$$\bar{x}_g = \begin{bmatrix} P_g \\ Q_g \end{bmatrix}, \bar{x} = \begin{bmatrix} \bar{x}_g \\ \bullet \end{bmatrix} \quad \forall g$$

vektor proměnných:
0….. závislý zdroj
g=1:n_g ..nezávislé zdroje

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model zátěže: $S_l(U_l,\omega); l \dots čítací\ index\ zátěže$

nezávislá zátěž : $P_l = P_l^{(nom)} \quad ; \quad Q_l = Q_l^{(nom)}$

napěťově a frekvenčně závislá zátěž:

$$P_l = P_l(U,\omega) = P_l^{(nom)}.U_l^{\alpha_{lP}}.\omega^{\alpha_{l\omega}} \quad \rightarrow (\partial P_l/\partial U_l) \neq 0$$

$$Q_l = Q_l(U,\omega) = Q_l^{(nom)}.U_l^{\beta_{lP}}.\omega^{\beta_{l\omega}} \rightarrow (\partial Q_l/\partial U_l) \neq 0$$

generované	odebírané
$P_G = \sum_{\forall g} P_g;$	$P_L = \sum_{\forall l} P_l \Rightarrow \frac{\partial P_L}{\partial P_g} = \sum_{\forall l} \underbrace{\left(\frac{\partial P_l}{\partial U_l}\right)}_{zátěž} . \underbrace{\left(\frac{\partial U_l}{\partial P_g}\right)}_{síť}$
$Q_G = \sum_{\forall g} Q_g$	$Q_L = \sum_{\forall l} Q_l \Rightarrow \frac{\partial Q_L}{\partial Q_g} = \sum_{\forall l} \underbrace{\left(\frac{\partial Q_l}{\partial U_l}\right)}_{zátěž} . \underbrace{\left(\frac{\partial U_l}{\partial Q_g}\right)}_{síť}$

$$\mathcal{R}_p = P_0 + P_G - P_L - P_\Delta = 0 \dots\dots\ bilance\ činných\ výkonů$$

$$\mathcal{R}_q = Q_0 + Q_G - Q_L - Q_\Delta = 0 \dots\dots bilance\ jalových\ výkonů$$

Poměrné přírůstky nákladů a ztrát.

$$\eta_g^{pp} = (\partial P_\Delta / \partial P_g), \qquad \eta_g^{pq} = (\partial P_\Delta / \partial Q_g),$$

$$\eta_g^{qp} = (\partial Q_\Delta / \partial P_g), \qquad \eta_g^{qq} = (\partial Q_\Delta / \partial Q_g).$$

Derivace omezujících podmínek

$\frac{\partial \mathcal{R}_P}{\partial P_g} = \left(\frac{\partial P_0}{\partial P_g}\right) + \underbrace{\left(\frac{\partial P_g}{\partial P_g}\right)}_{1} - \left(\frac{\partial P_L}{\partial P_g}\right) - \eta_g^{pp};$	$\frac{\partial \mathcal{R}_P}{\partial Q_g} = \left(\frac{\partial P_0}{\partial Q_g}\right) + \underbrace{\left(\frac{\partial P_g}{\partial Q_g}\right)}_{0} - \left(\frac{\partial P_L}{\partial Q_g}\right) - \eta_g^{pq}$
$\frac{\partial \mathcal{R}_Q}{\partial P_g} = \left(\frac{\partial Q_0}{\partial P_g}\right) + \underbrace{\left(\frac{\partial Q_g}{\partial P_g}\right)}_{0} - \left(\frac{\partial Q_L}{\partial P_g}\right) - \eta_g^{qp}$	$\frac{\partial \mathcal{R}_Q}{\partial Q_g} = \left(\frac{\partial Q_0}{\partial Q_g}\right) + \underbrace{\left(\frac{\partial Q_g}{\partial Q_g}\right)}_{1} - \left(\frac{\partial Q_L}{\partial Q_g}\right) - \eta_g^{qq};$

$$C(\bar{P}) = C_0(P_0) + \sum_{g=1}^{n_g} C_g(P_g) \quad \text{náklady zdrojů}$$

