Transmission power lines

A substitute diagram of single phase power line with equally distributed parameters is in Fig. 1. The power line has length *l* (R_1 is longitudinal resistance, X_1 longitudinal reactance, G_1 cross conductance and B_1 cross susceptance, all is per kilometre length (for each phase)).

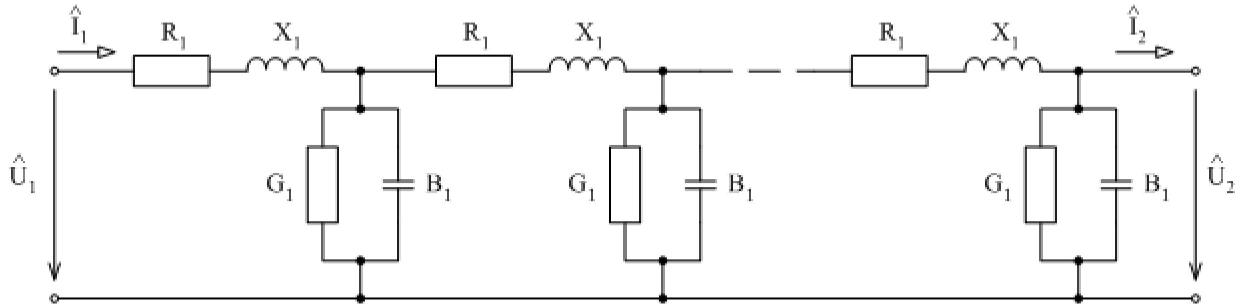

Fig. 1: The substitute diagram of single phase power line with equally distributed parameters

The symbols $\hat{U}_1$ and $\hat{I}_1$ are voltage and current phasors at the beginning of power line, $\hat{U}_2$ and $\hat{I}_2$ are phasors at the end of power line. The system frequency is *f*. The basic equations expressing the relationship between the quantities at the beginning and end of the power line have the formula:

$$\begin{pmatrix} \hat{U}_1 \\ \hat{I}_1 \end{pmatrix} = \begin{pmatrix} \cosh \hat{\gamma} l & \hat{Z}_v \sinh \hat{\gamma} l \\ \frac{1}{\hat{Z}_v} \sinh \hat{\gamma} l & \cosh \hat{\gamma} l \end{pmatrix} \cdot \begin{pmatrix} \hat{U}_2 \\ \hat{I}_2 \end{pmatrix} \qquad (1)$$

where $\hat{Z}_v$ is surge impedance

$$\hat{Z}_v = \sqrt{\frac{R_1 + j \cdot X_1}{G_1 + j \cdot B_1}} \quad (\Omega) \qquad (2)$$

and $\hat{\gamma}$ is propagation constant

$$\hat{\gamma} = \sqrt{(R_1 + j \cdot X_1) \cdot (G_1 + j \cdot B_1)} \quad (m^{-1}) \qquad (3)$$

Propagation constant $\hat{\gamma}$ has a complex character and it can be written as $\hat{\gamma} = \alpha + j\beta$ where the real part α is the specific damping and imaginary part β is the specific phase shift.

The longitudinal impedance $\hat{Z}_l$ and cross admittance $\hat{Y}_q$ per length *l* are:

$$\hat{Z}_l = (R_1 + j \cdot X_1) \cdot l \quad (\Omega) \qquad (4)$$

$$\hat{Y}_q = (G_1 + j \cdot B_1) \cdot l \quad (S) \qquad (5)$$

The figure 1 can be redrawn as shown (Fig. 2). It is two-port π-network.

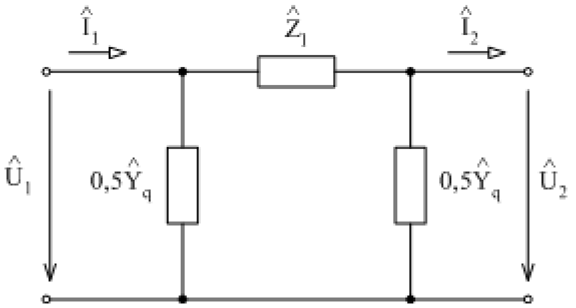

Fig. 2: Substitute π-network of long power line

The basic equation for the voltage and current at the beginning and end of power line:

$$\begin{pmatrix}\hat{U}_1\\ \hat{I}_1\end{pmatrix}=\begin{pmatrix}1+\dfrac{\hat{Z}_l\cdot\hat{Y}_q}{2} & \hat{Z}_l\\ \hat{Y}_q+\dfrac{\hat{Z}_l\cdot\hat{Y}_q^2}{4} & 1+\dfrac{\hat{Z}_l\cdot\hat{Y}_q}{2}\end{pmatrix}\begin{pmatrix}\hat{U}_2\\ \hat{I}_2\end{pmatrix}=\begin{pmatrix}\hat{A} & \hat{B}\\ \hat{C} & \hat{D}\end{pmatrix}\cdot\begin{pmatrix}\hat{U}_2\\ \hat{I}_2\end{pmatrix} \tag{6}$$

Example 1

Simple three phase power line with resistance R_1 = 0,0715 Ω/km, inductance X_1 = 0,426 Ω/km, conductance G_1 = 0 S/km, susceptance B_1 = 2,635 μS/km, the length of power line is l = 400 km.

Calculate the phase voltage and current at the beginning for power line open end. Calculate the charging power of power line. The voltage at the end is U_2 = 220 kV.

Longitudinal impedance:

$$\hat{Z}_{l1}=R_1+j\cdot X_1=0{,}0715+j0{,}426=0{,}432\cdot e^{j80{,}47^\circ}\ \Omega/\text{km}$$

Cross admittance:

$$\hat{Y}_{q1}=G_1+j\cdot B_1=0+j2{,}635\cdot10^{-6}=2{,}635\cdot10^{-6}\cdot e^{j90^\circ}\ \text{S/km}$$

Surge impedance:

$$\hat{Z}_v=\sqrt{\frac{\hat{Z}_{l1}}{\hat{Y}_{q1}}}=404{,}9\cdot e^{-j4{,}764^\circ}\ \Omega=403{,}5-j33{,}63\ \Omega$$

Propagation constant:

$$\hat{\gamma}=\sqrt{\hat{Z}_{l1}\cdot\hat{Y}_{q1}}=1{,}067\cdot10^{-3}\cdot e^{j85{,}24^\circ}\ \text{km}^{-1}=(0{,}0889+j1{,}0632)\cdot10^{-3}\ \text{km}^{-1}$$

Auxiliary mathematical derivation:

$$\sinh(\alpha + j\beta) = \frac{1}{2}\left(e^{\alpha + j\beta} - e^{-(\alpha + j\beta)}\right) = \frac{1}{2}\left[e^{\alpha}(\cos\beta + j\sin\beta) - e^{-\alpha}(\cos\beta - j\sin\beta)\right] =$$
$$= \frac{1}{2}\cos\beta\left(e^{\alpha} - e^{-\alpha}\right) + \frac{1}{2}j\sin\beta\left(e^{\alpha} + e^{-\alpha}\right) = \sinh\alpha \cdot \cos\beta + j\cosh\alpha \cdot \sin\beta$$

Similarly:

$$\cosh(\alpha + j\beta) = \frac{1}{2}\left(e^{\alpha + j\beta} + e^{-(\alpha + j\beta)}\right) = \frac{1}{2}\left[e^{\alpha}(\cos\beta + j\sin\beta) + e^{-\alpha}(\cos\beta - j\sin\beta)\right] =$$
$$= \frac{1}{2}\cos\beta\left(e^{\alpha} + e^{-\alpha}\right) + \frac{1}{2}j\sin\beta\left(e^{\alpha} - e^{-\alpha}\right) = \cosh\alpha \cdot \cos\beta + j\sinh\alpha \cdot \sin\beta$$

The phase-to-ground voltage at the end of power line:

$$\hat{U}_{f2} = U_{f2} = \frac{220}{\sqrt{3}} \doteq 127 \text{ kV}$$

The voltage phasor at the beginning of power line with open end ($I_2 = 0$):

$$\hat{U}_{f10} = \hat{U}_{f2}\cosh\hat{\gamma}l = 127 \cdot 0{,}9116 \cdot e^{j0{,}92^{\circ}} = 115{,}77 \cdot e^{j0{,}92^{\circ}} \text{ kV} = U_{f10} \cdot e^{j\vartheta_0}$$

The Ferranti effect is evident on power line, i.e. $U_{f10} < U_{f2}$!

