0242 \cdot 4,2} \cdot \log \frac{32,32^2 - 17,32^2}{32,32 \cdot 11,25} = 3,063 \text{ km}/\mu\text{F}$$

$$\begin{aligned} \delta' &= \frac{1}{0,0242 \cdot \epsilon_r} \cdot \log \sqrt{\frac{1}{3} \cdot \left(1 + \frac{R^2}{a^2} + \frac{a^2}{R^2}\right)} = \\ &= \frac{1}{0,0242 \cdot 4,2} \cdot \log \sqrt{\frac{1}{3} \cdot \left(1 + \frac{32,32^2}{17,32^2} + \frac{17,32^2}{32,32^2}\right)} = 0,99 \text{ km}/\mu\text{F} \end{aligned}$$

Partial capacity to the cable screen:

$$k_0 = \frac{1}{\delta + 2 \cdot \delta'} = \frac{1}{3,063 + 2 \cdot 0,99} = 0,198 \mu\text{F}/\text{km}$$

Partial mutual capacity:

$$k = \frac{\delta'}{(\delta + 2 \cdot \delta') \cdot (\delta - \delta')} = \frac{0,99}{(3,063 + 2 \cdot 0,99) \cdot (3,063 - 0,99)} = 0,095 \mu\text{F/km}$$

Operational capacity:

$$C = \frac{1}{\delta - \delta'} = \frac{1}{3,063 - 0,99} = 0,483 \mu\text{F/km}$$

or

$$C = k_0 + 3 \cdot k' = 0,198 + 3 \cdot 0,095 = 0,483 \mu\text{F/km}$$