$\partial_p, \partial_q \dots stupeň\ polynomu\ nákladů$

$$C_g(P_g) = \sum_{j=0}^{\partial p} C_{jg}.P_g^j, \qquad G_g(Q_g) = \sum_{j=0}^{\partial q} G_{jg}.Q_g^j$$

poměrné přírůstky nákladů:

$$b_g(P_g) = \frac{\partial C_g(P_g)}{\partial P_g} = \sum_{j=0}^{\partial p-1} b_{jg} P_g^j \quad na\ př. \quad b_{0g} + b_{1g} P_g + b_{2g} P_g^2,$$

$$h_g(P_g) = \frac{\partial G_g(Q_g)}{\partial Q_g} = \sum_{j=0}^{\partial q-1} h_{jg} Q_g^j \quad na\ př. \quad h_{0g} + h_{1g}.Q_g + h_{2g}.Q_g^2$$

Ekonomický dispečink výkonů

Výkony bilančního uzlu a jejich derivace

$$P_0 = P_\Delta + P_L - P_G, \qquad Q_0 = Q_\Delta + Q_L - Q_G$$

$$\frac{\partial P_0}{\partial P_g} = \eta_g^{pp} + \left(\frac{\partial P_L}{\partial P_g}\right) - \underbrace{\left(\frac{\partial P_g}{\partial P_g}\right)}_{1}, \qquad \frac{\partial P_0}{\partial Q_g} = \eta_g^{pq} + \left(\frac{\partial P_L}{\partial Q_g}\right) - \overbrace{\left(\frac{\partial P_g}{\partial Q_g}\right)}^{0}$$

$$\frac{\partial Q_0}{\partial Q_g} = \eta_g^{pp} + \left(\frac{\partial Q_L}{\partial Q_g}\right) - \underbrace{\left(\frac{\partial Q_g}{\partial Q_g}\right)}_{1}, \qquad \frac{\partial Q_0}{\partial P_g} = \eta_g^{qp} + \left(\frac{\partial Q_L}{\partial P_g}\right) - \overbrace{\left(\frac{\partial Q_g}{\partial P_g}\right)}^{0}$$

Lagrangeova funkce L(•):

$$L(\bar{x},\bar{\lambda}) = \underbrace{C_0(x_0) + \sum_{g=1}^{n_g} C_g(x_g)}_{\text{cílová funkce}} + \underbrace{\lambda_P.\mathcal{R}_P + \lambda_Q.\mathcal{R}_Q}_{\text{obsluha omezujících podmínek}}$$

Derivace L(•) podle výkonů bilančního uzlu:

$$\frac{\partial L}{\partial P_0} = b_0 + \lambda_P = 0; \quad \frac{\partial L}{\partial Q_0} = h_0 + \lambda_Q = 0$$

Ekonomický dispečink výkonů

Derivace L podle výkonů zdrojů

$$\frac{\partial L}{\partial P_g} = \underbrace{\left(\frac{\partial C_0(x_0)}{\partial P_0}\right)}_{b_0=-\lambda_P} . \left(\frac{\partial P_0}{\partial P_g}\right) + b_g + \lambda_P.\left(\partial\mathcal{R}_P/\partial P_g\right) + \lambda_Q\left(\partial\mathcal{R}_Q/\partial P_g\right)$$

$$\frac{\partial L}{\partial P_g} = b_g + \lambda_P.\left\{-\left(\partial P_0/\partial P_g\right) + \left(\partial\mathcal{R}_P/\partial P_g\right)\right\} + \lambda_Q\left(\partial\mathcal{R}_Q/\partial P_g\right)$$

