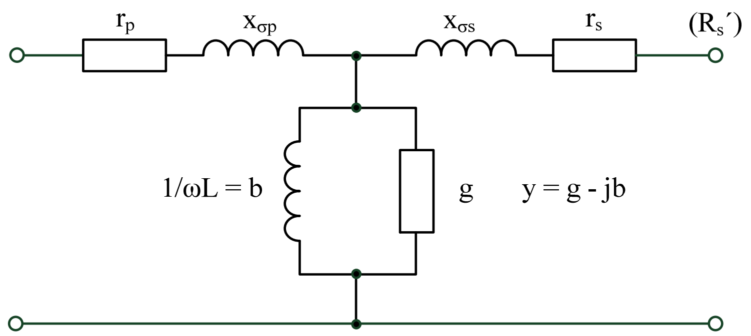
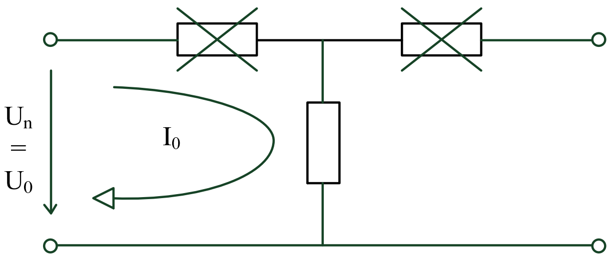


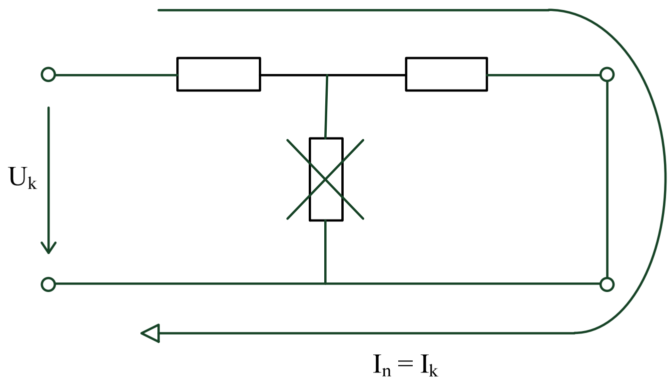
$$z = r + jx$$

$$\frac{1}{j\omega L} = -j\frac{1}{\omega L}$$

- No-load transformer:



- Short-circuit transformer:



$$\boxed{S_n, U_n, i_o, u_k, \Delta P_0, \Delta P_k}$$

$$S_n = \sqrt{3}\, U_n\, I_n$$

$$Z_n = \frac{U_n}{\sqrt{3} I_n} = \frac{U_n}{\frac{\sqrt{3} S_n}{\sqrt{3} U_n}} = \frac{U_n^2}{S_n}$$

$$R = \frac{U^2}{\Delta P} \rightarrow G = \frac{\Delta P_0}{U_n^2}$$

$$g = \frac{G}{Y_n} = \frac{\Delta P_0}{U_n^2}\frac{U_n^2}{S_n} = \frac{\Delta P_0}{S_n}$$

$$Y_q = \frac{I}{U} = \frac{i_0\, I_n}{\underbrace{u_0}_{1}\, \frac{U_n}{\sqrt{3}}} = i_0 \frac{1}{Z_n} = i_0 \frac{S_n}{U_n^2}$$

$$y_q = Y_q\, Z_n = i_0$$

$$b = \sqrt{y_q^2 - g^2}$$

$$B = b\, Y_n$$

$$R = \frac{\Delta P}{I^2} = \frac{\Delta P_K / 3}{I_n^2} = \frac{\Delta P_K\, U_n^2}{S_n^2}$$

$$r = \frac{R}{Z_n} = \frac{\Delta P_K}{S_n}$$

$$Z_K = \frac{U}{I} = \frac{\frac{u_K U_n}{\sqrt{3}}}{\underbrace{i_K}_{1}\, I_n} = u_K\, Z_n = u_K \frac{U_n^2}{S_n}$$

