Voltage Distribution along Transformer Winding

Introduction

- Transformers connected to long outdoor lines are often exposed to atmospheric overvoltage
- Should the respective overvoltage protection malfunction, a voltage wave will appear on transformer input terminals
- Such a wave might cause insulation damage and therefore it is necessary to analyze the subsequent voltage transient inside the transformer

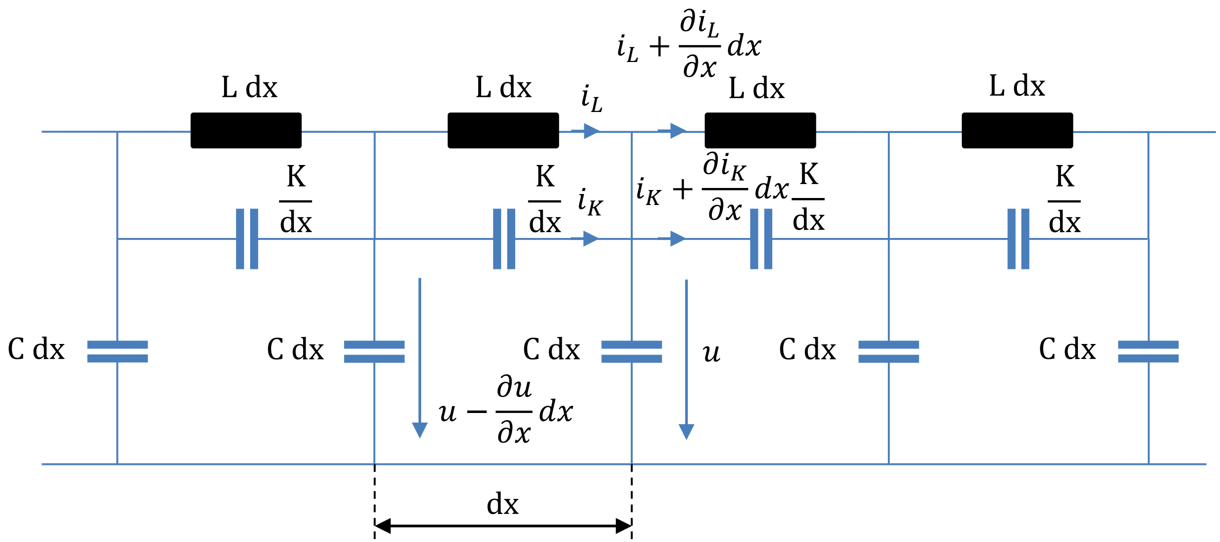
Winding behaviour

- Winding is a group of resistors, coils and capacitors
- In the beginning of the transient, the waveform is determined primarily by the capacity of the winding, whereas the inductance governs the waveform at the end
- Let us find the distribution of the voltage during the whole transient

Voltage distribution

- The simplest theory that determines the voltage distribution was presented by Wagner
- The theory works with the following assumptions:
  - The winding coil has only one layer
  - The resistance of the winding is minimal, i. e. equal to zero in the calculation
  - Mutual inductive and capacitive coupling is neglected
  - A voltage unit step is applied to the input terminals

Equivalent circuit with distributed parameters



- *L* (H/m) is the total inductance relative to length
- *C* (F/m) is the capacity between the coil wire and the ground relative to length
- *K* (F·m) is inter-turn capacity relative to length

Voltage distribution

- Kirchhoff's current law for the upper-right node:

$$i_L + i_K = i_L + \frac{\partial i_L}{\partial x} dx + i_K + \frac{\partial i_K}{\partial x} dx + C dx \frac{\partial u}{\partial t} \qquad (1)$$

- Current passing through the longitudinal capacity:

$$i_K = i_L + \frac{K}{dx} \frac{\partial}{\partial t} \left(u - \frac{\partial u}{\partial x} dx - u \right) = -K \frac{\partial^2 u}{\partial x \partial t} \quad (2)$$

- Kirchhoff's voltage law for the central loop:

$$u - \frac{\partial u}{\partial x} dx = L dx \frac{\partial i_L}{\partial t} + u \qquad (3)$$

Voltage distribution

- First time derivative of (1) is:

$$\frac{\partial i_L}{\partial x} + \frac{\partial i_K}{\partial x} + C\frac{\partial u}{\partial t} = 0 \quad (4)$$

- Applying another time derivative on (1) provides us with:

$$\frac{\partial^2 i_L}{\partial x \partial t} + \frac{\partial^2 i_K}{\partial x \partial t} + C\frac{\partial^2 u}{\partial t^2} = 0 \qquad (5)$$

- The first term can be substituted from (2) as:

$$\frac{\partial^2 i_L}{\partial x \partial t} - K\frac{\partial^4 u}{\partial x^2 \partial t^2} + C\frac{\partial^2 u}{\partial t^2} = 0 \qquad (6)$$

Voltage distribution

- By substituting from (3), we receive next:

$$-\frac{1}{L}\frac{\partial^2 u}{\partial x^2} - K\frac{\partial^4 u}{\partial x^2 \partial t^2} + C\frac{\partial^2 u}{\partial t^2} = 0 \qquad (7)$$

