Waves on ideal power line

$$R = G = 0$$

$$\hat{Z}_V = Z_V$$

$$\hat{U} = \hat{U}_{post} + \hat{U}_{odr}$$

$$\hat{I} = \hat{I}_{post} + \hat{I}_{odr}$$

homogenous power line

$$-\frac{\partial u}{\partial x} = L\frac{\partial i}{\partial t} \qquad -\frac{\partial i}{\partial x} = C\frac{\partial u}{\partial t}$$

wave equations

$$\frac{\partial^2 u}{\partial x^2} = LC\frac{\partial^2 u}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 i}{\partial t^2}$$

$$v = \frac{1}{\sqrt{LC}} \quad (m \cdot s^{-1}; H, F)$$

overhead power lines ca. 300 000 km/s

cable ca. 50 000 - 100 000 km/s

voltage waves

$$\frac{\partial^2}{\partial t^2} = p^2 \quad \rightarrow \quad \frac{\partial^2 u}{\partial x^2} = \frac{p^2}{v^2} u$$

$$u(x,t) = K_1 \cdot e^{-\kappa x} + K_2 \cdot e^{+\kappa x}$$

$$\kappa = \frac{p}{v}$$

$$u(x,t) = u_1(x,t) + u_2(x,t)$$

$$u(x,t) = u_1(0,t) \cdot e^{-\frac{p}{v}x} + u_2(0,t) \cdot e^{+\frac{p}{v}x}$$

$$u(x,t) = u_1(0, t - \frac{x}{v}) + u_2(0, t + \frac{x}{v})$$

u_1 – progressive wave
u_2 – reflected wave (only after reflection at the end or at a boundary)

$$u = u_p + u_1$$

<u>current waves</u>

$$-\frac{\partial u}{\partial x} = L\frac{\partial i}{\partial t} \quad \rightarrow \quad \frac{\partial i}{\partial t} = -\frac{1}{L}\frac{\partial u}{\partial x}$$

$$i = -\frac{1}{L}\int \frac{\partial u}{\partial x}dt$$

$$i = -\frac{1}{L}\int\left(-\frac{p}{v}\cdot K_1 \cdot e^{-\frac{p}{v}x} + \frac{p}{v}\cdot K_2 \cdot e^{+\frac{p}{v}x}\right)dt$$

$$i = \frac{1}{Lv}\int \frac{\partial}{\partial t}\left(K_1 \cdot e^{-\frac{p}{v}x} - K_2 \cdot e^{+\frac{p}{v}x}\right)dt$$