$$\frac{\partial L}{\partial P_g} = b_g + \lambda_P.\left\{1 - \left(\frac{\partial P_L}{\partial P_g}\right) - \eta_g^{pp}\right\} + \lambda_Q.\left\{\left(\frac{\partial Q_0}{\partial P_g}\right) - \left(\frac{\partial Q_L}{\partial P_g}\right) - \eta_g^{qp}\right\}$$

$$\frac{\partial L}{\partial Q_i} = \underbrace{\left(\frac{\partial C_0(x_0)}{\partial Q_0}\right)}_{h_0=-\lambda_Q} . \left(\frac{\partial Q_0}{\partial Q_i}\right) + h_i + \lambda_P.\left(\frac{\partial\mathcal{R}_P}{\partial Q_i}\right) + \lambda_Q\left(\frac{\partial\mathcal{R}_Q}{\partial Q_i}\right)$$

$$\frac{\partial L}{\partial Q_i} = h_i + \lambda_P.\left\{\left(\frac{\partial P_0}{\partial Q_i}\right) - \left(\frac{\partial P_L}{\partial Q_i}\right) - \eta_i^{pq}\right\} + \lambda_Q\left\{1 - \left(\frac{\partial Q_L}{\partial Q_i}\right) - \eta_i^{qq}\right\}$$

$$\begin{bmatrix} 1 - \left(\frac{\partial P_L}{\partial P_i}\right) - \eta_i^{pp} & \left(\frac{\partial Q_0}{\partial P_i}\right) - \left(\frac{\partial Q_L}{\partial P_i}\right) - \eta_i^{qp} \\ \left(\frac{\partial P_0}{\partial Q_i}\right) - \left(\frac{\partial P_L}{\partial Q_i}\right) - \eta_i^{pq} & 1 - \left(\frac{\partial Q_L}{\partial Q_i}\right) - \eta_i^{qq} \end{bmatrix} \begin{bmatrix} \lambda_P \\ \lambda_Q \end{bmatrix} = -\begin{bmatrix} b_i \\ h_i \end{bmatrix}$$

maticová rovnice pro λ_P,λ_Q

Case 1: bezeztrátová soustava

$$P_\Delta = Q_\Delta = \eta_i^{pp} = \eta_i^{pq} = \eta_i^{qp} = \eta_i^{qq} = 0,$$

nezávislá zátěž:

$$\left(\frac{\partial P_L}{\partial P_g}\right) = \left(\frac{\partial Q_L}{\partial P_g}\right) = \left(\frac{\partial P_L}{\partial Q_g}\right) = \left(\frac{\partial Q_L}{\partial Q_g}\right) = 0$$

$$P_0 = P_L - \sum_{\forall g} P_g; \qquad Q_0 = Q_L - \sum_{\forall g} Q_g$$

$$L = C_0\{x_0\} + \sum_{\forall g} C_g\left(x_g\right) +$$

$$+\lambda_P.\left(P_0 + P_G - P_L\right) + \lambda_Q\left(Q_0 + Q_G - Q_L\right)$$

$$\frac{\partial L}{\partial P_g} = \underbrace{\left(\frac{\partial C_0}{\partial P_0}\right)}_{b_0}\underbrace{\left(\frac{\partial P_0}{\partial P_g}\right)}_{-1} + b_g + \lambda_P.\left\{\overbrace{\underbrace{\left(\partial P_0/\partial P_g\right)}_{-1} + 1 - 0}^{0}\right\} + \lambda_Q\left(0\right) = 0$$

$$\frac{\partial L}{\partial Q_g} = \underbrace{\left(\frac{\partial C_0}{\partial Q_0}\right)}_{h_0}\underbrace{\left(\frac{\partial Q_0}{\partial Q_g}\right)}_{-1} + h_i + \lambda_P.\{0\} + \lambda_Q\left(0\right) = 0$$

$$\textit{výsledek}: \qquad b_0 = b_g = b, \quad h_0 = h_g = h \qquad \forall g$$

