Calculation of voltage in a small electrical network

Electrical DC network is shown (Fig. 1). Calculate the voltage in the nodes 2, 3 and 4. Calculate the node current in the balance node 1. The loads in the nodes, the balance node voltage and resistances of each branch are shown in Fig. 1.

Calculate it: 1) with node voltage method NVM

2) with gradual simplification method

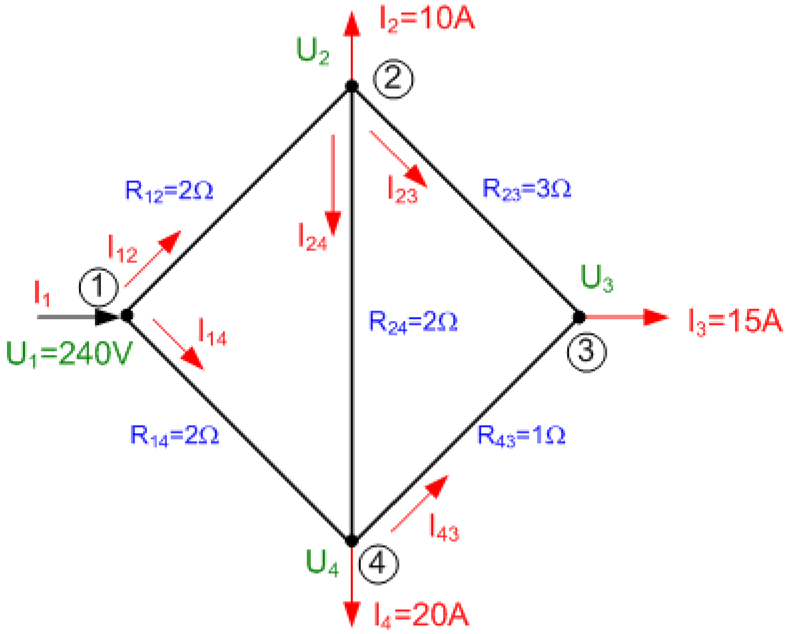

Fig. 1

1. Node voltage method (NVM)

First admittance matrix elements can be calculated based on the knowledge of each element parameters and the system topology. The principle is following:

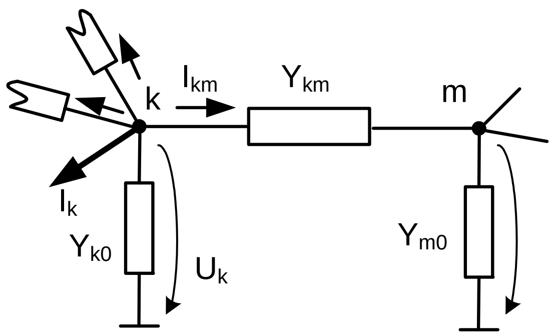

Fig. 2: Model of a system branch

Node current in node *k*:

$$\hat{I}_k + \sum_{m \neq k} \hat{I}_{km} = 0 \tag{1}$$

$$\hat{I}_k = -\sum_{m \neq k} \hat{I}_{km} = -\sum_{m \neq k} \hat{Y}_{km} \cdot \left(\hat{U}_k - \hat{U}_m \right) = -\sum_{m \neq k} \hat{Y}_{km} \cdot \hat{U}_k + \sum_{m \neq k} \hat{Y}_{km} \cdot \hat{U}_m \qquad (2)$$

Previous equations are matrix multiplications where $\hat{Y}_{(k,k)}$ is nodal self-admittance (diagonal element) and $\hat{Y}_{(k,m)}$ is between nodes admittance (non-diagonal elements).

$$\hat{I}_k = -\hat{U}_k \cdot \hat{Y}_{(k,k)} + \sum_{m \neq k} \hat{U}_m \cdot \hat{Y}_{(k,m)} \qquad (3)$$

where $\hat{Y}_{(k,k)}$, $\hat{Y}_{(k,m)}$ are the system admittance matrix elements.
Diagonal elements are negative and non-diagonal elements are positive.
Because of 4 nodes – admittance matrix type is 4 x 4.

