

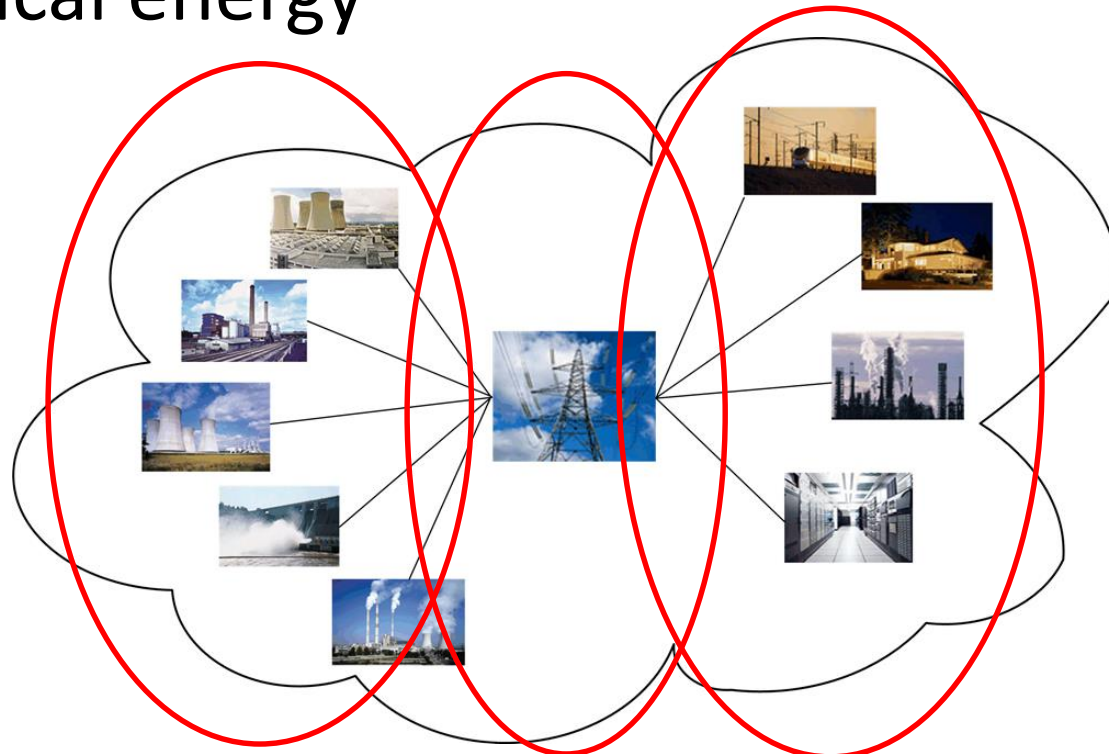
## Fundamental terms and definitions

# Power engineering

- A scientific discipline that focuses on:
  - Generation of electrical energy (EE)
  - Transmission and distribution of EE
  - Consumption of EE
  - Power grid operation and dispatch
  - Safety and development of the power grid

# Electric power system

- A system that provides generation, transmission, distribution and consumption of electrical energy



# Fundamental tasks of electric power system

- Provision of sufficient amount of electrical energy (EE) in required time
- Assurance of EE quality
- Reliability of EE delivery
- Economy optimization of EE delivery