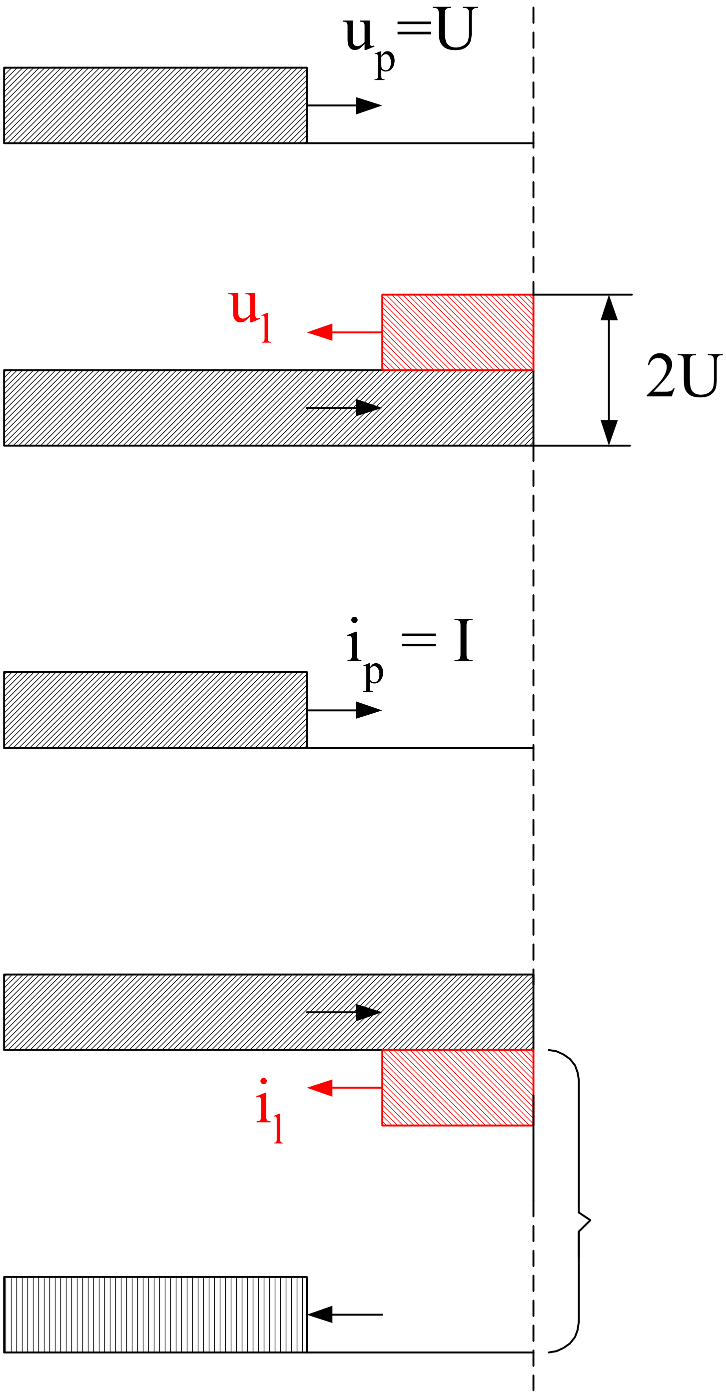
$$i = \frac{\sqrt{LC}}{L}\left(u_p - u_1\right) = \sqrt{\frac{C}{L}}\left(u_p - u_1\right) = i_p + i_1$$

$$i_p = \frac{u_p}{Z_V}$$ progressive wave

$$i_1 = -\frac{u_1}{Z_V}$$ reflected wave

Note: Further we consider rectangular shape waves (with a sharp edge).