algoritmus pro ∂_p=2 :

$$b = b_g = b_{0g} + b_{1g}P_g \Rightarrow P_g = \frac{b - b_{0g}}{b_{1g}}$$

$$\sum_{g=0}^{n_g} P_g = P_L = b.\sum_{\forall g}\left(\frac{1}{b_{1g}}\right) - \sum_{\forall g}\left(\frac{b_{0g}}{b_{1g}}\right)$$

$$1.\ b = \frac{P_L + \sum_{\forall i}\left(\frac{b_{0g}}{b_{1g}}\right)}{\sum_{\forall i}\left(\frac{1}{b_{1g}}\right)} \textit{.....výpočet společ. } b$$

$$2.\ P_g = \frac{b - b_{0g}}{b_{1g}}, \quad \forall g \textit{.....rozdělění výkonů}$$

Case 2: uvažování ztrát, nezávislá zátěž.

$$P_\Delta ; Q_\Delta ; \eta_g^{pp} ; \eta_g^{pq} ; \eta_g^{qp} ; \eta_g^{qq} \neq 0,$$

$$\begin{bmatrix} 1-\eta_g^{pp} & -\eta_g^{qp} \\ -\eta_g^{pq} & 1-\eta_g^{qq} \end{bmatrix} \begin{bmatrix} \lambda_P \\ \lambda_Q \end{bmatrix} = -\begin{bmatrix} b_g \\ h_g \end{bmatrix} ; b_g \gg h_g$$

$$\det = \left(1-\eta_g^{pp}\right)\left(1-\eta_g^{qq}\right) - \eta_g^{qp}\eta_g^{pq}$$

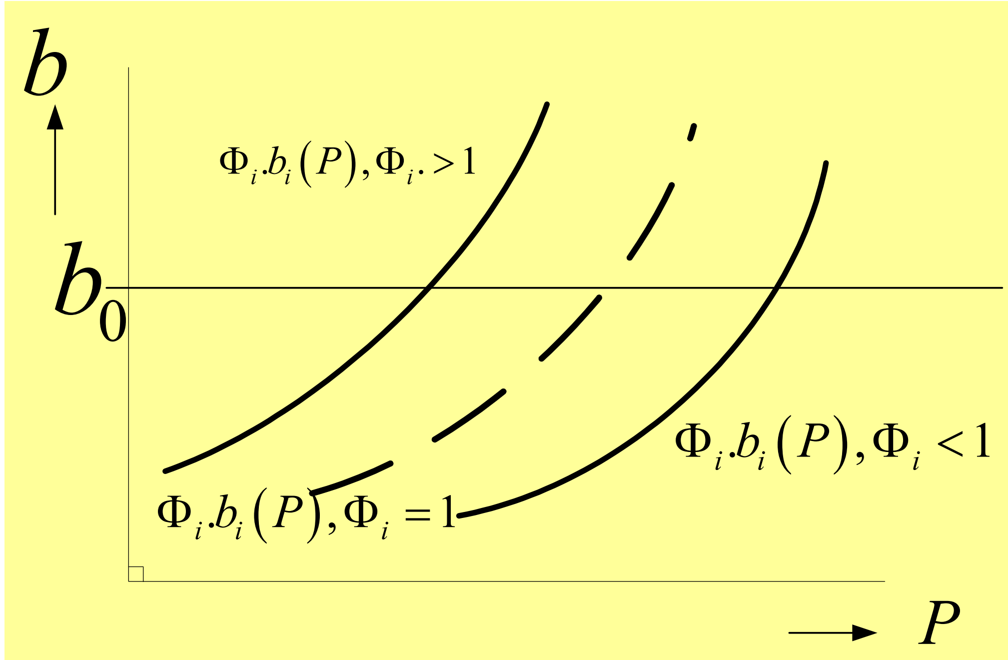
$$\eta_g^{pp}\eta_g^{qq} - \eta_g^{qp}\eta_g^{pq} = 0 \Rightarrow \det = 1-\eta_g^{pp} - \eta_g^{qq}$$