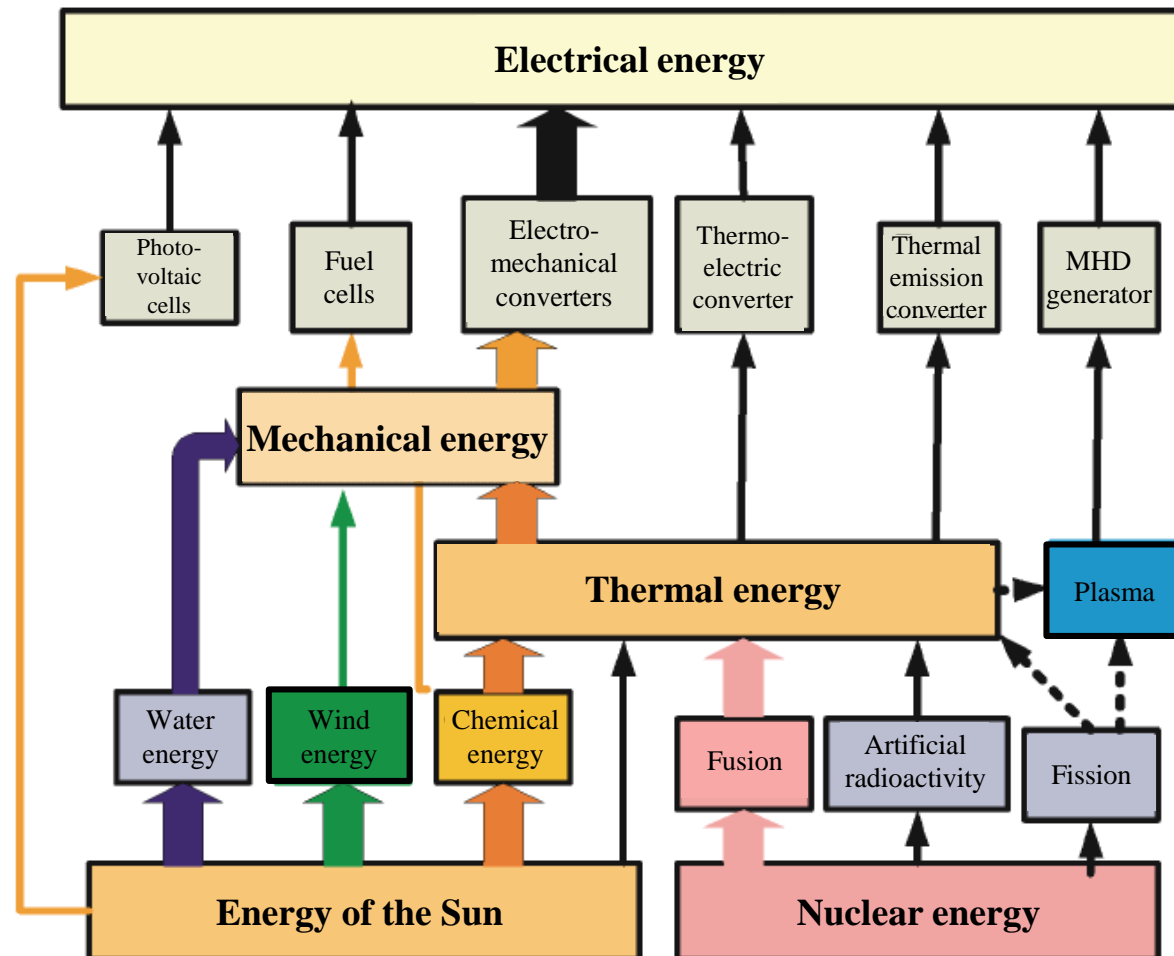
# Electrical energy

- Advantages
  - Relatively simple conversion to other kinds of energy
  - Possibility of long distance transportation
  - Possibility of generation in large units
- Disadvantages
  - Difficult to store

# Methods of EE acquisition

- Energy conversion from primary sources to electricity
  - Energy of the Sun
  - Energy of the Earth
  - Energy of the Earth-Moon interaction

# Intermediate steps of different types of energy conversions



# Types of electrical energy

- Based on means of production
  - Primary sources (mining, extraction)
  - Produced sources (refinement)
  - Secondary sources (from conversion losses)
- Based on renewability
  - Non-renewable sources
    - Available in finite quantity
  - Renewable sources
    - With possibility of partial or complete renewal either naturally or by human activity

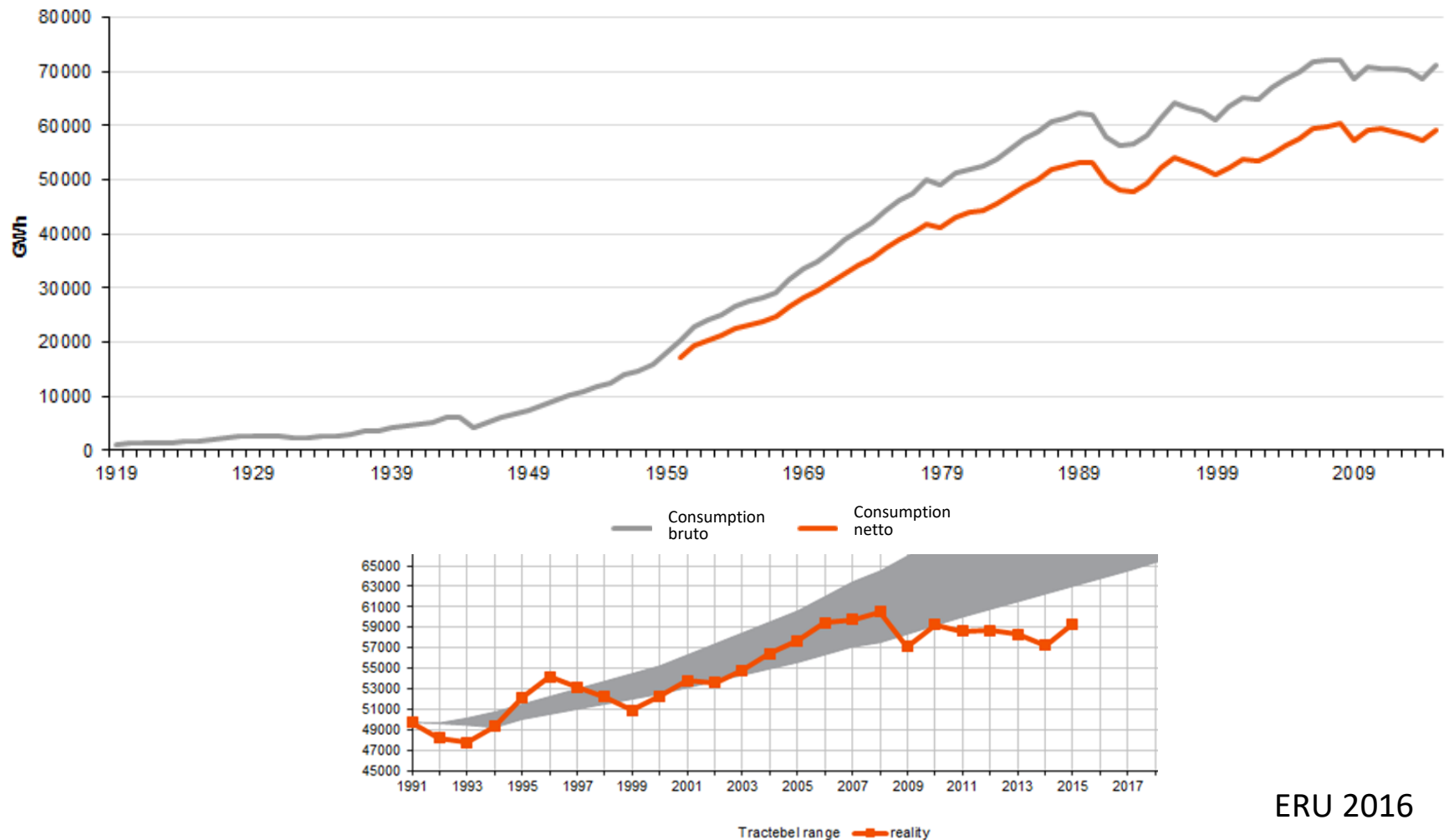


# Types of electrical energy

- Non-renewable
  - Fossil fuels
  - Nuclear fuels
- Renewable
  - Water energy
  - Wind energy
  - Solar irradiance
  - Biogas, biomass
  - Geothermal energy
  - Sea waves energy, tide