The first element Y_{11} is calculated as the negative sum of all admittances connected to the node 1.

$$Y_{11} = -(Y_{12} + Y_{14}) = -\left(\frac{1}{2} + \frac{1}{2}\right) = -1 \qquad (4)$$

The second element Y_{12} is the same as Y_{21} and it is calculated as the sum of admittances between the nodes 1 and 2:

$$Y_{12} = Y_{21} = \frac{1}{2} \qquad (5)$$

The next element Y_{13} is the same as Y_{31} and since the nodes 1 and 3 are not connected, the admittance is equal to 0. And further:

$$Y_{22} = -(Y_{12} + Y_{14} + Y_{24}) = -\frac{4}{3} \qquad (6)$$

$$Y_{33} = -(Y_{23} + Y_{34}) = -\frac{4}{3} \qquad (7)$$

$$Y_{44} = -(Y_{14} + Y_{24} + Y_{34}) = -2 \qquad (8)$$

$$Y_{14} = Y_{14} = \frac{1}{2} \qquad (9)$$

$$Y_{23} = Y_{32} = \frac{1}{3} \qquad (10)$$

$$Y_{24} = Y_{42} = \frac{1}{2} \qquad (11)$$

$$Y_{34} = Y_{43} = 1 \qquad (12)$$

Admittance matrix Y looks like:

$$[Y]=\begin{pmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{pmatrix}=\begin{pmatrix} -1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & -\frac{4}{3} & \frac{1}{3} & \frac{1}{2} \\ 0 & \frac{1}{3} & -\frac{4}{3} & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 & -2 \end{pmatrix} \tag{13}$$

First we do the general derivation for the specified network. The matrix equation is: (current is equal to product of admittance and voltage)

$$[\mathrm{I}]=[\mathrm{Y}]\cdot[\mathrm{U}] \tag{14}$$

To calculate voltage it is necessary to multiply currents by the inverse matrix $[\mathrm{Y}]^{-1}$ from the left. This is not possible because [Y] is a singular matrix (det Y = 0). Therefore the matrix and column vectors are divided to sub-matrixes for known and unknown quantities:

$$\overset{[\mathrm{I}_1]}{\underset{[\mathrm{I}_{odb}]}{\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix}}} = \overset{[\mathrm{Y}_1]\qquad\qquad [\mathrm{Y}_2]}{\underset{[\mathrm{Y}_3]\qquad\qquad [\mathrm{Y}_4]}{\left(\begin{array}{c|ccc} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ \hline Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{array}\right)}} \cdot \overset{[\mathrm{U}_1]}{\underset{[\mathrm{U}_{odb}]}{\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix}}} \tag{15}$$

$$\begin{bmatrix} [\mathrm{I}_1] \\ [\mathrm{I}_{odb}] \end{bmatrix} = \begin{bmatrix} [\mathrm{Y}_1] & [\mathrm{Y}_2] \\ [\mathrm{Y}_3] & [\mathrm{Y}_4] \end{bmatrix} \cdot \begin{bmatrix} [\mathrm{U}_1] \\ [\mathrm{U}_{odb}] \end{bmatrix} \tag{16}$$

where:

$[\mathrm{I}_1]$ .... unknown current value in the node 1

$[\mathrm{U}_1]$ .... known voltage value in the node 1

$[\mathrm{I}_{odb}]$ .... known load current values in calculated nodes

$[\mathrm{U}_{odb}]$........... unknown voltage values in calculated nodes

Dimensions of matrixes:

$[\mathrm{I}_1] \in R^{1x1}$, $[\mathrm{U}_1] \in R^{1x1}$, $[\mathrm{I}_{odb}] \in R^{3x1}$, $[\mathrm{U}_{odb}] \in R^{3x1}$

