Calculate initial sub-transient short-circuit current and initial sub-transient short-circuit power for three-phase short circuit according to Fig 1.

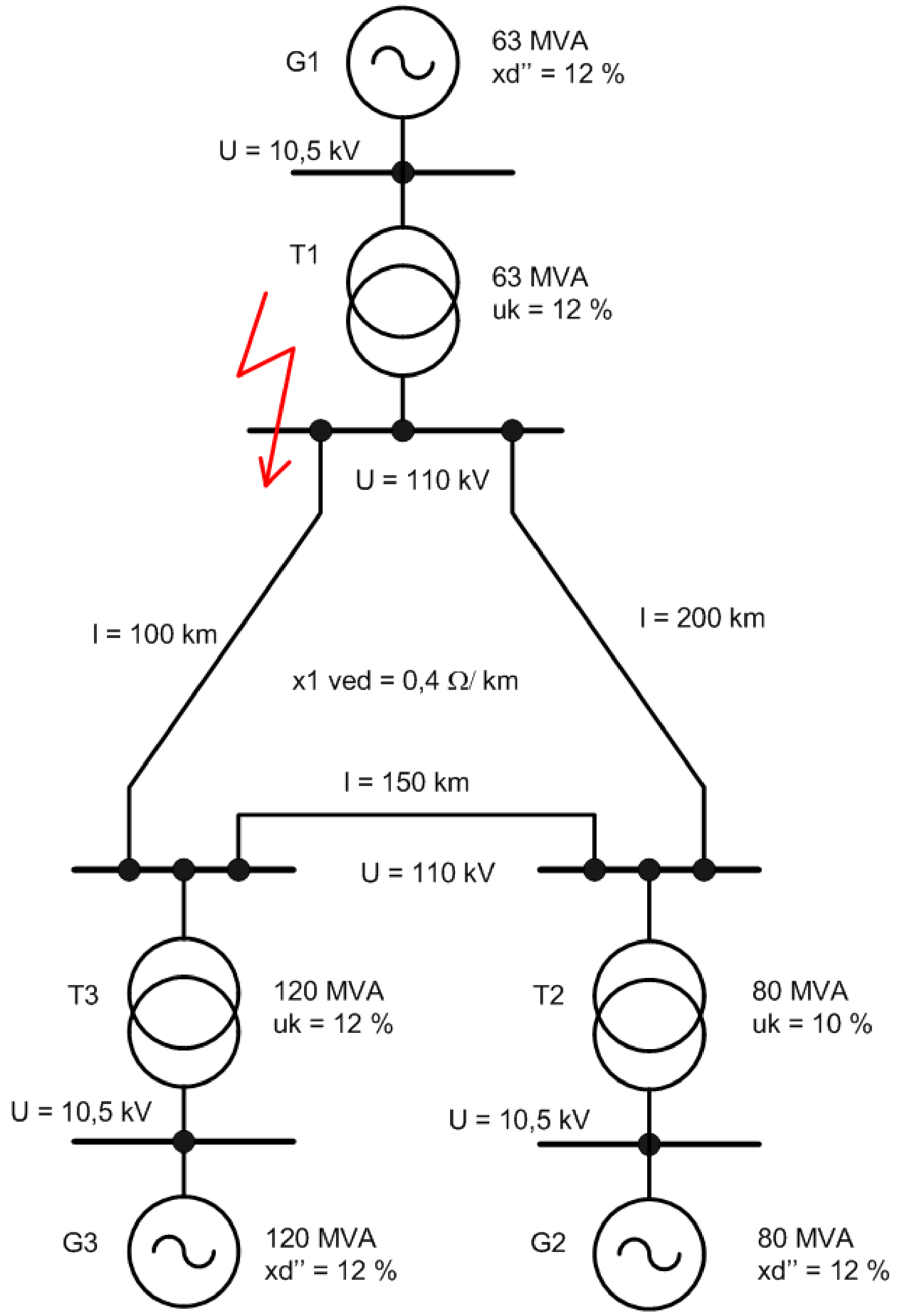

Fig. 1

Base power: $S_v = 120\ \mathrm{MVA}$ and base voltage: $U_v = 110\ \mathrm{kV}$

Hence base current: $I_v = \dfrac{S_v}{\sqrt{3} \cdot U_v} = 630\ \mathrm{A}$.