The current phasor at the beginning of power line with open end:

$$\hat{I}_{10} = \frac{\hat{U}_{f2}}{\hat{Z}_v}\sinh\hat{\gamma}l = \frac{127}{404{,}9 \cdot e^{-j4{,}764^{\circ}}} \cdot 0{,}4129 \cdot e^{j85{,}53^{\circ}} = 129{,}51 \cdot e^{j90{,}29^{\circ}} \text{ A} = I_{10} \cdot e^{j\delta}$$

The phase shift between voltage and current at the beginning of power line:

$$\varphi_{10} = \delta - \vartheta_0 = 90{,}29^{\circ} - 0{,}92^{\circ} = 89{,}37^{\circ}$$

The power line with open end is almost as capacity load.

Three phase charging power:

$$\hat{S}_{10} = 3\hat{U}_{f10}\hat{I}^{*}_{10} = 3 \cdot 115{,}77 \cdot 10^{3} \cdot e^{j0{,}92^{\circ}} \cdot 129{,}51 \cdot e^{-j90{,}29^{\circ}} = 44{,}6 \cdot e^{-j89{,}37^{\circ}} \text{ MVA}$$

$$P_{10} = 0{,}49 \text{ MW}, \quad Q_{10} = 44{,}6 \text{ MVAr capacitive}$$

Example 2

Three phase transposed power line with nominal voltage 220 kV has length 400 km and parameters $R_1 = 0{,}0715\ \Omega/km$, $X_1 = 0{,}426\ \Omega/km$, $G_1 = 0$ S/km, $B_1 = 2{,}635\ \mu S/km$.

Calculate the phase-to-ground voltage and current at the beginning when at the end is loaded with $P_2 = 125$ MW and $\cos\varphi = 1$, voltage at the end is $U_2 = 220$ kV. Solve the problem with π-network and symmetrical parameters.

Preliminary calculations:

$$\hat{U}_{f2} = U_{f2} = \frac{220}{\sqrt{3}} \doteq 127\ \text{kV}$$

$$\hat{Z}_l = (R_1 + j \cdot X_1) \cdot l = (0{,}0715 + j0{,}426) \cdot 400 = 28{,}6 + j170{,}4\ \Omega$$

$$\frac{\hat{Y}_q}{2} = \frac{1}{2}(G_1 + j \cdot B_1) \cdot l = \frac{1}{2} j2{,}635 \cdot 10^{-6} \cdot 400 = j0{,}527 \cdot 10^{-3}\ \text{S}$$

The current phasor for the load:

$$\hat{I}_2 = \left(\frac{\hat{S}_2}{3\hat{U}_{f2}}\right)^* = \left(\frac{125 + j0}{3 \cdot 127}\right)^* = 328\ \text{A}$$

The voltage phasor at the beginning of power line:

$$\hat{U}_{f1} = \hat{A} \cdot \hat{U}_{f2} + \hat{B} \cdot \hat{I}_2 = 137{,}7 \cdot e^{j24{,}82^\circ}\ \text{kV}$$

The current phasor at the beginning of power line:

$$\hat{I}_1 = \hat{C} \cdot \hat{U}_{f2} + \hat{D} \cdot \hat{I}_2 = 325{,}828 \cdot e^{j24{,}05^\circ}\ \text{A}$$

The complex power at the beginning of power line:

$$\hat{S}_1 = 3\hat{U}_{f1}\hat{I}_1^* = 3 \cdot 137{,}7 \cdot 10^3 \cdot e^{j24{,}82^\circ} \cdot 325{,}828 \cdot e^{-j24{,}05^\circ} = 134{,}6 \cdot e^{j0{,}77^\circ}\ \text{MVA}$$

$$P_1 = 134{,}59\ \text{MW}, \quad Q_1 = 1{,}81\ \text{MVAr inductive}$$

The power factor at the beginning of power line:

$$\cos\varphi_1 = \cos 0{,}77^\circ = 0{,}9999\ \text{ind.}$$

π-network is only the replacement of correctly considered homogeneous line. The elements A, B, C, D of matrix include only a few (1 to 2) initial elements of Taylor series for hyperbolical function describing homogeneous line. For large power line lengths considerable errors are generated.

The following figures show the comparison of values solved in example 2 for changing lengths of power line and for various networks (π, T, Γ) compared to homogenous line.