$[\mathrm{Y}_1] \in R^{1x1}$, $[\mathrm{Y}_2] \in R^{1x3}$, $[\mathrm{Y}_3] \in R^{3x1}$, $[\mathrm{Y}_4] \in R^{3x3}$

After few modifications and particular multiplications:

$$\mathbf{[I_1] = [Y_1].[U_1] + [Y_2].[U_{odb}]} \tag{17}$$

$$[I_{odb}] = [Y_3].[U_1] + [Y_4].[U_{odb}] \qquad /.[Y_4]^{-1} \tag{18}$$

$$[Y_4]^{-1}.[I_{odb}] = [Y_4]^{-1}.[Y_3].[U_1] + [U_{odb}] \tag{19}$$

$$\mathbf{[U_{odb}] = [Y_4]^{-1}.[I_{odb}] - [Y_4]^{-1}.[Y_3].[U_1]} \tag{20}$$

First there are calculated unknown voltage values from matrix equation (20) and then there is calculated the current value from matrix equation (17)

The final voltages in nodes 2, 3 and 4:

$$[U_{odb}] = \begin{bmatrix} U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} 199{,}375 \\ 181{,}563 \\ 190{,}625 \end{bmatrix} \ (V) \tag{21}$$

The current I_1 is (based on equation 17):

$$I_1 = -\,45 \text{ A} \tag{22}$$

A negative value is for delivery (if all node currents have the same orientation). Currents between individual nodes are:

$$I_{12} = (U_1 - U_2).Y_{12} = 20{,}31 \text{ A} \tag{23}$$

$$I_{14} = (U_1 - U_4).Y_{14} = 24{,}69 \text{ A} \tag{24}$$

$$I_{24} = (U_2 - U_4).Y_{24} = 4{,}38 \text{ A} \tag{25}$$

$$I_{23} = (U_2 - U_3).Y_{23} = 5{,}94 \text{ A} \tag{26}$$

$$I_{43} = (U_4 - U_3).Y_{34} = 9{,}06 \text{ A} \tag{27}$$

2. Gradual simplification method

For further simplifications it is needed to remove Δ between the nodes 1, 2 and 4 (by transfiguration D → Y):

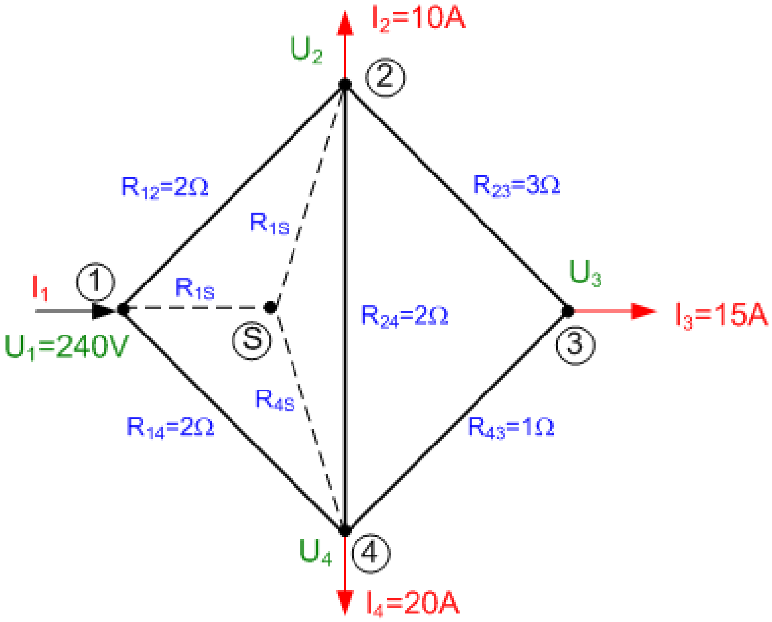

Fig. 3: Power system with transfiguration

The resistances recalculations:

$$R_{1S} = \frac{R_{12}.R_{14}}{R_{12} + R_{14} + R_{24}} = \frac{2}{3}\Omega \tag{28}$$

$$R_{2S} = \frac{R_{12}.R_{24}}{R_{12} + R_{14} + R_{24}} = \frac{2}{3}\Omega \tag{29}$$

$$R_{4S} = \frac{R_{14}.R_{24}}{R_{12} + R_{14} + R_{24}} = \frac{2}{3}\Omega \tag{30}$$