Reflection at the open end



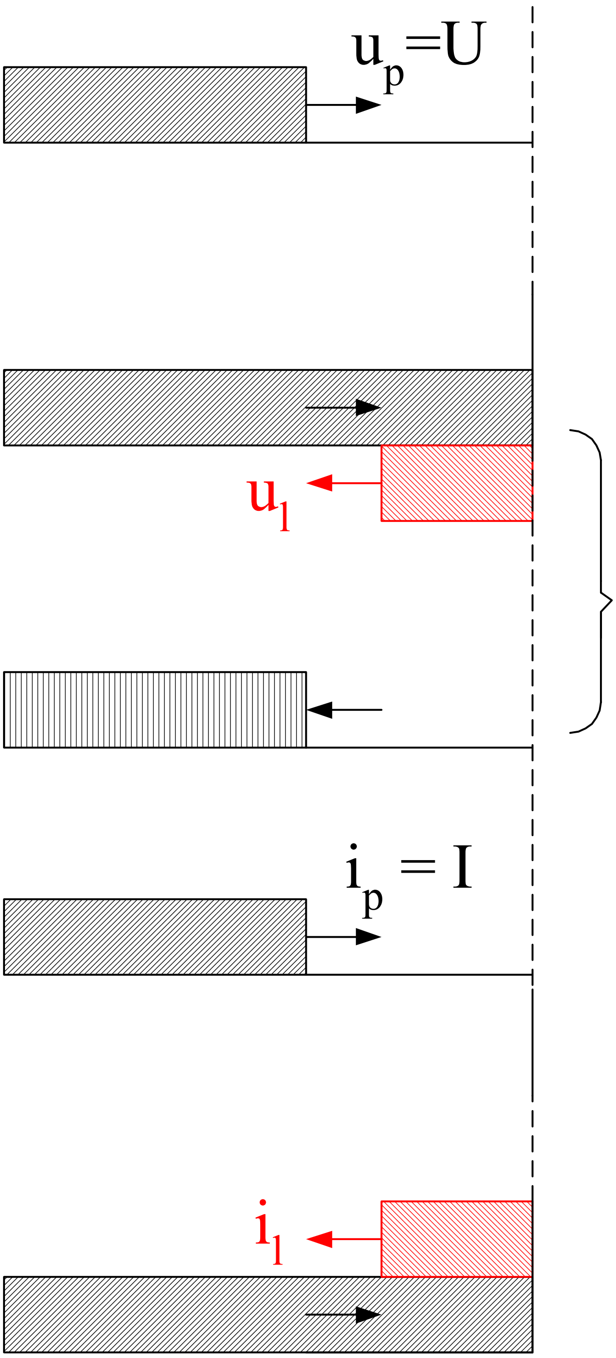
$$i = i_p + i_1 = 0$$

$$\underline{i_1 = -i_p}$$

$$i = \frac{u_p}{Z_v} - \frac{u_1}{Z_v} = 0$$

$$\underline{u_1 = u_p}$$

Reflection at the short-circuit end



$$u = u_p + u_1 = 0$$

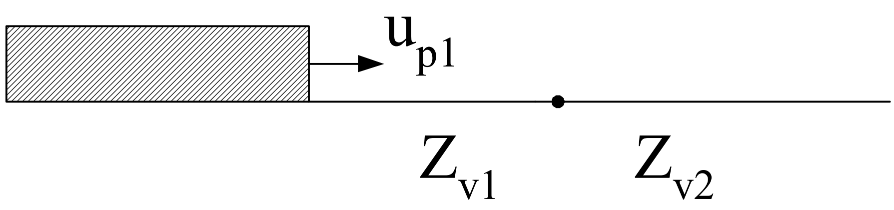
$$\underline{u_1 = -u_p}$$

$$i_1 = -\frac{u_1}{Z_v} = +\frac{u_p}{Z_v} = i_p$$

$$\underline{i_1 = i_p}$$

Boundary passing

Boundary = step change of wave impedance



$$u_1 = u_2$$
$$i_1 = i_2$$

$$u_{p1} + u_{l1} = u_{p2} + u_{l2} \qquad u_{l2} = 0$$
$$i_{p1} + i_{l1} = i_{p2} + i_{l2} \qquad i_{l2} = 0$$

$$u_{p1} + u_{l1} = u_{p2}$$
$$\frac{u_{p1}}{Z_{v1}} - \frac{u_{l1}}{Z_{v1}} = \frac{u_{p2}}{Z_{v2}}$$

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reflection coefficients

$$u_{l1} = \frac{Z_{v2} - Z_{v1}}{Z_{v1} + Z_{v2}} u_{p1} \qquad i_{l1} = \frac{Z_{v1} - Z_{v2}}{Z_{v1} + Z_{v2}} i_{p1}$$