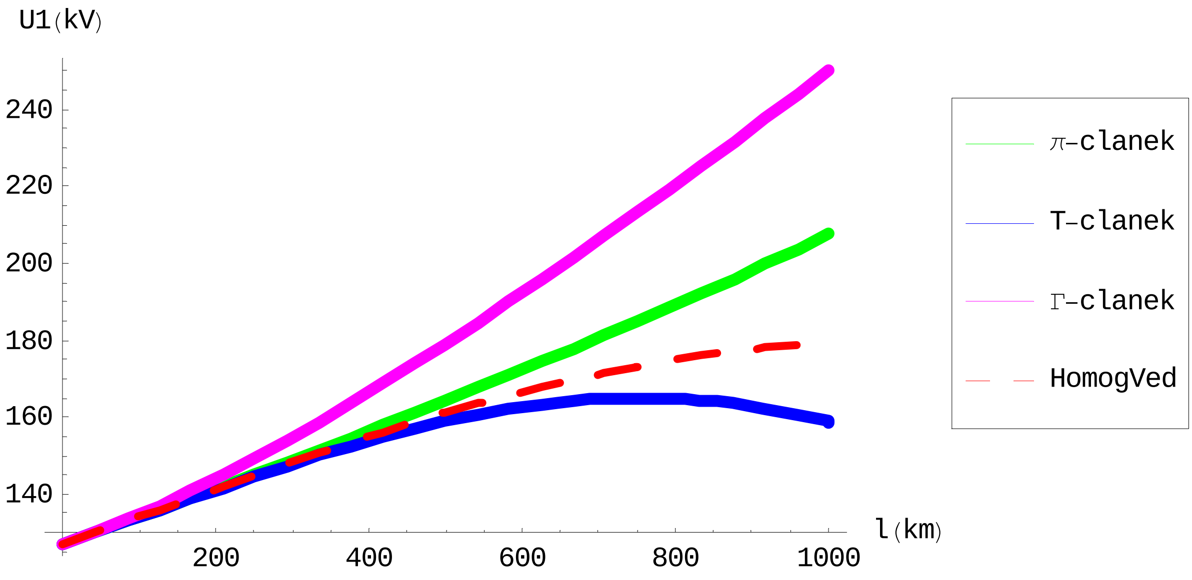
U1(kV)
240
220
200
180
160
140
200
400
600
800
1000
l(km)
π-clanek
T-clanek
Γ-clanek
HomogVed

Fig. 3

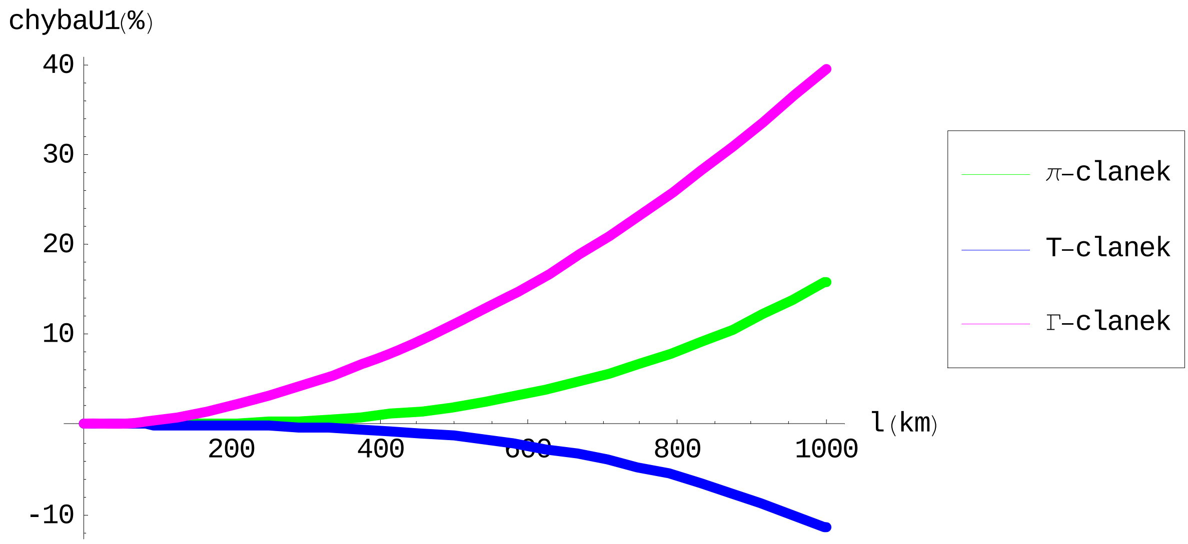
chybaU1(%)
40
30
20
10
-10
200
400
600
800
1000
l(km)
π-clanek
T-clanek
Γ-clanek