$$-\lambda_q = \frac{h_g\left(1-\eta_g^{pp}\right) + \eta_g^{pq}.b_g}{1-\eta_g^{pp}-\eta_g^{qq}} \doteq \frac{\eta_g^{pq}.b_g}{1-\eta_g^{pp}-\eta_g^{qq}}$$

$$-\lambda_p = \frac{b_g\left(1-\eta_g^{qq}\right) + \eta_g^{qp}.h_g}{1-\eta_g^{pp}-\eta_g^{qq}} = \frac{b_g\left(1-\eta_g^{qq} + \overbrace{\eta_g^{qp}.h_g / b_g}^{mal\acute{e}}\right)}{1-\eta_g^{pp}-\eta_g^{qq}}$$

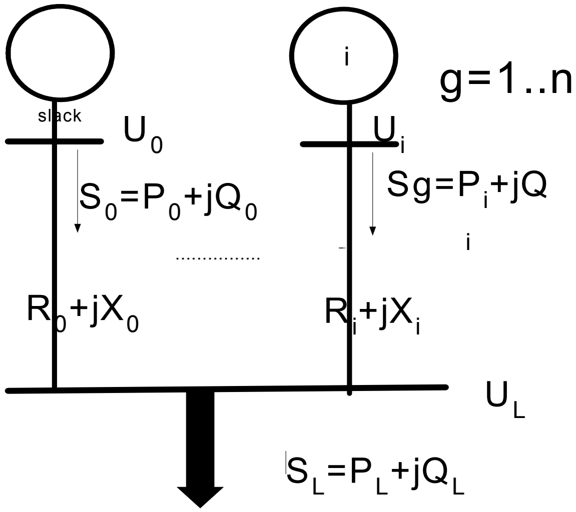
$$-\lambda_p = \frac{b_g\left(1-\eta_g^{qq}\right)}{1-\eta_g^{qq}-\eta_g^{pp}} = \underbrace{\left\{\frac{1}{1-\eta_g^{pp}/\left(1-\eta_g^{qq}\right)}\right\}}_{penal.\,faktor\ \Phi_g} b_g = b_0$$

grafické řešení rozdělování výkonů



1. Rozdělení P bez uvažování ztrát
2. Výp. chodu , Φ a rozdělení P
3. je-li bilance true pak konec jinak 2

Optimalizace v hvězdicové ekvivalentní síti



$$\mathcal{R}_P: \quad \sum_{g=0}^{n_g}(P_g - P_{\Delta i}) - P_L = 0, \quad \mathcal{R}_Q: \quad \sum_{g=0}^{n_g}(Q_g - Q_{\Delta i}) - Q_L = 0$$

$$S_{\Delta g} = (R_g + jX_g)\frac{S_g S_g^*}{\hat{U}_g \hat{U}_g^*} \Rightarrow P_{\Delta g} = R_g \frac{S_g^2}{U_g^2} = R_g \frac{P_g^2 + Q_g^2}{U_g^2}$$

$$\frac{\partial P_{\Delta g}}{\partial P_g} = \frac{2 P_g R_g}{U_g^2} \qquad b_g = \lambda.(1 - \frac{\partial P_\Delta}{\partial P_g})$$

$$b_{2g} P_g^2 + (b_{1g} + \frac{2.\lambda.R_g}{U_g^2}).P_g + b_{0g} - \lambda = 0 \rightarrow P_g = f(\lambda)$$

Algoritmus projekce λ

1. **k=0, počáteční nastavení** λ_k
2. **k=k+1, výpočet** $P_i \ \forall i$
3. **if kontrola bilance: konec , jinak :**
4. **if k=1 zvol jiné** λ **a jdi na 2**
   if k >1 projekce λ**, návrat do 2**