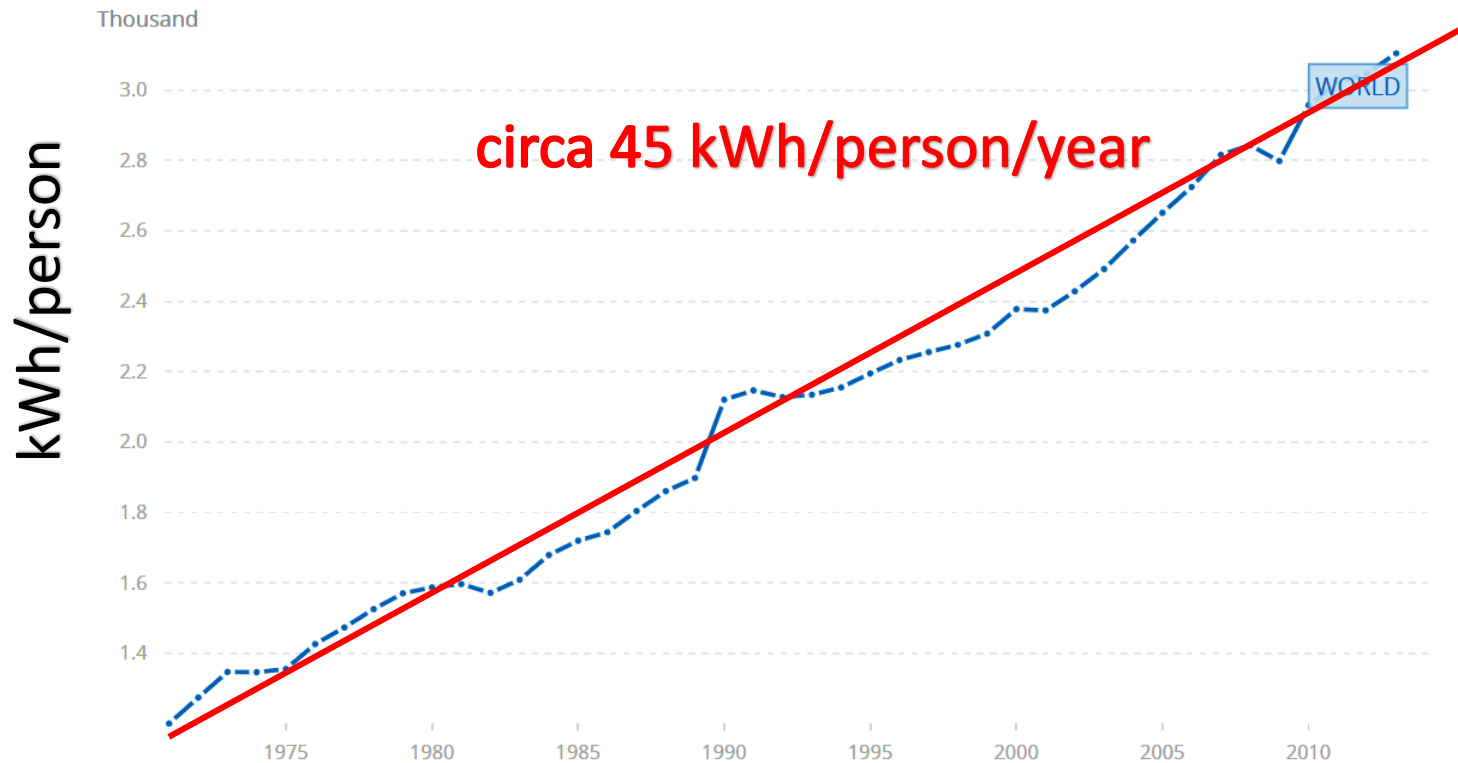


# Long-term development of power consumption in CZ



ERU 2016

# Long-term development of power consumption in the world



World bank 2016

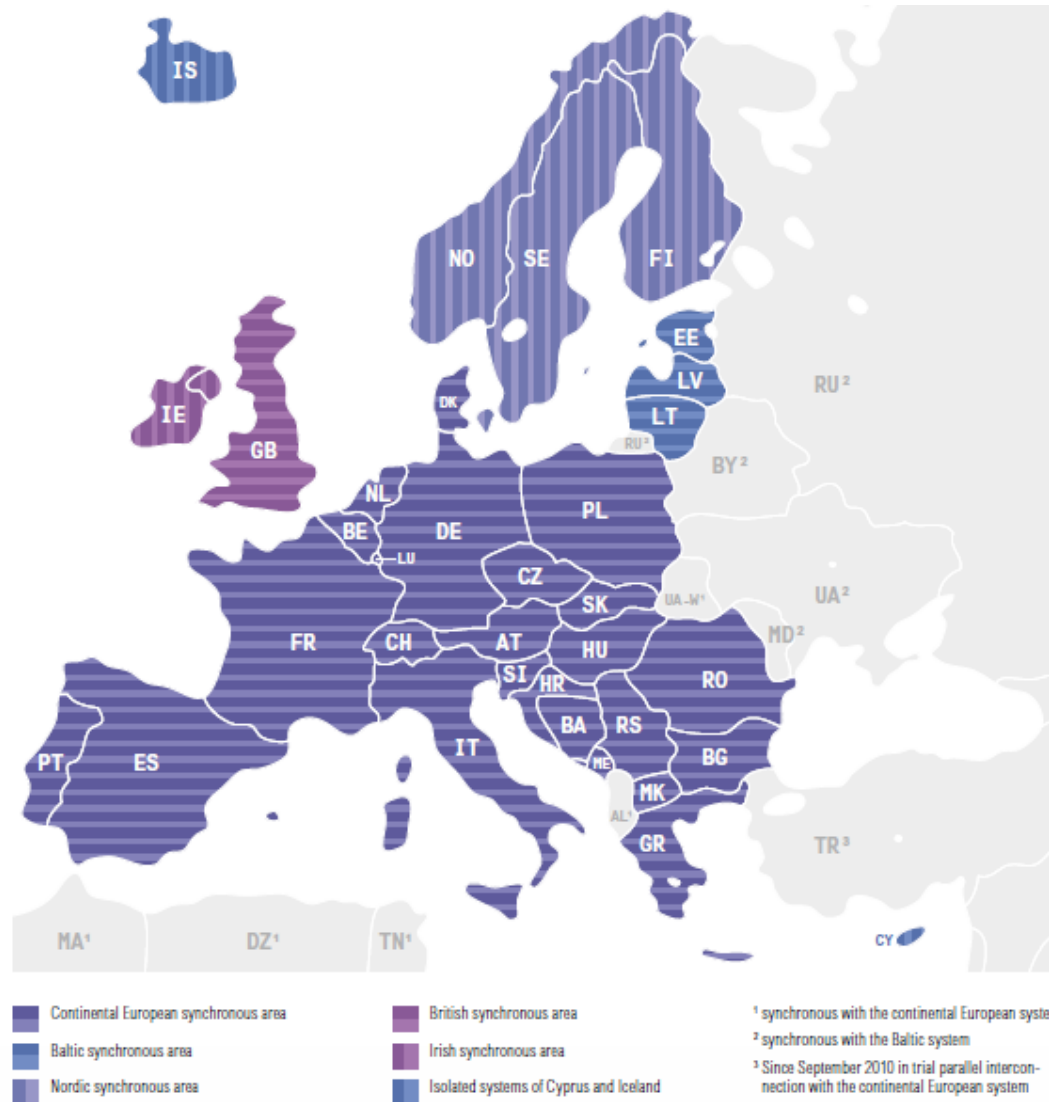
# Electric power capacity terminology

- **Nominal capacity  $P_N$**  – the highest permanent electric power of a machine; determined by its design
- **Installed power capacity  $P_i$**  – the total of individual nominal capacities of all similar machines inside an object
- **Maximum achievable power  $P_d$**  - the highest electric power of a machine achievable in its normal state and normal operating conditions
- **Instantaneous power  $P_p$**  - the highest electric power of a machine achievable in its actual state and actual operating conditions
