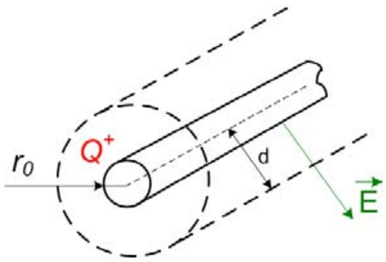
# Calculation of capacities

- voltage from cylindrical conductor




$$\oiint_S \vec{D}.d\vec{S} = Q$$

$\varepsilon.E.2\pi.l.r = Q$ (the entire conductor length)

$$E = \frac{Q_1}{2\pi.\varepsilon.r} \quad (V/m)$$

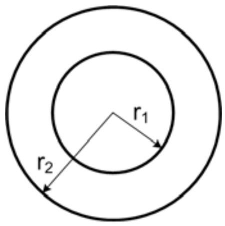
$$\vec{E} = -\text{grad}\ \varphi$$

$$U^+ = -\int_{r_0}^{d} \vec{E}.d\vec{l} = -\int_{r_0}^{d} \frac{Q_1}{2\pi\varepsilon.r} dr = -\frac{Q_1}{2\pi\varepsilon}\cdot[\ln r]_{r_0}^{d} = \frac{Q_1}{2\pi\varepsilon}\cdot \ln\frac{r_0}{d^+}$$

$$U^- = -\int_{r_0}^{d} \vec{E}.d\vec{l} = -\int_{r_0}^{d} \frac{-Q_1}{2\pi\varepsilon.r} dr = \frac{Q_1}{2\pi\varepsilon}\cdot[\ln r]_{r_0}^{d} = \frac{Q_1}{2\pi\varepsilon}\cdot \ln\frac{d^-}{r_0}$$

$$U = U^+ + U^- = \frac{Q_1}{2\pi\varepsilon}\cdot \ln\frac{d^-}{d^+} \quad (V)$$

- cable capacity (cylindrical capacitor):




$$Q = C.U$$

inside conductor $Q^+$, voltage form $r_1$ to $r_2$ $U = -U^+$

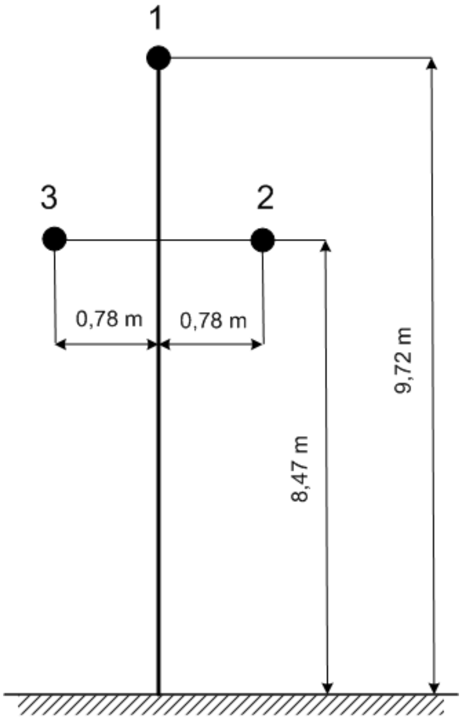
$$U = \frac{Q_1}{2\pi\varepsilon}\cdot \ln\frac{r_2}{r_1}$$

$$C = \frac{Q}{U} = \frac{2\pi\varepsilon}{\ln\frac{r_2}{r_1}} = \frac{2\pi.8{,}854.10^{-12}.\varepsilon_r}{\frac{\log\frac{r_2}{r_1}}{\log e}} = \frac{24{,}2.10^{-12}.\varepsilon_r}{\log\frac{r_2}{r_1}} \quad (F/m)$$

$$C = \frac{0{,}0242.\varepsilon_r}{\log\frac{r_2}{r_1}} \quad (\mu F/km)$$

# Power line parameters – Capacities calculation

**Ex. 1:** Three-phase transposed power line is in the figure. Phase conductors are made of AlFe 70 mm$^2$, with radius $r = 5{,}8$ mm. Calculate partial capacity to the ground of one conductor, partial mutual capacity and operational capacity (all capacities per 1 kilometre of overhead line).