- Multiplying the previous equation by –*L* gives us the ultimate expression:

$$\frac{\partial^2 u}{\partial x^2} + LK\frac{\partial^4 u}{\partial x^2 \partial t^2} - LC\frac{\partial^2 u}{\partial t^2} = 0 \qquad (8)$$

- In other words, we obtained a wave equation for a coil with unknown function of voltage $u(t,x)$
- Since (8) is a fourth order partial differential equation, its analytical solution will be rather difficult
- Such equations are generally solved by numerical methods

Voltage distribution – solution

- Let us focus on the analytical solution of the two simplest cases, i.e., for $t = 0$ and $t \to \infty$
- In the case $t = 0$, inductance effectively behaves as an open circuit, allowing no current to pass through it
- Therefore, we can omit inductances in the schematic and work only with capacitances
- Also, since time is constant, the function of voltage reduces to $u(0,x) = u_0$

Voltage distribution – solution for u_0

- The first step of deriving the expression for u_0 utilizes Kirchhoff‘s current law. Since time is fixed, the current transforms into charge, as follows:

$$q_0 - (q_0 + dq_0) - Cdxu_0 = 0, \qquad (9)$$

where q_0 is the charge on capacity K/dx

- In (9), q_0 vanishes after subtraction. If we multiply the equation by 1/dx, we obtain:

$$-\frac{dq_0}{dx} = Cu_0 \qquad (10)$$

Voltage distribution – solution for u_0

- Kirchhoff's law for voltage loop provides us with:

$$u_0 - du_0 = \frac{\frac{q_0}{K}}{dx} + u_0 \qquad (11)$$

- Charge q_0 can be expressed from (11) as:

$$q_0 = -K\frac{du_0}{dx} \qquad (12)$$

- By deriving (12) by time, we receive the following expression:

$$\frac{dq_c}{dx} = -K\frac{d^2u_0}{dx^2} = -Cu_0 \qquad (13)$$

- Finally, by substituting the right-hand term with (10), we obtain:

$$\frac{d^2u_0}{dx^2} = \frac{C}{K}u_0 \qquad (14)$$

Voltage distribution – solution for u_0

- Equation (14) represents a second order ordinary differential equation with solution in form:

$$u_0 = A_0 e^{\gamma x} + B_0 e^{-\gamma x} \qquad (15)$$

where $\boxed{\gamma = \sqrt{C/K}}$

Coefficients A_0 and B_0 can be determined from boundary conditions, i.e., $u_0(x = 0) = 1$ and $u_0(x = l) = 0$ (for grounded end of winding of total length l):

$$A_0 + B_0 = 1 \;\rightarrow\; B_0 = 1 - A_0 \qquad (16)$$

$$A_0 e^{\gamma l} + B_0 e^{-\gamma l} = 0 \qquad (17)$$

Voltage distribution – solution for u_0

- Substituting (16) into (17) gives us:

$$A_0 e^{\gamma l} + (1 - A_0) e^{-\gamma l} = 0 \rightarrow$$

$$A_0 = \frac{-e^{-\gamma l}}{e^{\gamma l} - e^{-\gamma l}},\ B_0 = \frac{e^{\gamma l}}{e^{\gamma l} - e^{-\gamma l}} \quad (18)$$

- Using the terms from (18), we obtain new equation for u_0 as per (15):

$$u_0 = \frac{-e^{-\gamma l}}{e^{\gamma l} - e^{-\gamma l}} e^{\gamma x} + \frac{e^{\gamma l}}{e^{\gamma l} - e^{-\gamma l}} e^{-\gamma x} \qquad (19)$$

Voltage distribution – solution for u_0

- By converting right-hand side of (19) into one term, we receive the following expression:

$$u_0 = \frac{e^{\gamma(l-x)} - e^{-\gamma(l-x)}}{e^{\gamma l} - e^{-\gamma l}} \qquad (20)$$

- Equation (20) can be rewritten using hyperbolic function identity:

$$\sinh x = \frac{1}{2}(e^{x} - e^{-x}), \qquad (21)$$

as

$$\boxed{u_0 = \frac{\sinh \gamma(l-x)}{\sinh \gamma l}} \qquad (22)$$

Voltage distribution – solution for u_F

- Let us examine the other case, where time t approaches infinity (we search for final values of voltage u_F)
- As the transient certainly becomes steady state for times approaching infinity, we assume that all functions are independent on time, and hence their time derivatives are zero
- Applying the previous to equation (8), we obtain:

$$\frac{d^2 u_F}{dx^2} = 0 \qquad (23)$$

Voltage distribution – solution for u_F

- The solution of (23), which is an implicit ordinary differential equation of second order, is in form:

$$u_F = A_F \cdot x + B_F \quad (24)$$

- Once again, A_F and B_F is determined by boundary conditions (grounded winding):

$$u_F(0) = 1 \;\rightarrow\; B_F = 1 \quad (25)$$

and

$$u_F(l) = 0 \;\rightarrow\; A_F \cdot l + 1 = 0 \rightarrow A_F = -\frac{1}{l} \quad (26)$$