- **Technical power minimum  $P_{TM}$**  – the lowest permanent electric power at which can a machine operate without the risk of damage

# Transmission network

- Interconnected set of power lines and devices that transmits electrical energy from generating units to distribution networks
- **In CZ:** power lines, transformers and substations operating at voltage of 400 kV and 220 kV, with 2 additional substations and 105 km power lines at 110 kV.
- The transmission network operator in CZ is ČEPS, a. s.
- The Czech transmission system is integrated into European transmission system (ENTSO-E) via cross-border power lines

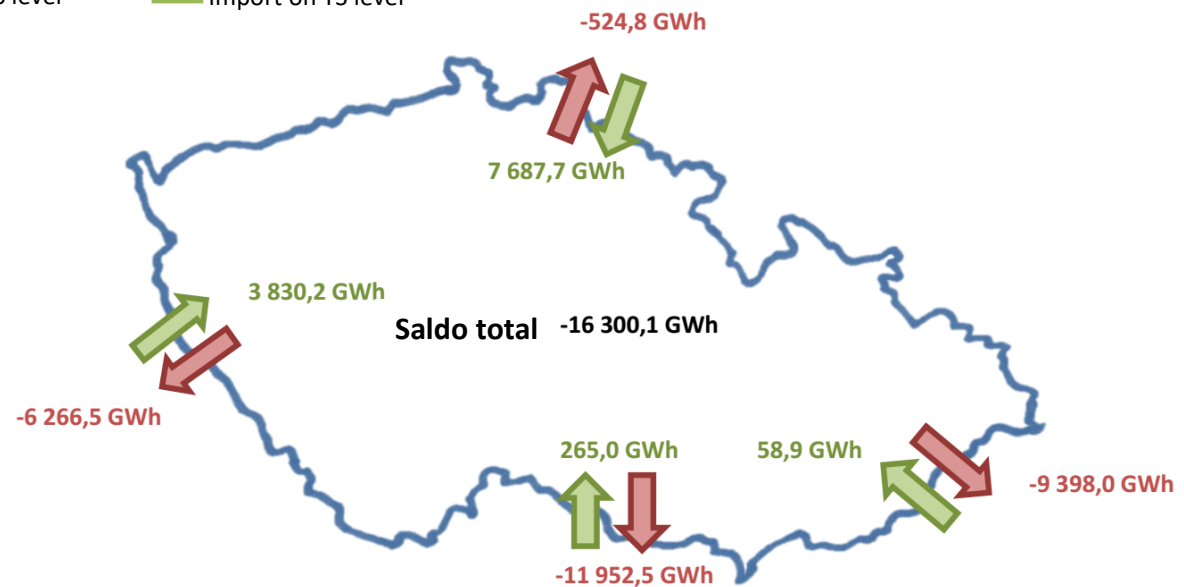
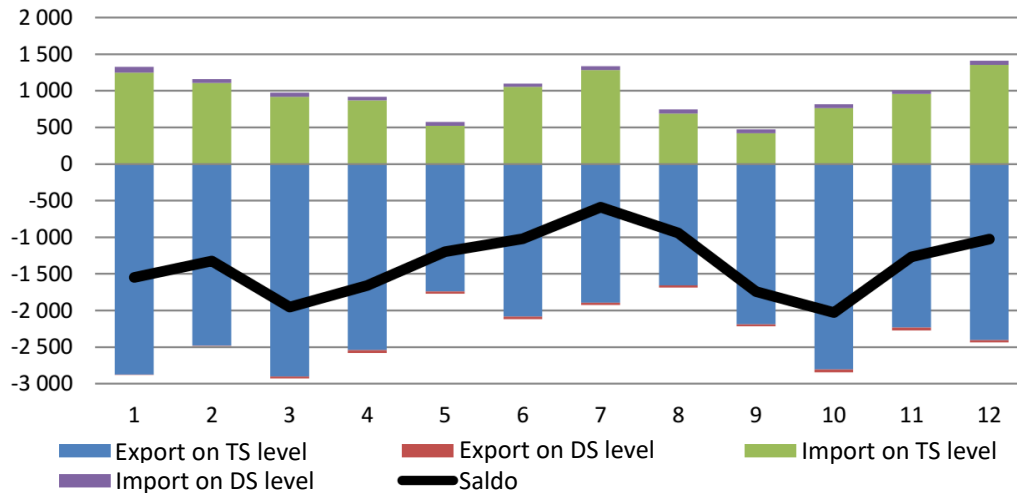
# European transmission networks