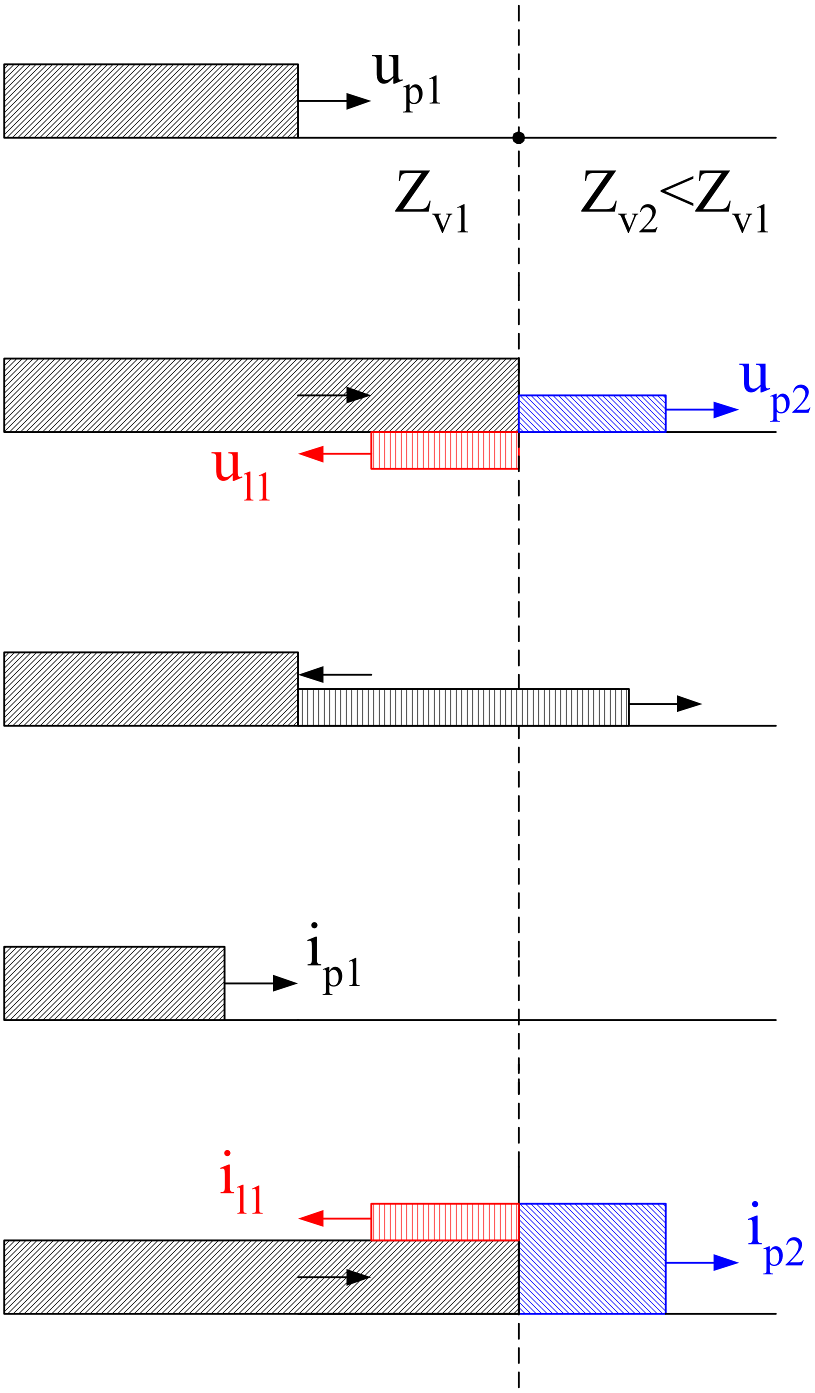
penetration coefficients

$$u_{p2} = \frac{2Z_{v2}}{Z_{v1} + Z_{v2}} u_{p1} \qquad i_{p2} = \frac{2Z_{v1}}{Z_{v1} + Z_{v2}} i_{p1}$$

Note: Power line branching is equal to a parallel combination of Z_{v2}, Z_{v3},…