Mean geometric height of phase conductors above the ground:

$$h = \sqrt[3]{h_1 \cdot h_2 \cdot h_3} = \sqrt[3]{9{,}72 \cdot 8{,}47 \cdot 8{,}47} = 8{,}86 \text{ m}$$

Mean geometric distance between phase conductors:

$$d = \sqrt[3]{d_{12} \cdot d_{13} \cdot d_{23}} =$$

$$= \sqrt[3]{\sqrt{0{,}78^2 + 1{,}25^2} \cdot \sqrt{0{,}78^2 + 1{,}25^2} \cdot (0{,}78 + 0{,}78)} = 1{,}5 \text{ m}$$

Potential coefficients:

$$\delta = \frac{1}{0{,}0242} \cdot \log \frac{2h}{r} = \frac{1}{0{,}0242} \cdot \log \frac{2 \cdot 8{,}86 \cdot 10^3}{5{,}8} = 144 \text{ km}/\mu\text{F}$$

$$\delta' = \frac{1}{0{,}0242} \cdot \log \frac{\sqrt{4 \cdot h^2 + d^2}}{d} = \frac{1}{0{,}0242} \cdot \log \frac{4 \cdot 8{,}86^2 + 1{,}5^2}{1{,}5} = 44{,}4 \text{ km}/\mu\text{F}$$

Partial capacity to the ground per 1 kilometre of the line length:

$$k_0 = \frac{1}{\delta + 2 \cdot \delta'} = \frac{1}{144 + 2 \cdot 44{,}4} = 0{,}00429 \ \mu\text{F}/\text{km}$$

Partial mutual capacity per 1 kilometre of the line length:

$$k = \frac{\delta'}{(\delta + 2 \cdot \delta') \cdot (\delta - \delta')} = \frac{44{,}4}{(144 + 2 \cdot 44{,}4) \cdot (144 - 44{,}4)} = 0{,}00191 \ \mu\text{F}/\text{km}$$

Operational capacity of one conductor per 1 kilometre of the line length:

$$C = \frac{1}{\delta - \delta'} = \frac{1}{144 - 44{,}4} = 10{,}02 \cdot 10^{-3} \ \mu\text{F}/\text{km}$$

or $C = k_0 + 3 \cdot k' = 0{,}00429 + 3 \cdot 0{,}00191 = 10{,}02 \cdot 10^{-3} \ \mu\text{F}/\text{km}$

$$\hat{C}_A = \frac{k_0 \cdot \hat{U}_A + k' \cdot \left(\hat{U}_A - \hat{U}_B\right) + k' \cdot \left(\hat{U}_A - \hat{U}_C\right)}{\hat{U}_A} = k_0 + k' \cdot \left(1 - \hat{a}^2\right) + k' \cdot \left(1 - \hat{a}\right) =$$

$$= k_0 + k' \cdot \left(2 - \hat{a}^2 - \hat{a}\right) = k_0 + 3k'$$

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$$\left(\delta_{km}\right) = \begin{pmatrix} \delta & \delta' & \delta' \\ \delta' & \delta & \delta' \\ \delta' & \delta' & \delta \end{pmatrix}$$

$$\left(c_{km}\right) = \left(\delta_{km}\right)^{-1} = \begin{pmatrix} c & c' & c' \\ c' & c & c' \\ c' & c' & c \end{pmatrix}$$

$$c = \frac{\delta + \delta'}{\delta^2 + \delta \cdot \delta' - 2\delta'^2} = \frac{\delta + \delta'}{\left(\delta - \delta'\right)\left(\delta + 2\delta'\right)}$$

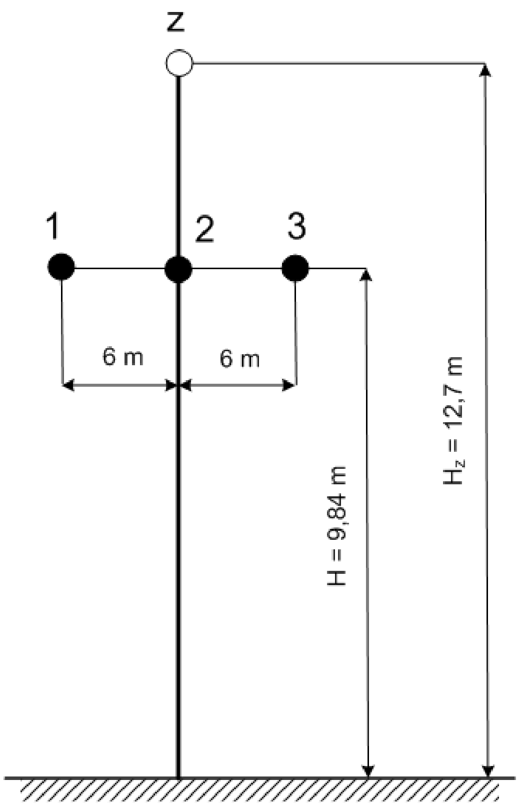
$$c' = \frac{-\delta'}{\delta^2 + \delta \cdot \delta' - 2\delta'^2} = \frac{-\delta'}{\left(\delta - \delta'\right)\left(\delta + 2\delta'\right)}$$