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# Cross-border power flows

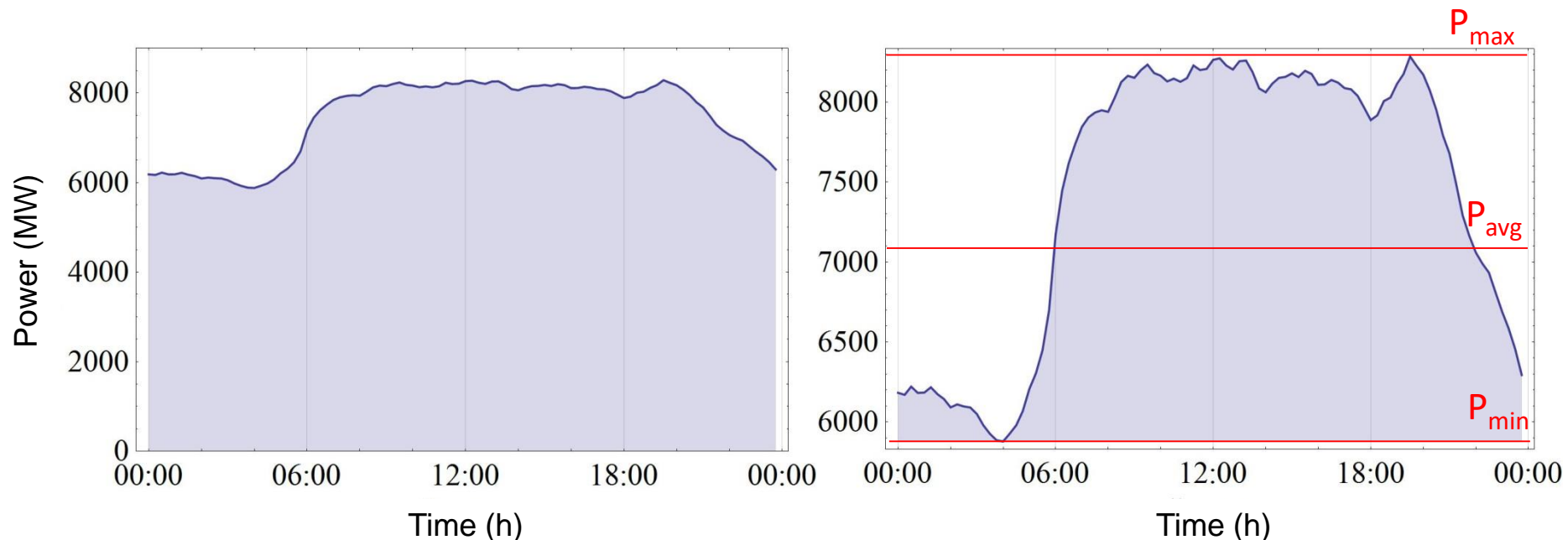
Cross-border power flows (GWh) – example for the Czech power grid





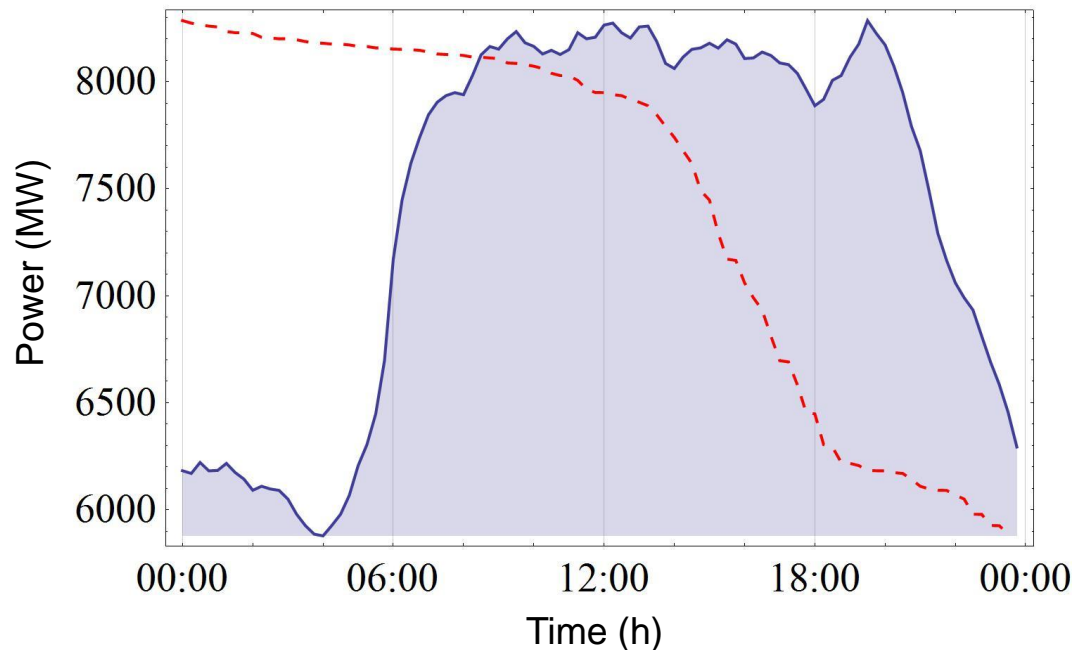
# Daily load profile

- Dependency of electrical load  $P$  (MW) on time  $t$  (h). Typically processed for whole network or for single units (factories, homes)