Fig. 4

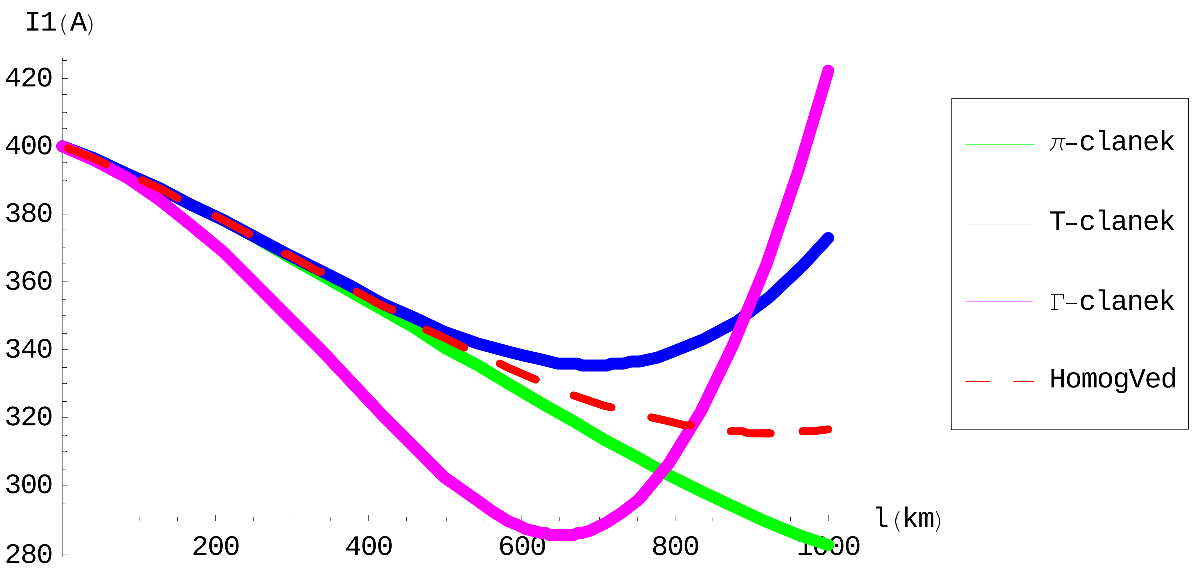
I1(A)
420
400
380
360
340
320
300
280
200
400
600
800
1000
l(km)
π-clanek
T-clanek
Γ-clanek
HomogVed

Fig. 5

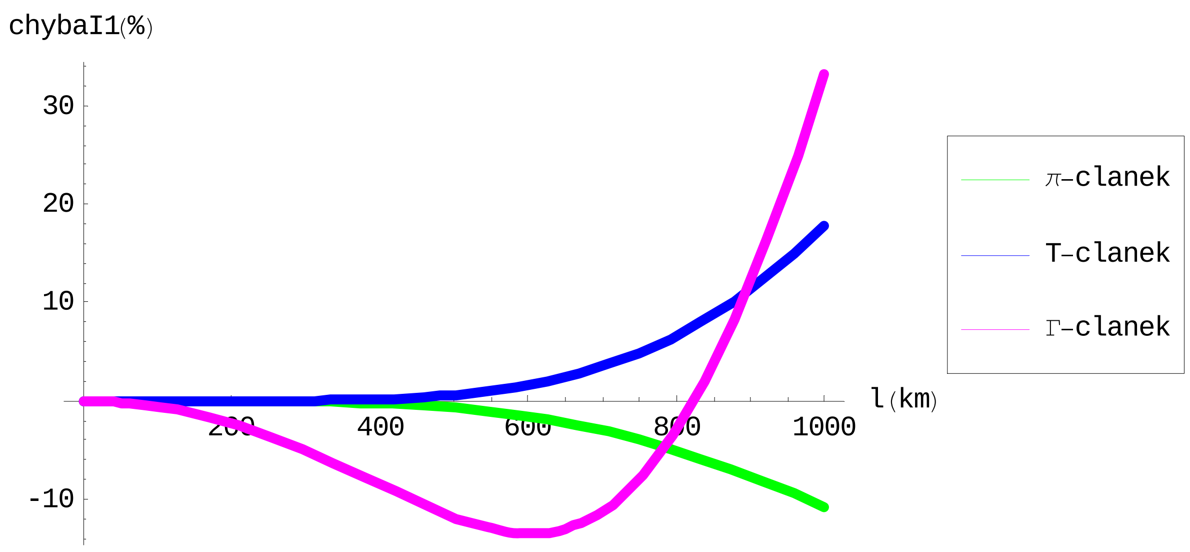
chybaI1(%)
30
20
10
-10
200
400
600
800
1000
l(km)
π-clanek
T-clanek
Γ-clanek