$$z_K = \frac{Z_K}{Z_n} = u_K$$

$$x_0 = \sqrt{z^2 - r^2}$$

$$X_0 = x_0\, Z_n$$

$$z = \frac{Z}{Z_n} = \frac{R + jX}{Z_n} = \frac{R}{Z_n} + j\frac{X}{Z_n} = r + jx$$

$$\left.\begin{aligned} Z_{K22} &= z_K \frac{\overbrace{U_n^2}^{22000}}{S_n} \\ Z_{K0,4} &= z_K \frac{\overbrace{U_n^2}^{400}}{S_n} \end{aligned}\right\} \underline{Z_{K0,4} = Z_{K22}\left(\frac{0,4}{22}\right)^2}$$

Example 1

Calculate parameters of two-port network for a two-winding transformer with these technical data: Nominal power S_{nom} = 1600 kVA; nominal voltage 22 kV, 400/231 V; connection of phase windings D/yn1; short-circuit voltage $u_{k\%}$ = 6 %; no-load current $I_{0\%}$ = 0,7 %; no-load losses ΔP_0 = 2,41 kW; short-circuit losses ΔP_k = 16 kW. Calculate:
a) parameters in relative values (related to nominal power and nominal voltage)
b) parameters in denominated values (related to one nominal voltage).

Solution:

a) Calculation in relative values related to the base power $S_{nom} = 1{,}6 \cdot 10^6\ VA$, voltage is not included in the equations:

Relative shunt conductance:

$$g_q = \frac{\Delta P_0}{S_{nom}} = \frac{2410}{1{,}6 \cdot 10^6} = 0{,}0015$$

Relative shunt admittance:

$$y_q = \frac{I_{0\%}}{100} = \frac{0{,}7}{100} = 0{,}007$$

Relative shunt susceptance:

$$b_q = \sqrt{y_q^2 - g_q^2} = \sqrt{0{,}007^2 - 0{,}0015^2} = 0{,}006836$$

Relative shunt admittance:

$$\hat{y}_q = g_q - jb_q = 0{,}0015 - j0{,}006836 = 0{,}007 \cdot e^{-j1{,}354}$$

Relative resistance of both windings:

$$r_k = \frac{\Delta P_k}{S_{nom}} = \frac{16000}{1{,}6 \cdot 10^6} = 0{,}01$$

Relative series impedance (short-circuit):

$$z_k = \frac{u_{k\%}}{100} = \frac{6}{100} 0{,}06$$

Relative leakage reactance of both windings:

$$x_k = \sqrt{z_k^2 - r_k^2} = \sqrt{0{,}06^2 - 0{,}01^2} = 0{,}05916$$

Series impedance (short-circuit):

$$\hat{z}_k = r_k + jx_k = 0{,}01 + j0{,}05916 = 0{,}06 \cdot e^{j1{,}031}$$

Hence relative series admittance:

$$\hat{y}_k = \hat{z}_k^{-1} = 0{,}06^{-1} \cdot e^{-j1{,}403} = 16{,}6 \cdot e^{-j1{,}403} = 2{,}77 - j16{,}4336$$

Substitute two-port network with series impedance is in Fig. 1 and with shunt admittance in Fig. 2. T-network has one half of the series impedance in the primary circuit and the secondary circuit which is not a real situation precisely (Fig. 3); π-network has one half of the shunt admittance in the primary circuit and the secondary circuit and the longitudinal branch includes the series admittance (Fig. 4).