# Load duration curve

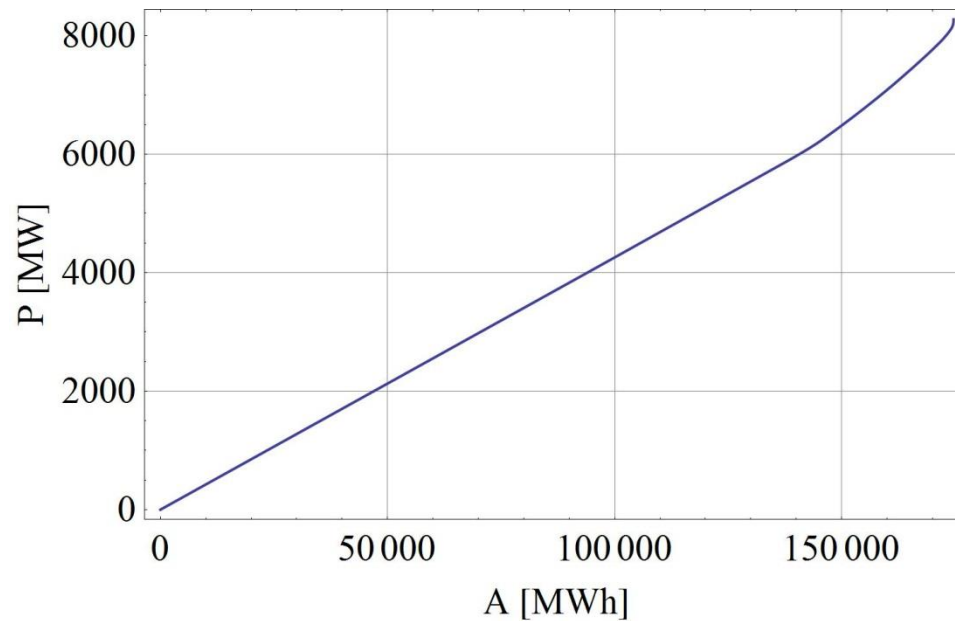
- $P = f(t)$  characteristic
- Constructed by sorting electrical load values from load profile in a descending order



# Production curve

- Cumulative integral of load duration curve with respect to electrical load  $P$

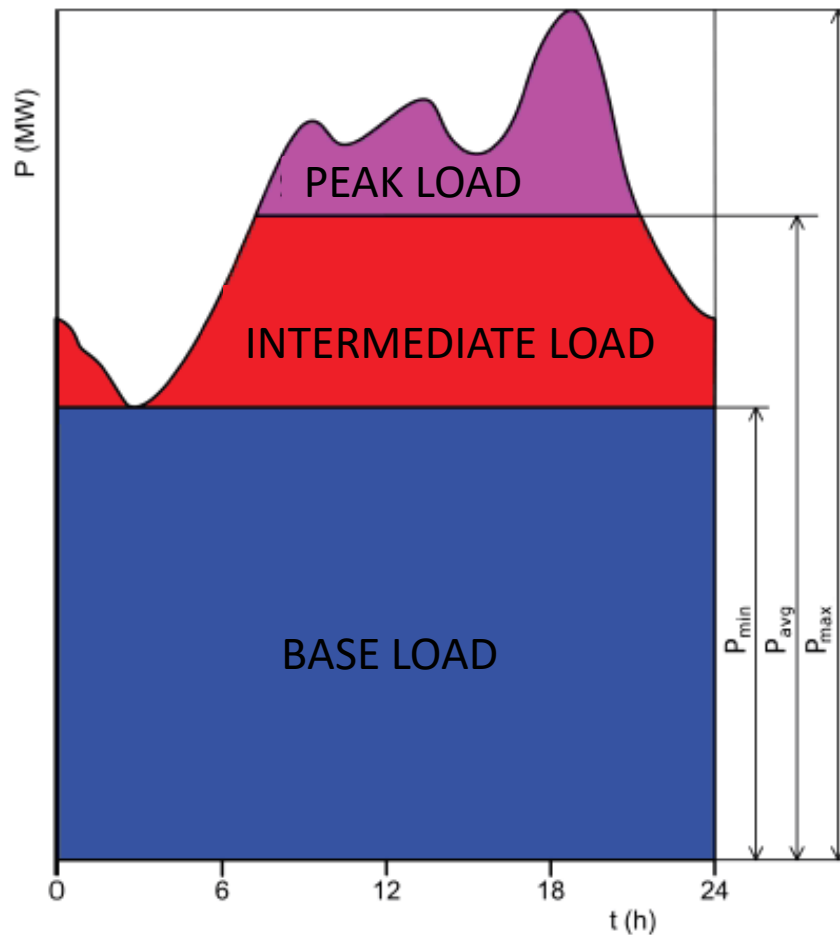
$$A_c = \int_0^{P_{max}} t(P) dP$$



# Types of electrical load inside a load profile

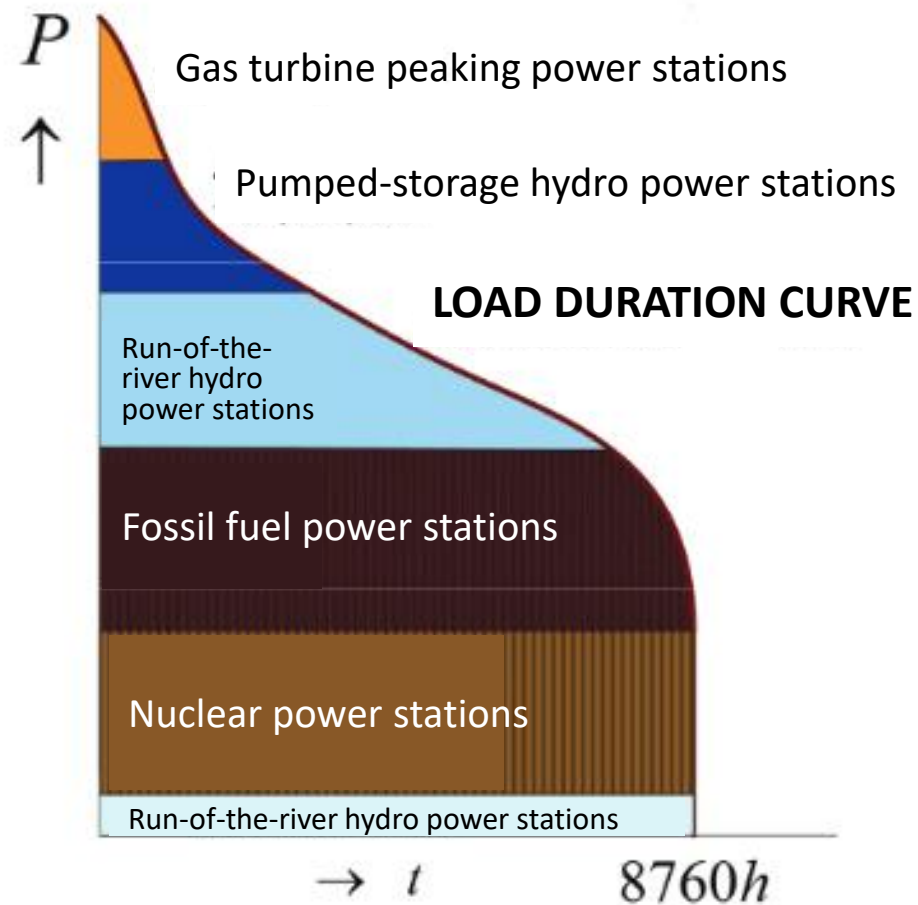
- **Base load** – part of the load profile below the minimum load
- **Intermediate load** – part of the load profile between base load and average load
- **Peak load** – part of the load profile above average load
- **Average load**  $P_{\text{avg}}$  – permanent load at which a machine would produce the same amount of work as in the load diagram