Then we recalculate relative reactances of all elements for base values:

- Generators

G1: $x_{G1} = x_d'' \dfrac{S_v}{S_n} = 0{,}12 \cdot \dfrac{120}{63} = 0{,}23$

G2: $x_{G2} = 0{,}12 \cdot \dfrac{120}{80} = 0{,}18$

G3: $x_{G3} = 0{,}12 \cdot \dfrac{120}{120} = 0{,}12$

- Transformers

T1: $x_{T1} = u_k \dfrac{S_v}{S_n} = 0{,}12 \cdot \dfrac{120}{63} = 0{,}23$

T2: $x_{T2} = 0{,}10 \cdot \dfrac{120}{80} = 0{,}15$

T3: $x_{T3} = 0{,}12 \cdot \dfrac{120}{120} = 0{,}12$

- Power lines

V12: $x_{V12} = x_{1ved} \cdot 1 \cdot \dfrac{S_v}{U_v^2} = 0{,}4 \cdot 200 \cdot \dfrac{120}{110^2} = 0{,}8$

V13: $x_{V13} = 0{,}4 \cdot 100 \cdot \dfrac{120}{110^2} = 0{,}4$

V23: $x_{V23} = 0{,}4 \cdot 150 \cdot \dfrac{120}{110^2} = 0{,}6$

Recalculating relations can be obtained:

$$x_G = x_d'' \cdot \frac{Z_{nG}}{Z_v} \cdot p^2 = x_d'' \cdot \frac{U_{nG}^2}{S_{nG}} \cdot \frac{S_v}{U_v^2} \cdot \frac{U_v^2}{U_{nG}^2} = x_d'' \cdot \frac{S_v}{S_{nG}}$$

Where: Z_{nG}........ generator nominal impedance (Ω)

Z_v.......... base impedance (Ω)

p............ voltage ration of all transformers between the place of short-circuit (the place with the base voltage) and the generator place (-)

Note: In case that neighbouring voltage levels don't correspond to transformer ratio, the full recalculating relation must be used which can't be further simplified.

Equivalent diagram with recalculated reactances (after simplification) is in Fig. 2.

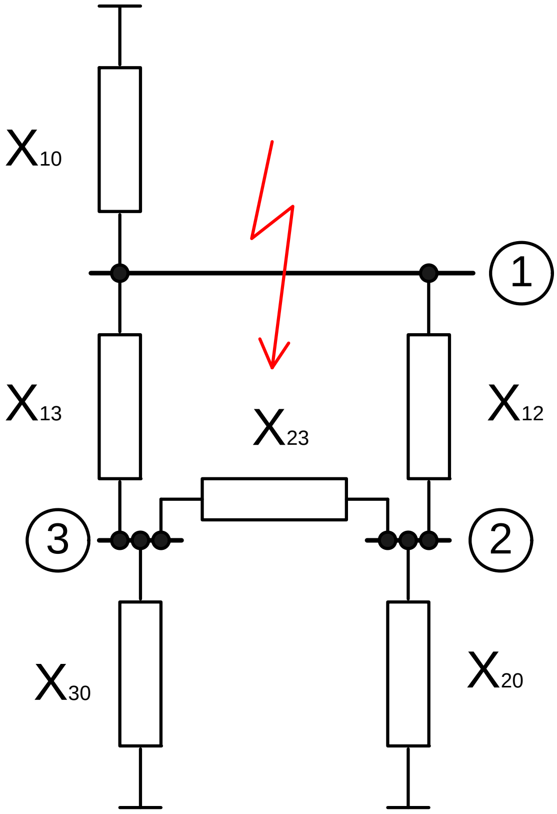

Fig. 2

$$x_{10} = x_{G1} + x_{T1} = 0{,}46 \qquad x_{12} = x_{V12} = 0{,}8$$
$$x_{20} = x_{G2} + x_{T2} = 0{,}33 \qquad x_{13} = x_{V13} = 0{,}4$$
$$x_{30} = x_{G3} + x_{T3} = 0{,}24 \qquad x_{23} = x_{V23} = 0{,}6$$

We convert recalculated reactances (impedances) to admittances for further calculation (inverse values):

$$y_{10} = 2{,}17 \qquad y_{12} = y_{21} = 1{,}25$$
$$y_{20} = 3{,}03 \qquad y_{13} = y_{31} = 2{,}5$$
$$y_{30} = 4{,}17 \qquad y_{23} = y_{32} = 1{,}67$$

Now we can create admittance matrix of the system in Fig. 2.