Boundary overhead line – cable

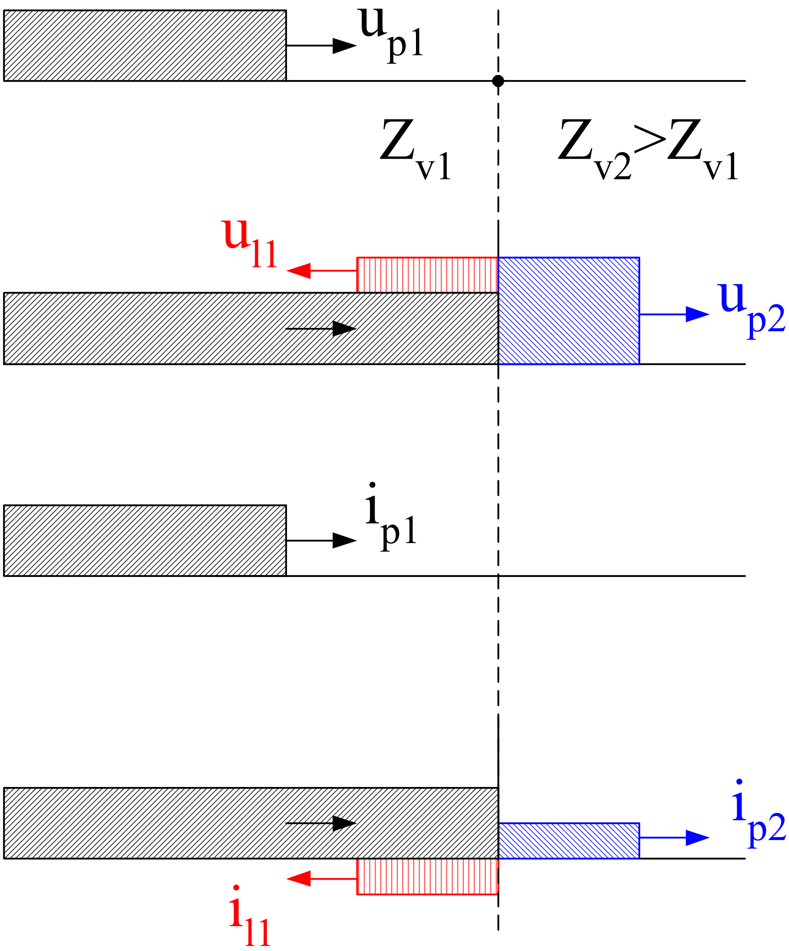
$(Z_{V1} \doteq 400\ \Omega;\ \ Z_{V2} \doteq 40\ \Omega)$



$$u_{p2} = \frac{2Z_{v2}}{Z_{v1} + Z_{v2}} u_{p1}$$

$$i_{p2} = \frac{2Z_{v1}}{Z_{v1} + Z_{v2}} i_{p1}$$

Boundary cable – overhead line

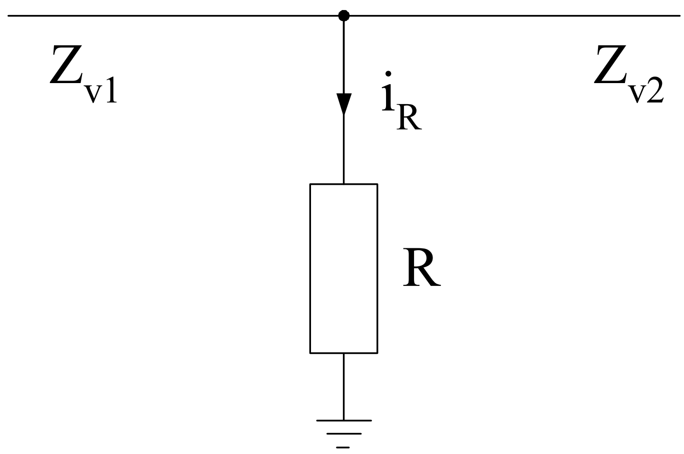


$$u_{p2} = \frac{2Z_{v2}}{Z_{v1} + Z_{v2}} u_{p1}$$

$$i_{p2} = \frac{2Z_{v1}}{Z_{v1} + Z_{v2}} i_{p1}$$

<u>Cross resistance</u>

(surge arrester)