The power network can be redrawn:

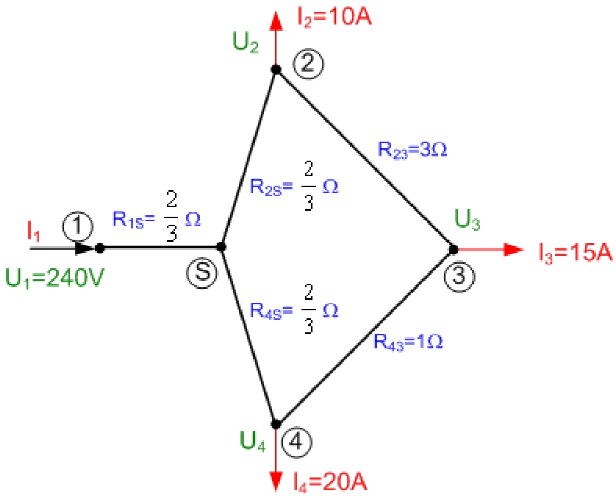

Fig. 4: Redrawn power network

The current reduction from nodes 2 and 4 is made further. This is done by distribution of loads to neighbouring nodes based on current divider principle. Currents I_2 and I_4 are distributed to the nodes S and 3 as follows:

$$I_{23} = \frac{R_{2S}}{R_{2S} + R_{23}} \cdot I_2 = \frac{20}{11} \text{ A} \tag{31}$$

$$I_{2S} = \frac{R_{23}}{R_{2S} + R_{23}} \cdot I_2 = \frac{90}{11} \text{ A} \tag{32}$$

$$I_{43} = \frac{R_{4S}}{R_{4S} + R_{34}} \cdot I_4 = 8 \text{ A} \tag{33}$$

$$I_{4S} = \frac{R_{34}}{R_{4S} + R_{34}} \cdot I_4 = 12 \text{ A} \tag{34}$$

The resulting nodal currents in the nodes S and 3 are:

$$I_S = I_{2S} + I_{4S} = \frac{222}{11} \text{ A} \tag{35}$$

$$I`_3 = I_{23} + I_{43} + I_3 = \frac{273}{11} \text{ A} \tag{35}$$

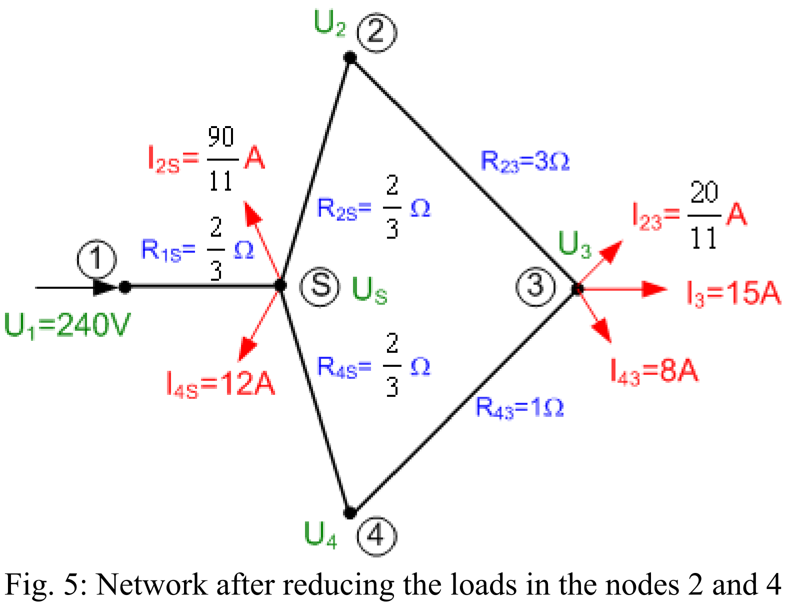

Fig. 5: Network after reducing the loads in the nodes 2 and 4

Simplifying of parts S – 2 – 3 and S – 4 – 3 (this parts are serial summed) and then a parallel combination:

$$R_{S3a} = R_{2S} + R_{23} \text{ and } R_{S3b} = R_{4S} + R_{43} \tag{36}$$

$$R_{S3} = \frac{R_{S3a}.R_{S3b}}{R_{S3a} + R_{S3b}} = \frac{55}{48}\ \Omega \tag{37}$$

Current I_1:

$$I_1 = I_2 + I_3 + I_4 = 10 + 15 + 20 = 45 \text{ A} \tag{38}$$

The whole network is simplified now as a radial network supplied from one side (Fig.6). The voltages in the nodes S and 3 are:

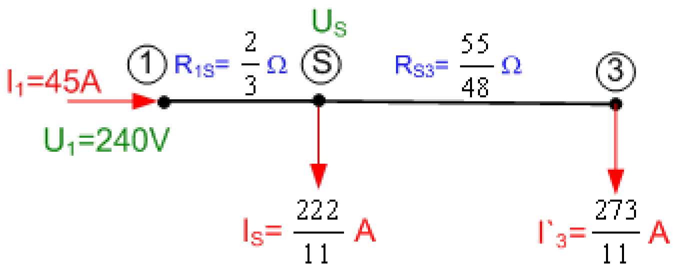

Fig. 6: Radial network supplied from one side

$$U_S = U_1 - I_1.R_{1S} = 240 - 45\cdot\frac{2}{3} = 210 \text{ V} \tag{39}$$

$$U_3 = U_S - I_{S3}.R_{S3} = 210 - \frac{273}{11}\cdot\frac{55}{48} = 181{,}56 \text{ V} \tag{39}$$

The voltages in the nodes 2, 3 and 4 are calculated now. The calculation is made for the same voltages of both feeders U_S (see Fig.4). First we calculate the currents I_{S1} and I_{S2} by means of the current moments and then voltages are calculated based on these currents.

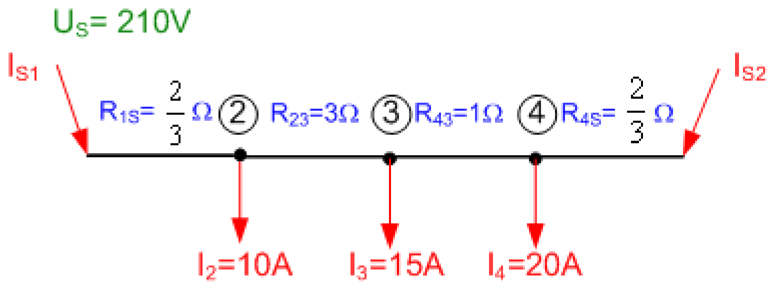

Fig. 7: The power network supplied from both sides

$$I_{S1} = \frac{R_{4S}.I_4 + (R_{4S} + R_{43}).I_3 + (R_{4S} + R_{43} + R_{23}).I_2}{R_{4S} + R_{43} + R_{23} + R_{1S}} = 15{,}94 \text{ A} \tag{40}$$

$$I_{S2} = \frac{R_{1S}.I_2 + (R_{1S} + R_{23}).I_3 + (R_{1S} + R_{23} + R_{43}).I_4}{R_{4S} + R_{43} + R_{23} + R_{1S}} = 29{,}06 \text{ A} \tag{41}$$

$$U_2 = U_S - I_{S1}.R_{1S} = 210 - 15{,}94\cdot\frac{2}{3} = 199{,}37 \text{ V} \tag{42}$$

$$U_4 = U_S - I_{S2}.R_{4S} = 210 - 29{,}06\cdot\frac{2}{3} = 190{,}63 \text{ V} \tag{42}$$