Voltage distribution – maximum stress

- Undamped EM oscillations occur between u_0 and u_F. Their trend is given by the solution of equation (8) (original PDE) for respective times t
- Winding insulation is stressed the most at time t = 0 and position x = 0. This can be expressed as:

$$E_0 = -\mathrm{grad}(u_0) = -\frac{du_0}{dx} = -\frac{d}{dx}\left(\frac{\sinh(\gamma(l-x))}{\sinh(\gamma l)}\right) = \frac{\gamma\cosh(\gamma(l-x))}{\sinh(\gamma l)} \rightarrow E_{0,\max} = \gamma \cdot \mathrm{cotgh}(\gamma l) \qquad (27)$$

- Practically $\gamma l > 3$, therefore $\mathrm{cotgh}(\gamma l) \approx 1$ and $E_{0,\max} \approx \gamma$

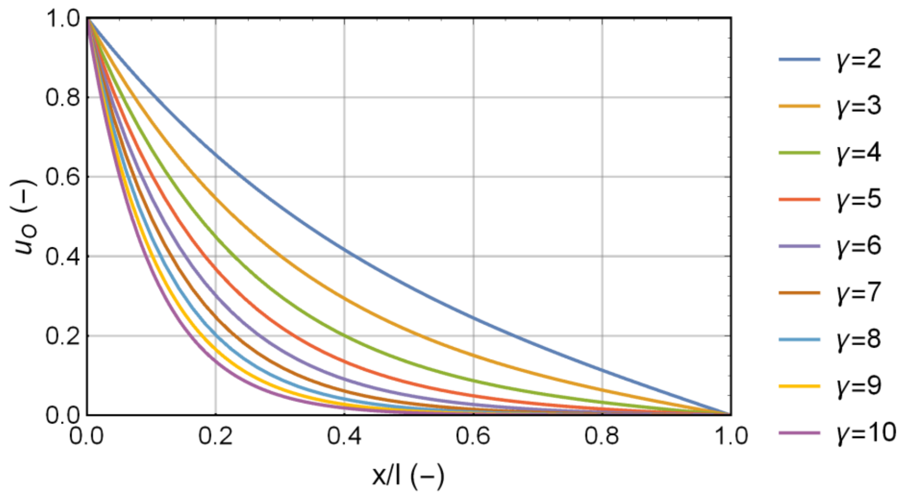
Voltage distribution – ungrounded winding

- Initial voltage distribution along an ungrounded coil for different values of γl
- The derivation of expression for u_0 for ungrounded coil is more complicated, as the second boundary condition is unknown ($u_0(0,l) \neq 0$)
- Therefore, let us state only the ultimate expression:

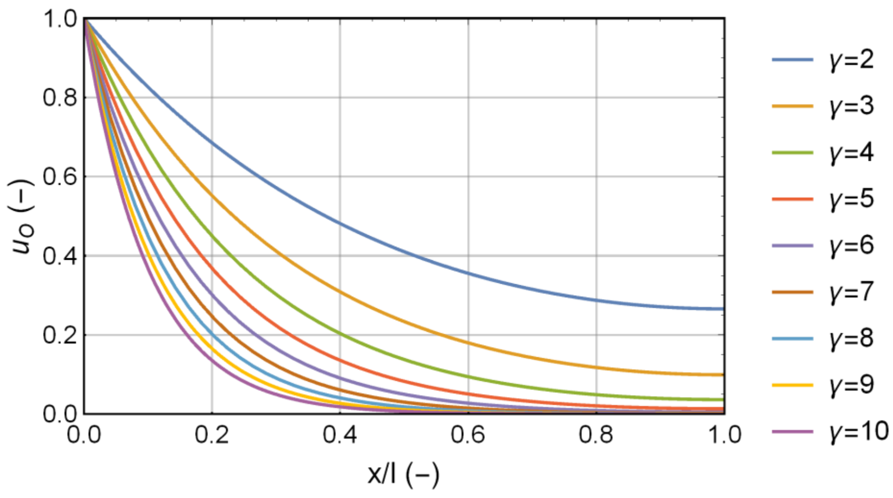
$$\boxed{u_0 = \frac{\cosh \gamma(l-x)}{\cosh \gamma l}} \qquad (28)$$

Voltage distribution plots

- Initial voltage distribution along a grounded and ungrounded coil for different values of γ



Grounded end of a coil



Ungrounded end of a coil

Voltage distribution – effects

- During the transient, undamped oscillations can reach values of up to 150/280 % of the initial voltage for grounded/ungrounded winding, respectively.
- To prevent such large values, several methods of equalizing the initial voltage distribution along the winding are employed:
  - Disc windings: ground capacity *C* can be compensated (capacitive screen) and/or the series capacity can be increased (turn interlacing)
  - Multi-layered windings: ground capacity *C* is present only for the first and the last layer. Moreover, the inter-layer capacity is much larger than the capacity between winding discs.