$$u_1 = u_2$$
$$i_1 = i_2$$

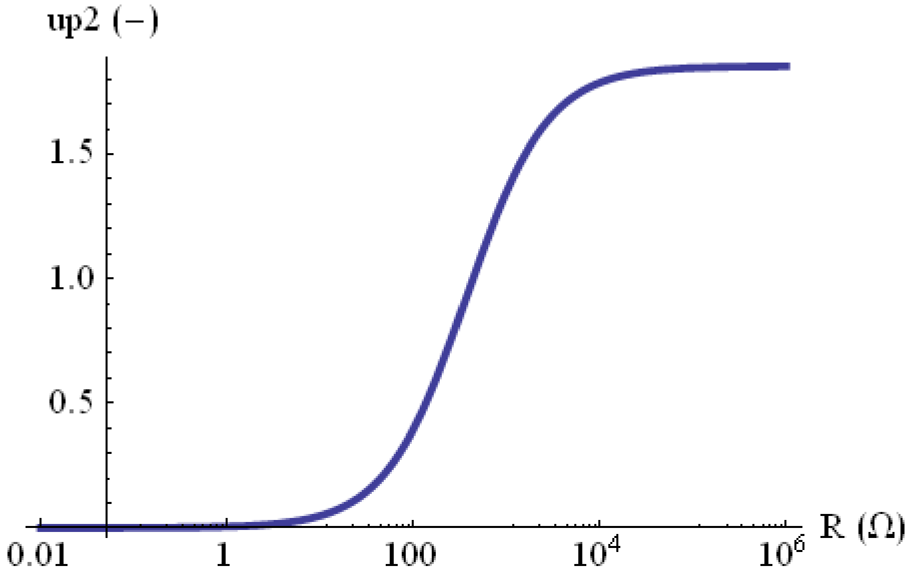
$$u_{p1} + u_{l1} = u_{p2}$$
$$i_{p1} + i_{l1} = i_{p2} + i_R$$

$$u_{p1} + u_{l1} = u_{p2}$$

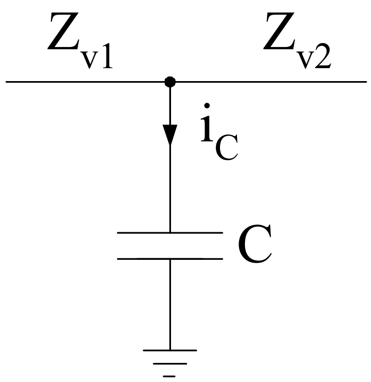
$$\frac{u_{p1}}{Z_{v1}} - \frac{u_{l1}}{Z_{v1}} = \frac{u_{p2}}{Z_{v2}} + \frac{u_{p2}}{R}$$

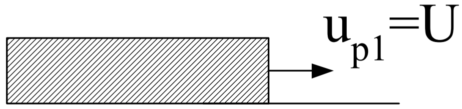
$$u_{p2} = \frac{\dfrac{2}{Z_{v1}}}{\dfrac{1}{Z_{v1}} + \dfrac{1}{Z_{v2}} + \dfrac{1}{R}} u_{p1}$$

ex.: $Z_{V1} = 400\ \Omega$, $Z_{V2} = 5000\ \Omega$, $u_{p1} = 1$



Cross capacity
(overhead line – capacitor bushing - transformer)





$$U_{p1}(p) = \frac{U}{p}$$

$$U_{p1}(p) + U_{l1}(p) = U_{p2}(p)$$
$$\underline{I_{p1}(p) + I_{l1}(p) = I_{p2}(p) + I_c(p)}$$

$$\frac{U_{p1}(p)}{Z_{v1}} - \frac{U_{l1}(p)}{Z_{v1}} = \frac{U_{p2}(p)}{Z_{v2}} + pCU_{p2}(p)$$

$$U_{p2}(p) = \frac{2Z_{v2}}{Z_{v1} + Z_{v2}} \frac{1}{1 + pT} U_{p1}(p)$$

$$T = \frac{CZ_{v1}Z_{v2}}{Z_{v1} + Z_{v2}} \quad (s; F, \Omega)$$

$$\underline{u_{p2} = \frac{2Z_{v2}}{Z_{v1}+Z_{v2}}\left(1-e^{-\frac{t}{T}}\right)U}$$