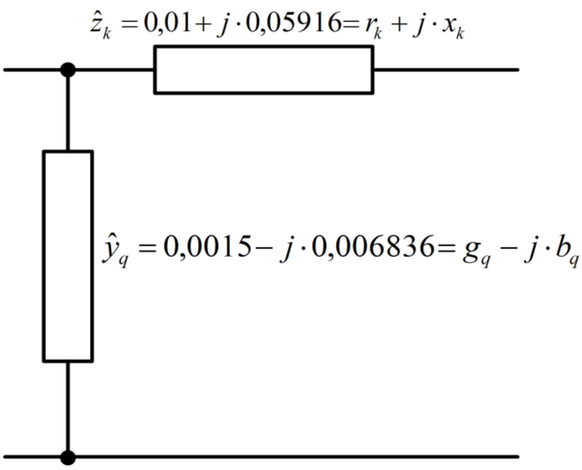


Fig. 1 - Substitute two-port gamma-network with series impedance

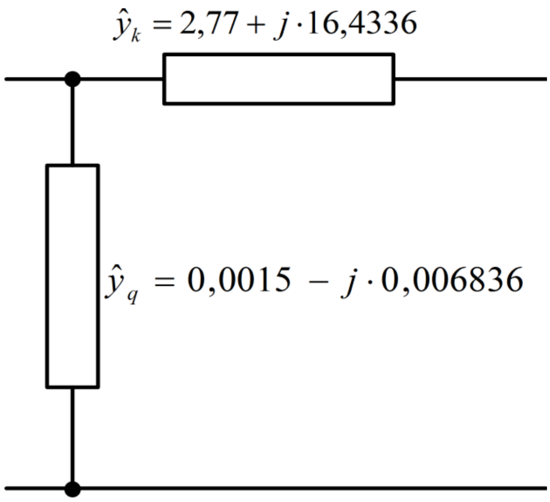


Fig. 2 - Substitute two-port gamma-network with shunt admittance

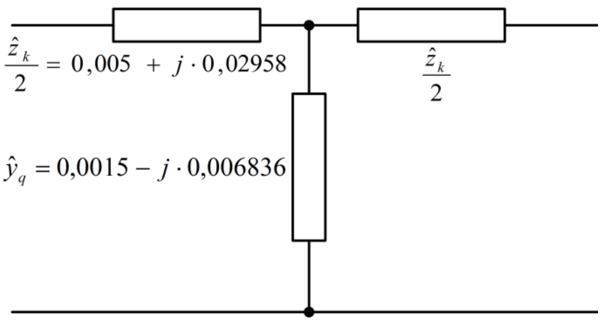


Fig. 3 – Substitute T-network

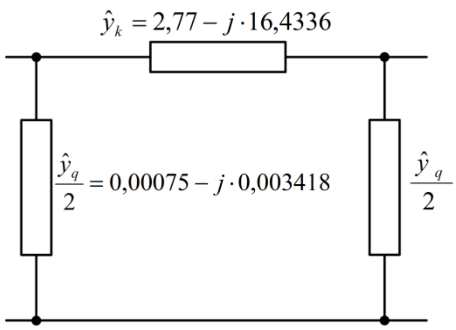


Fig. 4 – Substitute π-network

b) Calculation in denominated values related to the rated voltage 22 kV

Shunt admittance:

$$\hat{Y}_q = \hat{y}_q \frac{S_{nom}}{U_{nom}^2} = (0{,}0015 - j0{,}006836) \cdot \frac{1{,}6 \cdot 10^6}{(22 \cdot 10^3)^2} = (4{,}9793 - j22{,}5984) \cdot 10^{-6}\ S$$
$$= G_q - jB_q = (23{,}1405 \cdot 10^{-6}) \cdot e^{-j1354}\ S$$