Fig. 6

Ferranti effect

The Ferranti effect is an increase in voltage occurring at the receiving end in comparison to the voltage at the sending end for energized transmission lines with open end or little loaded. The situation is in the figure 7. The power line is shown as a T-network. If the active part of the cross admittance is neglected (G = 0), only a capacity current will flow through the T-network cross part for power line open end. Due to the open end the end current is zero ($I_2 = 0$) so there is no voltage drop on the right side of the longitudinal impedance and the voltage U_2 is on the cross admittance. The following current flows through the left side of the longitudinal impedance:

$$I_1 = I_B = B_k \cdot l \cdot U_2 \qquad (7)$$

The current I_1 results in voltage drop on a half of the longitudinal impedance $\frac{Z_k}{2}$ and on the cross admittance Y_k.

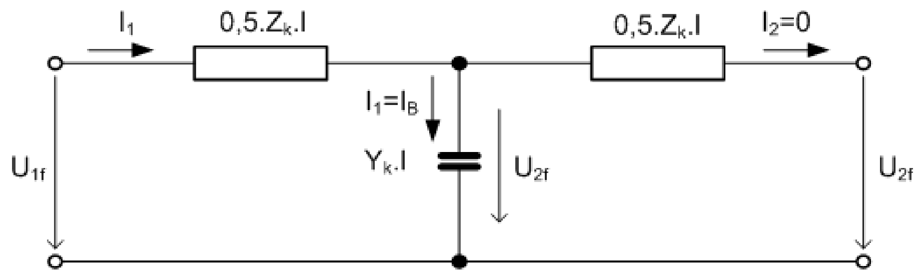

Fig. 7: The substitute diagram of T-network for open end of power line

A voltage increase occurs on the power line instead of voltage drop, i.e. the negative voltage drop. The difference between the voltage at the beginning and end is approximately (when neglecting resistance R_k):

$$\mathbf{U}_2 - \mathbf{U}_1 = I_B \cdot \frac{X}{2} = 0{,}5.B_k.l.U.X_k.l \qquad (8)$$

For copper and aluminium power line ($\mu_r = 1$) the equation (8) can be simplified:

$$\mathbf{U}_2 - \mathbf{U}_1 = 0{,}55.U.l^2.10^{-6} \quad (kV;kV,km) \qquad (9)$$

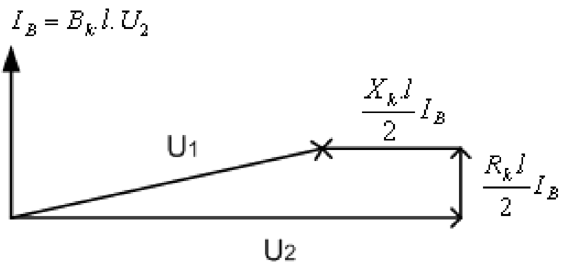

Fig. 8: Phasor diagram for power line with open end

Example: Calculate the voltage at the beginning when the power line lost its load at the end. The power line is 3 x 220 kV with length 500 km.

$\mathbf{U}_2 - \mathbf{U}_1 = 0{,}55.U.l^2.10^{-6} = 0{,}55.220.500.10^{-6} =$ **<u>30,25 kV</u>**

The voltage at the end increases approximately to 250 kV.