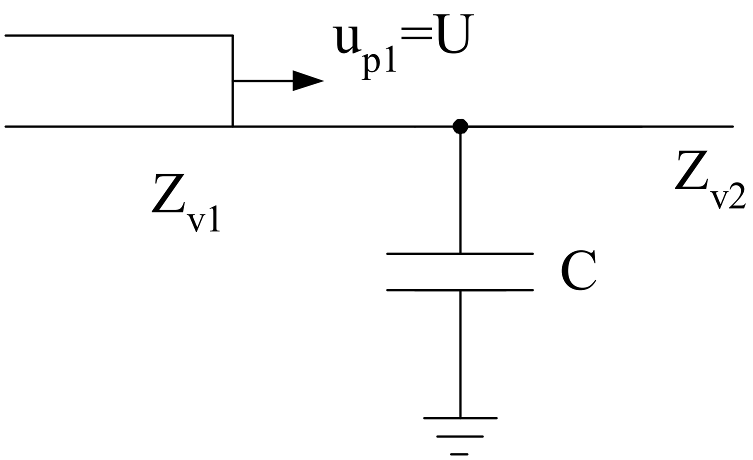
For $t = 0:$ $u_{p2} = 0$

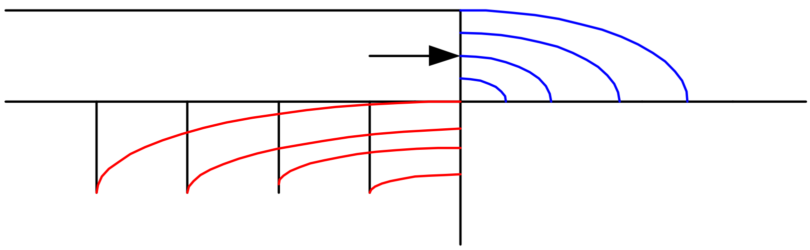
For $t \to \infty \, (t \geq 3T):$ $u_{p2} = \frac{2Z_{v2}}{Z_{v1}+Z_{v2}} U$

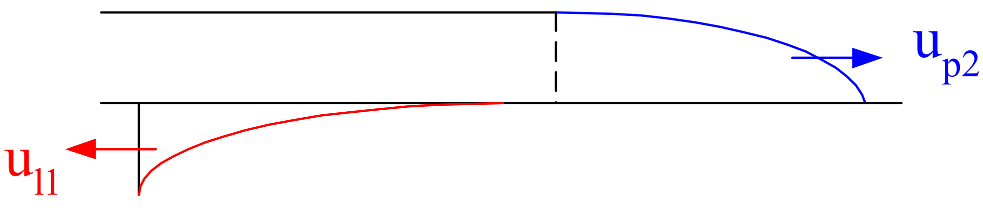
Note: $t = 0$ – as short-circuit end

$t \to \infty$ - as no capacity

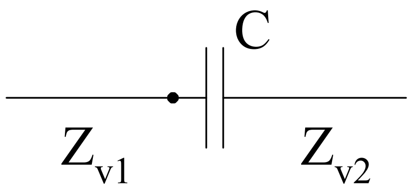
! $u_{l1} = u_{p2} - u_{p1}$

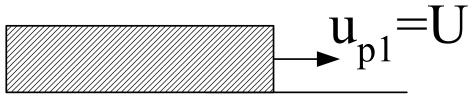






Longitudinal capacity
(series compensation)





$$U_{p1}(p)=\frac{U}{p}$$

$$U_{p1}(p)+U_{l1}(p)=U_c(p)+U_{p2}(p)$$
$$\underline{I_{p1}(p)+I_{l1}(p)=I_{p2}(p)}$$