$$k_0 = c + 2c' = \frac{\delta + \delta' - 2\delta'}{\left(\delta - \delta'\right)\left(\delta + 2\delta'\right)} = \frac{1}{\delta + 2\delta'}$$

$$k' = -c' = \frac{\delta'}{\left(\delta - \delta'\right)\left(\delta + 2\delta'\right)}$$

$$C = k_0 + 3k' = \frac{1}{\delta + 2\delta'} + \frac{3\delta'}{\left(\delta - \delta'\right)\left(\delta + 2\delta'\right)} = \frac{1}{\delta - \delta'}$$

**Ex. 2:** Three-phase transposed line is in the figure. There are marked suspension heights of phase conductors and of ground wire.




Phase conductors (unbundle):

AlFe 150 $mm^2$ , $2 \cdot r = 24$ mm

sag: p = 1,2 m

Ground wire:

Fe 70 $mm^2$ , $2 \cdot r_z = 11$ mm

sag: $p_z$ = 1 m

Calculate partial capacity to the ground of one conductor, partial mutual capacity and operational capacity (all capacities per 1 kilometre of overhead line).

Calculation height (phase conductors):

$$h_1 = h_2 = h_3 = H - 0{,}7 \cdot p = 9{,}84 - 0{,}7 \cdot 1{,}2 = 9\,\text{m}$$

Calculation height (ground wire):

$$h_z = H_z - 0{,}7 \cdot p_z = 12{,}7 - 0{,}7 \cdot 1 = 12\,\text{m}$$

Mean geometric height of phase conductors above the ground:

$$h = \sqrt[3]{h_1 \cdot h_2 \cdot h_3} = \sqrt[3]{9^3} = 9\,\text{m}$$

Mean geometric distance between phase conductors:

$$d = \sqrt[3]{d_{12} \cdot d_{13} \cdot d_{23}} = \sqrt[3]{6 \cdot 12 \cdot 6} = 7{,}56\,\text{m}$$

Mean geometric distance between ground wire and phase conductors:

$$d_{vz} = \sqrt[3]{d_{1z} \cdot d_{2z} \cdot d_{3z}} = \sqrt[3]{\sqrt{6^2 + 3^2} \cdot 3 \cdot \sqrt{6^2 + 3^2}} = 5{,}13\,\text{m}$$

Potential coefficients (phase conductors only):

$$\delta = \frac{1}{0{,}0242}\cdot\log\frac{2\cdot h}{r} = \frac{1}{0{,}0242}\cdot\log\frac{2\cdot 9\cdot 10^3}{12} = 131{,}24\ \mathrm{km/\mu F}$$

$$\delta' = \frac{1}{0{,}0242}\cdot\log\frac{\sqrt{4\cdot h^2 + d^2}}{d} = \frac{1}{0{,}0242}\cdot\log\frac{\sqrt{4\cdot 9^2 + 7{,}56^2}}{7{,}56} = 17{,}02\ \mathrm{km/\mu F}$$

Potential coefficients (ground wire only):

$$\delta_{zz} = \frac{1}{0{,}0242}\cdot\log\frac{2\cdot h_z}{r_z} = \frac{1}{0{,}0242}\cdot\log\frac{2\cdot 12\cdot 10^3}{5{,}5} = 150{,}41\ \mathrm{km/\mu F}$$

Potential coefficients (phase conductors + ground wire):

$$\delta_{vz} = \frac{1}{0{,}0242}\cdot\log\frac{\sqrt{4\cdot h\cdot h_z + d_{vz}^2}}{d_{vz}} = \frac{1}{0{,}0242}\cdot\log\frac{\sqrt{4\cdot 9\cdot 12 + 5{,}13^2}}{5{,}13} = 25{,}64\ \mathrm{km/\mu F}$$

Correction potential coefficient:

$$\delta_k = \frac{\delta_{vz}^2}{\delta_{zz}} = \frac{25{,}64^2}{150{,}41} = 4{,}37\ \mathrm{km/\mu F}$$