Diagonal elements ($y_{(i,i)}$) are negative sums of all admittances from the node i, off-diagonal elements ($y_{(i,j)}$) are admittances between the nodes i and j.

$$y_{(1,1)} = -(2{,}17 + 2{,}5 + 1{,}25) = -5{,}92$$
$$y_{(2,2)} = -(3{,}03 + 1{,}25 + 1{,}67) = -5{,}95$$
$$y_{(3,3)} = -(4{,}17 + 1{,}67 + 2{,}5) = -8{,}33$$

Admittance matrix is:

$$(y) = \begin{pmatrix} -5{,}92 & 1{,}25 & 2{,}5 \\ 1{,}25 & -5{,}95 & 1{,}67 \\ 2{,}5 & 1{,}67 & -8{,}33 \end{pmatrix}$$

Hence the impedance matrix $(\mathrm{z}) = (\mathrm{y})^{-1}$:

$$(\mathrm{z}) = -\begin{pmatrix} 0{,}216 & 0{,}067 & 0{,}078 \\ 0{,}067 & 0{,}199 & 0{,}060 \\ 0{,}078 & 0{,}060 & 0{,}155 \end{pmatrix}$$

(if the admittance matrix is symmetrical by the diagonal, the same applies even for impedance matrix)

(Note: these are only the imaginary parts of impedances and admittances)

Impedance matrix defines the relation between the vectors of node voltages and currents:

$$\begin{pmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \hat{u}_3 \end{pmatrix} = \begin{pmatrix} \hat{z}_{(1,1)} & \hat{z}_{(1,2)} & \hat{z}_{(1,3)} \\ \hat{z}_{(2,1)} & \hat{z}_{(2,2)} & \hat{z}_{(2,3)} \\ \hat{z}_{(3,1)} & \hat{z}_{(3,2)} & \hat{z}_{(3,3)} \end{pmatrix} \begin{pmatrix} \hat{i}_1 \\ \hat{i}_2 \\ \hat{i}_3 \end{pmatrix}$$

Node voltages are (all are relative values) voltages between nodes and the ground.

Node currents represent loads or supplies in individual nodes. $\hat{I}_2$ a $\hat{I}_3$ are zero in our case because there is no consumption (1. Kirchhoff law – the current sum is zero in a node). In node 1 there is a short-circuit which represents a consumption place (currents flow into this node). Therefore $\hat{I}_1$ is not zero and equal to the desired short-circuit current.

The reference voltage (equal to one) is in the source (generator) place in the real situation. The current is limited by the impedances where it flows (Fig. 3).

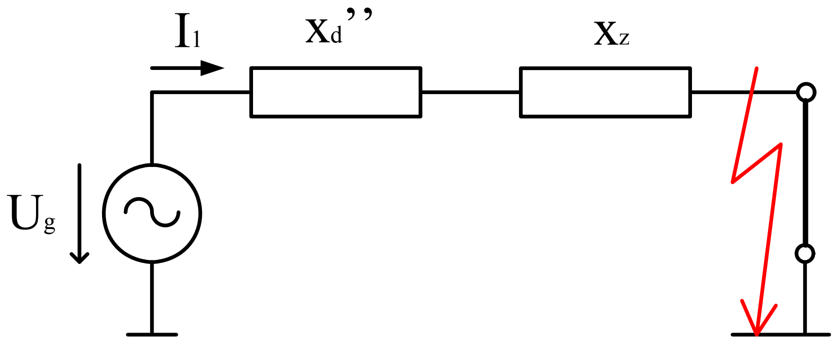

Fig. 3

For the calculation we can consider the reference voltage in the short-circuit place and the generator internal voltage as zero (source is substituted only by its sub-transient reactance). The flowing current must remain unchanged (see Fig. 4).