# Peak and load matching (load profile)



- Pumped-storage hydro power stations
- Conventional hydro power stations
- Conventional hydro power stations
- Gas turbine and combined cycle gas turbine power stations
- Coal and nuclear power stations
- Run-of-the-river hydro power stations

# Peak and load matching (load duration curve)



# Optimization of power station dispatch

- Optimal peak and load matching while maintaining lowest possible operating costs
- The operating costs of a power station have 2 basic components:
  - Fixed costs (**independent** on the power output)
  - Variable costs (**dependent** on the power output; typically fuel costs)
- The goal is to find the optimal power outputs of all dispatchable power stations

# Optimization of power station dispatch

- Therefore, it is necessary to find the minimum of cost function  $N$  (the sum of the individual operating costs) with constraint  $B$  (power balance):

$$N = \sum_{i=1}^M N_i \text{ \& } B = \sum_{i=1}^M P_i = P$$

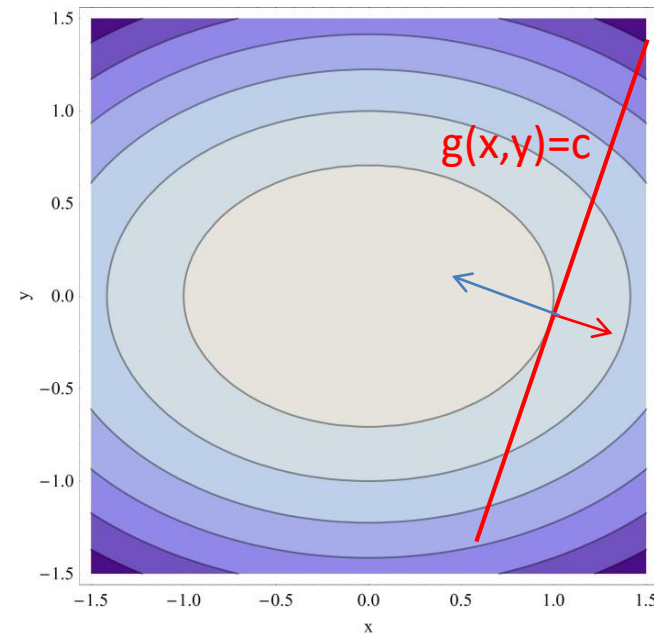
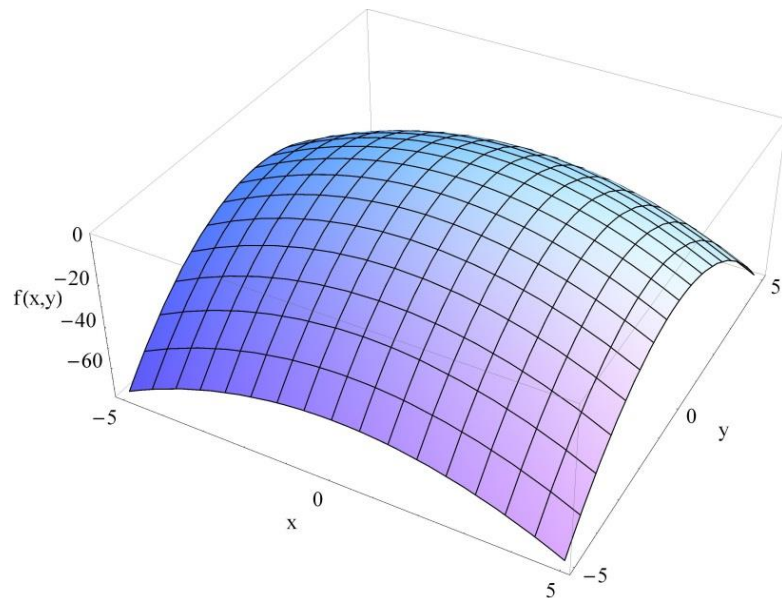
where  $N_i$  are cost functions of individual power stations,  $P_i$  are power outputs of individual power stations and  $P$  is the electrical load of the electrical network



# Constrained extreme values of multivariable functions

- Constrained extreme values
  - An optimization problem
    - We search for the extreme value of function  $f(x,y)$
    - The extreme value is constrained by function  $g(x,y)=c$
  - If we assume that  $f$  and  $g$  have continuous first order partial derivatives, then a Lagrange function
$$L(x, y, \lambda) = f(x, y) + \lambda(g(x, y) - c)$$
can be used to locate the extreme value by solving
$$\nabla L(x, y, \lambda) = 0$$

# Constrained extreme values of multivariable functions – geometrical point of view



Gradient of function  $f$  must be parallel to the gradient of function  $g$  if the  $f$  extreme is located inside the space constrained by  $g$ . This is defined mathematically as:

$$\begin{aligned}\nabla f &= -\lambda \nabla g \\ \nabla f + \lambda \nabla g &= 0\end{aligned}$$

# Optimization of power station dispatch

- Lagrange function for optimal distribution of power generation is:

$$L(P_1, P_2, \dots, P_n, \lambda) = N + \lambda B = \sum_{i=1}^M N_i + \lambda \left( \sum_{i=1}^M P_i - P \right)$$