Factors:

$$N = \delta - \delta_k = 131{,}24 - 4{,}37 = 126{,}87\ \mathrm{km/\mu F}$$

$$N' = \delta' - \delta_k = 17{,}02 - 4{,}37 = 12{,}65\ \mathrm{km/\mu F}$$

Partial capacity of one conductor to the ground:

$$k_0 = \frac{1}{N + 2\cdot N'} = \frac{1}{126{,}87 + 2\cdot 12{,}65} = 6{,}57\cdot 10^{-3}\ \mathrm{\mu F/km}$$

Partial mutual capacity:

$$k' = \frac{N'}{(N + 2\cdot N')\cdot(N - N')} = \frac{12{,}65}{(126{,}87 + 2\cdot 12{,}65)\cdot(126{,}87 - 12{,}65)} = 0{,}728\cdot 10^{-3}\ \mathrm{\mu F/km}$$

Operational capacity of one conductor per 1 kilometre of the line length:

$$C = \frac{1}{N - N'} = \frac{1}{126{,}87 - 12{,}65} = 8{,}755\cdot 10^{-3}\ \mathrm{\mu F/km}$$

or $C = k_0 + 3\cdot k' = (6{,}57 + 3\cdot 0{,}728)\cdot 10^{-3} = 8{,}755\cdot 10^{-3}\ \mathrm{\mu F/km}$

$$(\delta_{km}) = \begin{pmatrix} \delta & \delta' & \delta' & \delta_{vz} \\ \delta' & \delta & \delta' & \delta_{vz} \\ \delta' & \delta' & \delta & \delta_{vz} \\ \delta_{vz} & \delta_{vz} & \delta_{vz} & \delta_{zz} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$(\delta_{km})_{mod} = A - B \cdot D^{-1} \cdot C$$

$$(\delta_{kor}) = B \cdot D^{-1} \cdot C = \begin{pmatrix} \delta_{vz} \\ \delta_{vz} \\ \delta_{vz} \end{pmatrix} \left(\delta_{zz}^{-1}\right) \left(\delta_{vz} \quad \delta_{vz} \quad \delta_{vz}\right) = \begin{pmatrix} \frac{\delta_{vz}^2}{\delta_{zz}} & \frac{\delta_{vz}^2}{\delta_{zz}} & \frac{\delta_{vz}^2}{\delta_{zz}} \\ \frac{\delta_{vz}^2}{\delta_{zz}} & \frac{\delta_{vz}^2}{\delta_{zz}} & \frac{\delta_{vz}^2}{\delta_{zz}} \\ \frac{\delta_{vz}^2}{\delta_{zz}} & \frac{\delta_{vz}^2}{\delta_{zz}} & \frac{\delta_{vz}^2}{\delta_{zz}} \end{pmatrix} = \begin{pmatrix} \delta_{kor} & \delta_{kor} & \delta_{kor} \\ \delta_{kor} & \delta_{kor} & \delta_{kor} \\ \delta_{kor} & \delta_{kor} & \delta_{kor} \end{pmatrix}$$

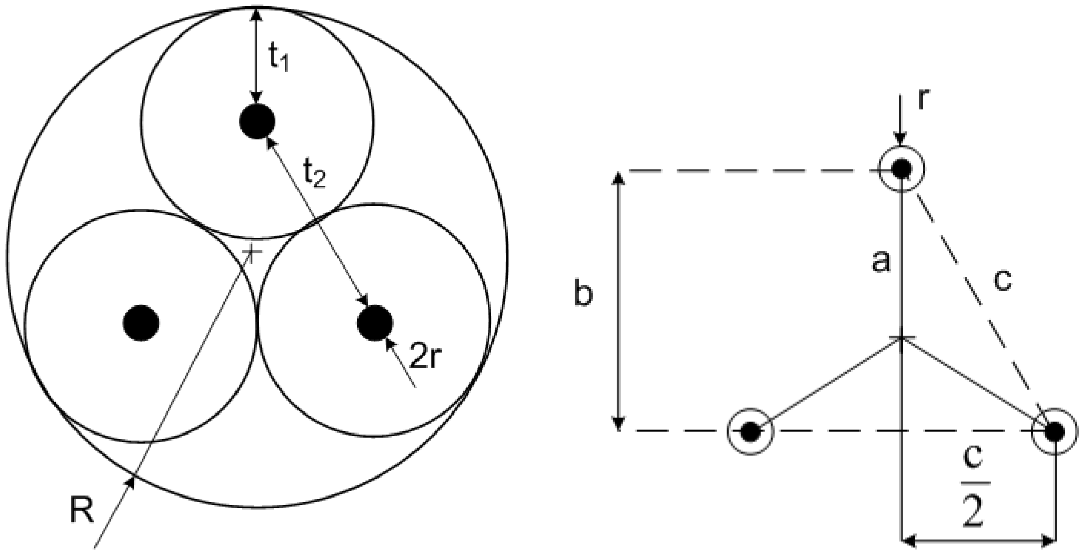
$$(\delta_{km})_{mod} = \begin{pmatrix} \delta - \delta_{kor} & \delta' - \delta_{kor} & \delta' - \delta_{kor} \\ \delta' - \delta_{kor} & \delta - \delta_{kor} & \delta' - \delta_{kor} \\ \delta' - \delta_{kor} & \delta' - \delta_{kor} & \delta - \delta_{kor} \end{pmatrix} = \begin{pmatrix} N & N' & N' \\ N' & N & N' \\ N' & N' & N \end{pmatrix}$$