Series impedance (short-circuit):

$$\hat{Z}_k = \hat{z}_k \frac{U_{nom}^2}{S_{nom}} = (0{,}01 + j0{,}05916) \cdot \frac{(22 \cdot 10^3)^2}{1{,}6 \cdot 10^6} = 3{,}025 + j17{,}8961 = R_k + jX_k$$
$$= 18{,}15 \cdot e^{j1{,}403}\ \Omega$$

and the corresponding series admittance:

$$\hat{Y}_k = \hat{Z}_k^{-1} = 0{,}0551 \cdot e^{-j1{,}403}\ S$$

These values enable to determine the substitute two-port networks gamma, T, π as in the previous case with relative values.

c) Calculation in denominated values related to the rated voltage 22 kV. Analogous as for b):

$$\hat{Y}_q = \hat{y}_q \frac{S_{nom}}{U_{nom}^2} = (0{,}0015 - j0{,}006836) \cdot \frac{1{,}6 \cdot 10^6}{400^2} = (0{,}0151 - j0{,}06836)$$
$$= 0{,}07 \cdot e^{-j1{,}354}\ S$$

$$\hat{Z}_k = \hat{z}_k \frac{U_{nom}^2}{S_{nom}} = (0{,}01 + j0{,}05916) \cdot \frac{400^2}{1{,}6 \cdot 10^6} = (0{,}001 + j0{,}005916) = 0{,}006 \cdot e^{j1{,}403}\ \Omega$$

$$\hat{Y}_k = \hat{Z}_k^{-1} = 166{,}6 \cdot e^{-j1{,}403}\ S$$

Example 2

Calculate the no-load reactive power consumption for the three phase transformer with the same parameters as in Ex.1. The machine is connected to 22 kV.

Solution:

a) we start with the two-port gamma-network in denominated values related to nominal primary voltage 22 kV. (Fig. 5)

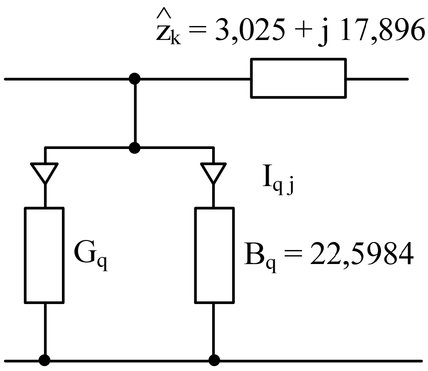


Fig. 5

No-load reactive power:

$$-j\,Q_{q0} = 3\,\hat{U}_f^*\,\hat{I}_{qj} = 3\,\hat{U}_f^*\,\hat{U}_f\left(-j\,B_q\right) = -j\,B_q\,U^2 = -j\,(22\,.\,10^3)^2\,.\,22{,}5984\,.\,10^{-6}$$
$$= -j\ 10937{,}6\ [VAr] = -j\ 10{,}9376\ [kVAr]$$

b) If I_o is the no-load current, no-load apparent power is:

$$S_{10} = 3\,U_{f\,nom}\,I_0 = 3\,U_{f\,nom}\,I_{nom}\,\frac{I_0}{I_{nom}}\frac{100}{100} = S_{nom}\,\frac{I_{0\%}}{100} = 1600\,\frac{0{,}7}{100} = 11{,}2\ [kVA]$$

because $I_{0\%} = I_0\,.\,100/I_{nom}$. Since no-load active power losses are $\Delta P = 2{,}41\ kW$, no-load reactive power is:

$$Q_{q0} = \sqrt{S_{10}^2 - \Delta P_0^2} = \sqrt{11{,}2^2 - 2{,}41^2} = 10{,}9376\ [kVAr]$$

It can be shown this is the smallest reactive power consumed by the transformer during its operation. Only this power can be compensated by any individual compensating device (parallel capacitor) connected usually to the lower voltage side.

Both calculation methods are equivalent. The first one is used if we know the transformer substitute two-port network. The second one uses machine technical data directly. If we would like to know an approximate value, considering $S_{10} \doteq Q_{q0}$ is sufficient. Here 2,3 % error is made. If we want to install a compensating capacitor, we must choose the closest lower value, i.e. 10 kVAr.