- The minimum must then meet the following conditions:

$$\begin{aligned} \frac{\partial L}{\partial P_1} &= \frac{\partial N_1}{\partial P_1} + \lambda \\ \frac{\partial L}{\partial P_2} &= \frac{\partial N_2}{\partial P_2} + \lambda \\ &\vdots \\ \frac{\partial L}{\partial P_M} &= \frac{\partial N_M}{\partial P_M} + \lambda \\ \Rightarrow -\lambda &= \frac{\partial N_1}{\partial P_1} = \frac{\partial N_2}{\partial P_2} = \dots = \frac{\partial N_M}{\partial P_M} \end{aligned}$$

# Example

- A power station has two blocks with installed capacity of  $P_{n1}=P_{n2}=100\pm 20$  MW. Operating costs of the first block are  $N_1 = 50000 + 250P_1 + 40P_1^2$  (Kč/h) and of the second block  $N_2 = 45000 + 150P_2 + 50P_2^2$  (Kč/h). Find the optimal way to secure output of  $P_c=210$  MW
  - Lagrange function

$$L = N_1(P_1) + N_2(P_2) + \lambda(P_1 + P_2 - P_c)$$

$$\frac{\partial L}{\partial P_1} = 250 + 80P_1 + \lambda$$

$$\frac{\partial L}{\partial P_2} = 150 + 100P_2 + \lambda$$

$$P_c = P_1 + P_2$$

# Example

$$\Rightarrow P_1 = 116 \text{ MW}, P_2 = 94 \text{ MW}, \lambda = -9539 \text{ Kč/MWh}$$

- The optimum is located inside the regulation ranges of both blocks.

# Load factor

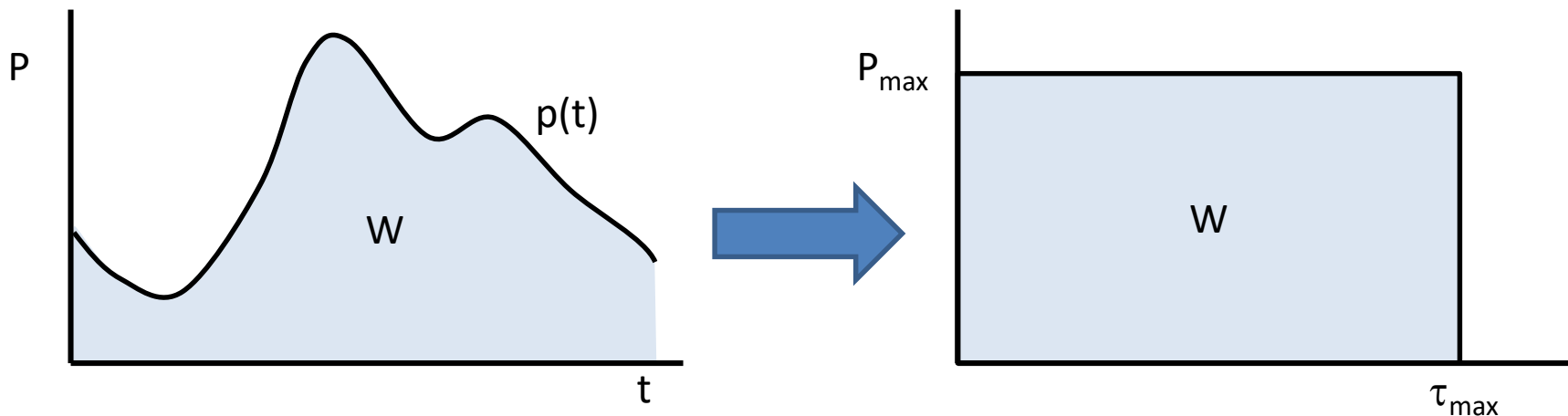
$$\xi = \frac{P_{avg}}{P_{max}} = \frac{\tau_{max}}{T}$$

- The ratio of the average electric power and the maximum electric power.
- Ideally  $\xi=1$ , the closer the load factor is to the ideal value, the more economical is the production unit

# Exploitation time

- The ratio of the actual generation of a power station and its theoretical maximum over the same period.

$$\tau_{max} = \frac{W}{P_{max}} = \frac{1}{P_{max}} \int_0^T p(t) dt$$



# Time of full losses

- The total time during which would a machine working at maximum load  $P_{\max}$  produce the same losses of electrical energy as during the actual period  $T$  at load  $p(t)$

Loss energy  $A_z$

$$A_z = R \int_0^T I^2(t) dt = RI_{ef}^2 T = RI_{ef}^2 T \left( \frac{\xi I_{\max}}{I_{avg}} \right)^2 = \overset{P_{zm}}{RI_{\max}^2} T \left( \frac{\xi I_{ef}}{I_{avg}} \right)^2$$

Time of full losses:

$$\tau_z = T \left( \frac{\xi I_{ef}}{I_{avg}} \right)^2 = T \left( \frac{I_{ef}}{I_{\max}} \right)^2 = \frac{\int_0^T i^2(t) dt}{I_{\max}^2} \quad \xi = \frac{I_{avg}}{I_{\max}}$$

If voltage  $U$  and power factor  $\cos \varphi$  are constant:

$$\tau_z = \frac{1}{P_{\max}^2} \int_0^T p(t)^2 dt$$



# Utilization and diversity factor

- Since the working machines do not necessarily need to run at full load, the following factors have been introduced to incorporate that fact into the power station design
- **Utilization factor** – the ratio of the actual power input of the working machines and the total of their power ratings

$$k_z = \frac{\sum P_s}{\sum P_n} \leq 1$$

- **Diversity factor** – the ratio of power ratings of working machines and the total of power ratings of all machines

$$k_s = \frac{\sum P_{na}}{\sum P_n} \leq 1$$

# Demand factor

- The ratio of maximum power output of a machine and its power rating

$$\beta = \frac{P_{max}}{P_i}$$

- Factor  $\beta$  can also be defined as:

$$\beta = \frac{k_s k_z}{\eta_r \eta_z}$$

where  $\eta_r$  is the power feed efficiency and  $\eta_z$  is the machine efficiency

# Design load and current

- Design load is used for generator, power station, power lines and feeding transformer design

$$P_p = P_i \beta$$

- Design current can be calculated by

$$I_p = \frac{P_p}{\sqrt{3} U \cos \varphi},$$

where  $U$  is line voltage and  $\cos \varphi$  is the power factor