$$k_0 = \frac{1}{N + N'} = \frac{1}{\delta - \delta_{kor} + 2(\delta' - \delta_{kor})} = \frac{1}{\delta + 2\delta' - 3\delta_{kor}} \qquad \text{higher}$$

$$k' = \frac{N'}{(N - N')(N + 2N')} = \frac{\delta' - \delta_{kor}}{(\delta - \delta')(\delta + 2\delta' - 3\delta_{kor})} \qquad \text{lower}$$

$$C = k_0 + 3k' = \frac{1}{N - N'} = \frac{1}{\delta - \delta'} \qquad \text{equal}$$

**Ex. 3:** Calculate capacities for three-core cable with Aluminium screen. Conductors: Al 300 $mm^2$ $(r = 11{,}25\,mm)$. Conductor insulation thickness is $t_1 = 3{,}75\,mm$, insulation thickness between conductors is $t_2 = 7{,}5\,mm$ and insulation permittivity is $\varepsilon_r = 4{,}2$.




From figure:

$$c = t_2 + 2 \cdot r = 7{,}5 + 2 \cdot 11{,}25 = 30\,mm$$

$$b = \sqrt{c^2 - \left(\frac{c}{2}\right)^2} = \frac{c}{2} \cdot \sqrt{3} = \frac{30}{2} \cdot \sqrt{3} = 26\,mm$$

$$a = \frac{2}{3} \cdot b = \frac{2}{3} \cdot 26 = 17{,}32\,mm$$

$$R = a + r + t_1 = 17{,}32 + 11{,}25 + 3{,}75 = 32{,}32\,mm$$

Potential coefficients:

$$\delta = \frac{1}{0{,}0242 \cdot \varepsilon_r} \cdot \log \frac{R^2 - a^2}{R \cdot r} = \frac{1}{0{,}0242 \cdot 4{,}2} \cdot \log \frac{32{,}32^2 - 17{,}32^2}{32{,}32 \cdot 11{,}25} = 3{,}063\,km/\mu F$$

$$\delta' = \frac{1}{0{,}0242 \cdot \varepsilon_r} \cdot \log \sqrt{\frac{1}{3} \cdot \left(1 + \frac{R^2}{a^2} + \frac{a^2}{R^2}\right)} =$$

$$= \frac{1}{0{,}0242 \cdot 4{,}2} \cdot \log \sqrt{\frac{1}{3} \cdot \left(1 + \frac{32{,}32^2}{17{,}32^2} + \frac{17{,}32^2}{32{,}32^2}\right)} = 0{,}99\,km/\mu F$$

Partial capacity to the cable screen:

$$k_0 = \frac{1}{\delta + 2 \cdot \delta'} = \frac{1}{3{,}063 + 2 \cdot 0{,}99} = 0{,}198\,\mu F/km$$

Partial mutual capacity:

$$k = \frac{\delta'}{(\delta + 2 \cdot \delta') \cdot (\delta - \delta')} = \frac{0{,}99}{(3{,}063 + 2 \cdot 0{,}99) \cdot (3{,}063 - 0{,}99)} = 0{,}095\ \mu F / km$$

Operational capacity:

$$C = \frac{1}{\delta - \delta'} = \frac{1}{3{,}063 - 0{,}99} = 0{,}483\ \mu F / km$$

or

$$C = k_0 + 3 \cdot k' = 0{,}198 + 3 \cdot 0{,}095 = 0{,}483\ \mu F / km$$