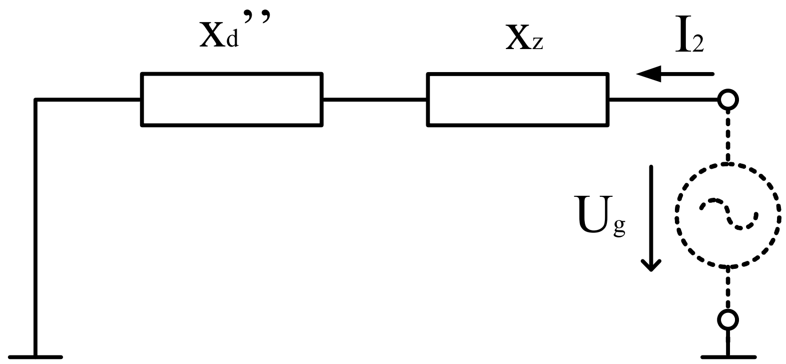

Fig. 4

Hence the matrix equation is simplified

$$\begin{pmatrix}\hat{u}_1\\ \hat{u}_2\\ \hat{u}_3\end{pmatrix}=\begin{pmatrix}\hat{z}_{(1,1)} & \hat{z}_{(1,2)} & \hat{z}_{(1,3)}\\ \hat{z}_{(2,1)} & \hat{z}_{(2,2)} & \hat{z}_{(2,3)}\\ \hat{z}_{(3,1)} & \hat{z}_{(3,2)} & \hat{z}_{(3,3)}\end{pmatrix}\begin{pmatrix}\hat{i}_1\\ 0\\ 0\end{pmatrix},$$

In relative values: $u_1 = 1$.

Since from matrix $\hat{u}_1 = \hat{z}_{(1,1)} \cdot \hat{i}_1$, the short-circuit current in node 1 in relative values is the inverse value of $\hat{z}_{(1,1)}$ element in the impedance matrix: $i_1 = -\dfrac{1}{0{,}216} = -4{,}63$.

(minus corresponds the current flowing outside the node, i.e. consumption)

In denominated values:

$I_1 = 4{,}63 \cdot I_v = 4{,}63 \cdot 0{,}63\ \text{kA} = 2{,}92\ \text{kA}$.

Sub-transient power is:

$S_1 = \sqrt{3} U_v I_1 = \sqrt{3} \cdot 110 \cdot 2{,}92\ \text{MVA} = 555\ \text{MVA}$

If we know all node currents and the impedance matrix, we can calculate node voltages:

$$\begin{pmatrix}\hat{u}_1\\ \hat{u}_2\\ \hat{u}_3\end{pmatrix}=\begin{pmatrix}\hat{z}_{(1,1)} & \hat{z}_{(1,2)} & \hat{z}_{(1,3)}\\ \hat{z}_{(2,1)} & \hat{z}_{(2,2)} & \hat{z}_{(2,3)}\\ \hat{z}_{(3,1)} & \hat{z}_{(3,2)} & \hat{z}_{(3,3)}\end{pmatrix}\begin{pmatrix}-4{,}63\\ 0\\ 0\end{pmatrix}=\begin{pmatrix}1\\ 0{,}312\\ 0{,}362\end{pmatrix}$$

Hence we can calculate short-circuit contributions from individual generators. They are equal to currents flowing through reactances x_{10}, x_{20}, x_{30}. They are calculated as ratio of node voltages difference and particular reactances.

$$i_{10} = \frac{u_1}{x_{10}} = \frac{1}{0{,}46} = 2{,}17 \rightarrow I_{10} = 1{,}37\ \text{kA}$$

$$i_{20} = \frac{u_2}{x_{20}} = \frac{0{,}312}{0{,}33} = 0{,}94 \rightarrow I_{20} = 0{,}60\ \text{kA}$$

$$i_{30} = \frac{u_3}{x_{30}} = \frac{0{,}362}{0{,}24} = 1{,}51 \rightarrow I_{30} = 0{,}95\ \text{kA}$$

It is also good to realize the meaning of voltage vector elements. There must be zero voltage in the short-circuit place. But we get $\hat{u}_1 = 1$. This is caused by the inverse idea in Fig. 3 and 4. Hence we can see that the resulting voltages are voltage drops from the rated level in real. The real node voltages are calculated as a complement to 1. We get:

$$\hat{u}_{skut} = \begin{pmatrix}0\\ 0{,}688\\ 0{,}638\end{pmatrix} \rightarrow \hat{U}_{skut} = \begin{pmatrix}0\\ 75{,}7\\ 70{,}2\end{pmatrix}\text{kV}$$