$$\frac{U_{p1}(p)}{Z_{v1}}-\frac{U_{l1}(p)}{Z_{v1}}=\frac{U_{p2}(p)}{Z_{v2}}$$

$$U_c(p)=\frac{I_{p2}(p)}{pC}=\frac{U_{p2}(p)}{pCZ_{v2}}$$

$$U_{p2}(p)=\frac{2Z_{v2}}{Z_{v1}+Z_{v2}}\frac{pT}{1+pT}U_{p1}(p)$$

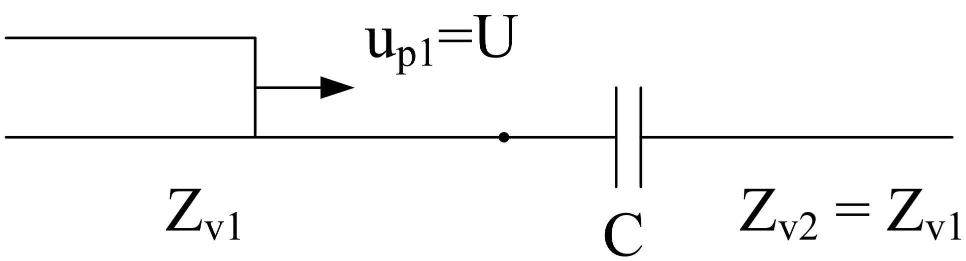
$$T=C(Z_{v1}+Z_{v2}) \qquad (s;F;\Omega)$$

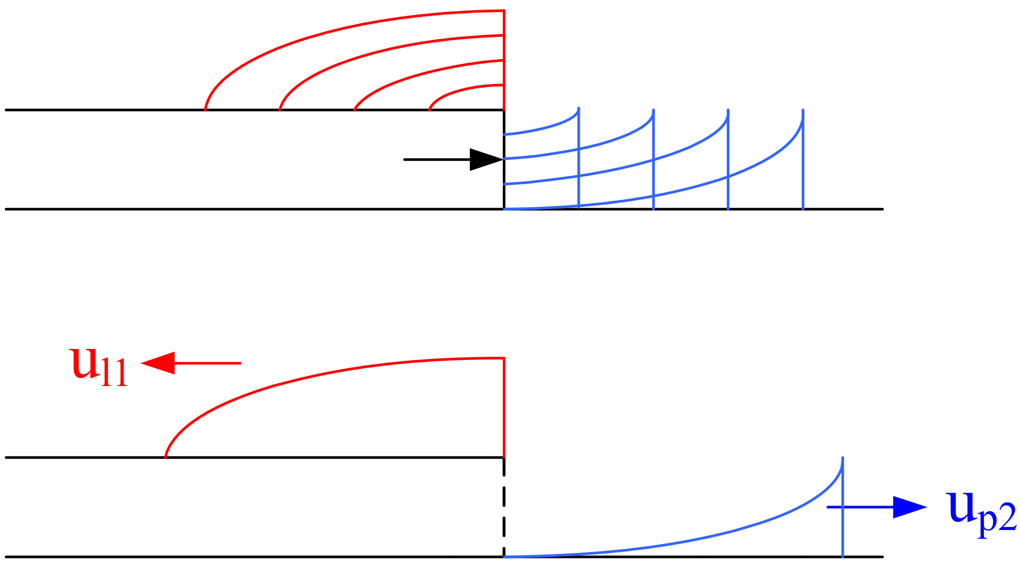
$$u_{p2} = \frac{2Z_{v2}}{Z_{v1} + Z_{v2}} \cdot e^{-\frac{t}{T}} \cdot U$$

For $t = 0$: $u_{p2} = \frac{2Z_{v2}}{Z_{v1} + Z_{v2}} U$

For $t \to \infty (t \geq 3T)$: $u_{p2} = 0$

Note: $t = 0$ – as no capacity
$t \to \infty$ - as open end





Note: Longitudinal inductance has the same characteristics as cross capacity. Cross inductance has the same characteristics as longitudinal capacity.